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VEC

THE PRINCIPLES

OF

STRUCTURAL DESIGN.

PART I.

BY

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Assoc. Inst. C.E., M. San. Inst.

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PREFACE TO PART I.

THIS treatise has been compiled for the use of the officers of the Royal Engineers studying at the School of Military Engineering. The great variety of designs which may ordinarily fall to the lot of such officers renders it imperative that any treatise professing to assist or guide them in any way, should be directed not to one branch of structural engineering only, but should, as far as possible, include all those that are likely to come within an officer's experience at home or abroad. The present volume, therefore, includes a good deal of miscellaneous matter. It has been prepared so as to correspond approximately with Part I. of the course of instruction, which aims at placing an officer in a position to design buildings of an ordinary nature, such as barracks, stables, store-houses, etc. Certain subjects, however, such as continuous beams, built-up iron or steel girders, and the theoretical consideration of retaining walls and arches, have been deferred to Part II., as they would have made the present volume of inconvenient size.

In Part II., besides the subjects above alluded to, the design of bridges of all sorts, of reservoir walls, and of sea defences, will be considered.

Most treatises on the subject in England are compiled for engineers practising at home. Engineer officers of the English army, however, have to do a large part of their work abroad, and have to fall back upon first principles in their designs. Every endeavour has been made in this volume to emphasize this fact by giving tables of the materials in use abroad, and by examples based on foreign experience. The tables, however, are most imperfect, and it is hoped that further information on the subject may, in the future, be forthcoming.

With regard to theoretical proofs, it will, no doubt, be universally conceded that without the assistance of mathematics very little advance could be made in any scientific design. On the one hand "a complex algebraical investigation is often not merely useful, but indispensable; but in expounding such a proposition as a part of practical science, and applying it to practical purposes, simplicity is of the first importance, and, in fact, the more thoroughly a man

has studied the higher mathematics, the more fully does he become aware of this truth."* On the other hand, to gain simplicity by the simple method of omitting all the steps which have led to the investigation is to deprive those students of science, who wish to examine for themselves the data on which the practice is founded, of the assistance which they naturally look for in forming a sound and logical conclusion. In this work, therefore, mathematical proofs have been given, where it has been possible to do so without unduly interfering with the context, in appendices to the chapter dealing with the practical application of the subject. It is hoped that, thus relegated to a subordinate place, and yet not wholly omitted, mathematical science is given its due place in relation to engineering. Most of these proofs have been taken from books on the subject; some, however, have been worked independently by Captain C. F. Close, R.E.

The following works have been consulted in the preparation of the text:—

General Wray's *Applications of Theory to the Practice of Construction* (revised by Colonel Seddon, R.E.).

Professor Rankine's *Applied Mechanics.*

" " *Civil Engineering.*

" " *Useful Rules and Tables.*

Professor de Lanza's *Applied Mechanics.*

Mr Claxton Fidler's *Practical Treatise on Bridge Construction.*

Professor Warren's *Engineering Construction in Iron, Steel and Timber.*

Rivington's *Building Construction.* Part IV.

Professors Johnson, Turneaure and Bryan's *Theory and Practice of Modern Framed Structures.*

Professor Unwin's *Testing of Materials of Construction.*

Mr. Tarn's *Mechanics of Architecture.*

Professor Baker's *Treatise on Masonry Construction.*

Mr. Anglin's *Design of Structures.*

Mr. Patton's *Practical Treatise on Foundations.*

The Roorkee Treatise of Civil Engineering.

The Military Works Handbook of Specifications, etc.,

As well as technical journals and the well-known pocketbooks of Molesworth, Trautwine, and Hurst.

G.K.S.M.

Chatham, April, 1897.

* Professor Rankine, *Applied Mechanics* p. 9.

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ERRATA.

Page 56, line 3, *for* "$\frac{p}{w_1}$" *read* "$\frac{P}{w_1}$, P being the load."

,, 129, ,, 19, *for* "$W = \frac{1}{48}$" *read* "$n = \frac{1}{48}$."

,, 129, ,, 25, *for* "v," *read* "V."

,, 139, ,, 2, *for* "W = weight" *read* "w = weight."

,, 147, ,, 14, *for* "$\frac{1\ (x \times 1)}{x\ (x-1)} = \frac{1 \times 15}{14 \times 13} \times W_s\ 2400$" *read* "$\frac{1\ (x+1)}{x\ (x-1)} = \frac{1 \times 15}{14 \times 13} \times 2400.$"

,, 147, ,, 18, *for* "$a = 26$" *read* "$a = 3{\cdot}36$."

,, 182, ,, 2, *for* "120^2" *read* "144^2."

,, 182, ,, 30, *for* "275" *read* "2·75."

,, 185, ,, 2, *for* "+336" *read* "×336."

,, 210, substitute *for* line 3 "$= \sqrt{M_h^2 + M_v^2} = \sqrt{717^2 + 260^2} = 760$, Mh and Mv being the moments of horizontal and vertical components, as on p. 207."

,, 210, line 6, *for* "3638" *read* "7460."

,, 212, ,, 16, *for* "$2w + c$" *read* "$2w \times c$."

,, 219, *Fig.* 196, *omit* central row of rivets.

,, 222, line 8, *for* "$3{\cdot}1416 \div 4 \times d^2 \times r_s$" *read* "$3{\cdot}1416 \times d^2 \times r_s \div 4$."

,, 222, ,, 14, *for* "n'" *read* "n''."

,, 223, ,, 8, *for* "fall" *read* "fail."

,, 241, ,, 34, *for* "LSW" *read* "LSw."

,, 255, ,, 27, *for* "$7 \div 4$" *read* "$7 \div 5$."

,, 256, ,, 8, *for* "2409" *read* "2400."

,, 256, ,, 15, *for* "Chapter IX." *read* "Chapter X."

,, 266, ,, 1, *for* "portions" *read* "positions."

,, 277, ,, 11, *for* "*Plate* V." *read* "*Plate* VII."

,, 282, ,, 10, *for* "$GA = 5100 \times 15 - 3400 \times 7{\cdot}5 \div 5{\cdot}9$" *read* "$GA = \frac{5100 \times 15 - 3400 \times 7{\cdot}5}{5{\cdot}9''}$."

,, 307, ,, 16, *for* "$Wl \times 12$" *read* "$Wl \div 12$."

,, 308, ,, 9, *for* "square" *read* "cubic."

THE PRINCIPLES OF STRUCTURAL DESIGN.

CHAPTER I.

INTRODUCTION.

Definitions. — Active and Passive Forces. — Tension, Compression, and Shearing. — Ultimate and Working Strength. — Factor of Safety, Fatigue, Elasticity, Stiffness, and Resilience.

AN engineering design may be looked upon as a problem in attack and defence. Just as a military commander on the defence seeks to be everywhere stronger than his enemy, not merely in troops, but also in defensive power, so the engineer in designing a structure must arrange it so that it shall be everywhere stronger than the force or forces it has to sustain, not merely in actual weight, but in the various influences which ensure stability. And as in warfare it is necessary for a general to study carefully his adversary, as well as to be acquainted with the capabilities of his own resources, so the engineer must study the external conditions likely to be brought to bear on his structure, as well as the capabilities of that structure for resistance.

The external, or what we may call the attacking powers, are sometimes called *active* forces, the resisting power the *passive* forces. The latter are, within the limits of the capacity of the material, only just sufficient to balance the former. Thus if a beam capable of bearing a load of 10 tons is loaded only with 1 ton, the internal resistance will be that due to 1 ton only. This may appear self-evident, but it is a principle of importance, and is of assistance in the investigation of certain cases. It is not, however, strictly accurate to speak of either as *force*, because there are other considerations, as will be

presently seen, which enter into the question of stability. It is possibly preferable to call these *active and passive influences.*

In the investigation of both, experimental science must necessarily play a large part. On careful observation of the behaviour of materials under various conditions, our knowledge of the stability of any structure formed of those materials must be founded. This is of the utmost importance. No design is practically worth considering, where the knowledge of the properties of its materials is in any way ignored.

Yet such knowledge must be utilized by the skill of the scientist. In the words of Luigi Cossa (quoted by Sir G. Molesworth): "It is needful to hold ourselves aloof equally from the so-called doctrinaires, who refuse the assistance of practice, and from the empiricists, who obstinately close their eyes to the light of theory." A French writer has stated "theory is the general, practice is the soldier," and as an army without officers would be a rabble, so practice alone in engineering will not tend to economy, but to extravagance on the one hand, or danger on the other. Theory and practice must work hand in hand.

Definition of Terms.

Before entering on the examination either of the general principles usually involved in stability, or of the properties of various materials, it is necessary to define various terms commonly used.

A *Structure*, as opposed to a *Machine*, is any work of human art which is ultimately intended to remain in equilibrium, without motion other than that of such a minor nature as vibration, or expansion and contraction due to changes in temperature, etc. The expression "structure" applies not merely to a work as a whole, such as a bridge, or a house, but to each part of it, as the arch, or the floor joists.

A *Stress* is the internal force set up in a structure by the action of external forces.

A *Strain* is the change of form produced by a stress. It may be that this change is only temporary, *i.e.*, when the force is removed, the structure may resume its original form. Or the stress may be so great that the structure does not regain its original form. It is then said to be permanently strained.

There are three kinds of stresses, viz. :—

(1). *Tensile* stress, or *tension.* In this case the tendency is to pull apart, or tear away, one particle of the substance from that adjacent to it. The fibres in the structure tend to become elongated.

(2). *Compressive* stress is the opposite of that just described. In this case the force pushes at both ends of the structure, tending to force the particles together, or crush the fibres of the material. In some cases the compressive stress is called the *bearing* stress, *e.g.*, in rivetted joints where the rivet tends to *crush* into the plate, remaining itself intact.

The two above classes of stress may be either *normal* or *oblique*, relative to the surface at which they act.

(3). *Shearing* stress is where the particles tend to slide past one another by the action of forces pulling in opposite directions. Thus in *Fig.* 1 the rivets are subject to a shearing stress, tending to cut them in two. This stress is also called *tangential*, as it acts in two contiguous bodies parallel to their surface of contact.

Fig. 1.

In addition to the above simple stresses there are various compound stresses. Of these the most common is *transverse* stress, in which the load, acting transversely to the axis of the structure, tends to produce compression in one set of fibres, tension in another set, and shearing stress throughout.

Where English standards are used, a stress is usually measured in tons, cwts., or lbs. The *intensity of stress* or stress on a unit area is lbs. or tons per square foot or square inch. With the Metric System, the stress intensity is usually expressed in kilogrammes per square centimètre. 1 kilo. per square centimètre = 14·2 lbs. per square inch.

The word *strength* is often used in connection with structures. It is useful to remember that it means the quantity of resistance to rupture which the structure is capable of affording, measured in any convenient way. The strength of a structure may be expressed by various compound expressions, to be noticed hereafter when treating of Resistance to Rupture (Chapter V.).

Besides the *stress intensity* there are the following expressions used with regard to stress :—

(1). *Ultimate stress*, sometimes called *limiting* or *breaking* stress, which, as the name implies, gives the stress at which rupture occurs.

(2). *Proof stress*, which is intended to be applied only for a short time to test a structure, is the greatest stress that can be applied without permanent injury.

(3). *Working stress, i.e.*, that which can be applied for any length of time without fear of injury.

Of these, the ultimate stress having been found by experiment, the working stress bears a certain definite ratio to it. This ratio is called the *factor of safety*. It is usually taken at about $\frac{1}{4}$ or $\frac{1}{5}$ of the ultimate stress, but varies, as shown in table below, for various materials, and various loads. Thus if wrought iron is proved to have an ultimate tensile strength of 24 tons to the square inch, and it be decided to arrange a structure built of this iron so that each square inch shall bear no greater stress than 6 tons, when in tension, the factor of safety will be $\frac{\text{ultimate stress} = 24}{\text{working stress} = 6} = 4$.

Load.

Just as *stress* is the internal force excited in a structure by outward influences, so *load* is the corresponding term for the external force acting upon the structure. The expressions ultimate load and working load are often (conveniently) used in place of ultimate stress and working stress.

A load may be *distributed* in various ways. It may be concentrated in the centre, as in *Fig.* 2, or concentrated at a given point, not at the centre, as in *Fig.* 3, or it may be uniformly distributed, as in *Fig.* 4, over a part or over the whole. The stresses produced in

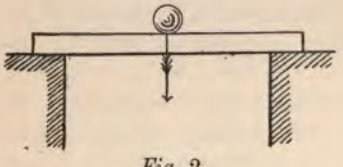


Fig. 2.

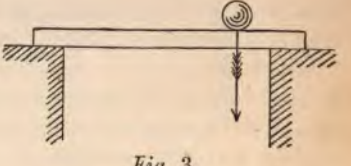


Fig. 3.

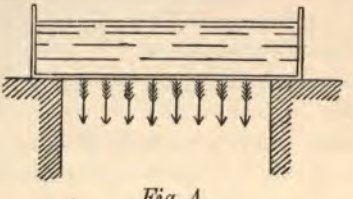


Fig. 4.

the beam, as we shall see in subsequent chapters, will vary with the distribution, though the load may be the same in every case.

Loads are commonly spoken of as *dead loads* and *live loads*. The former is that which is applied very gradually, and which remains steady. Thus water quiescent in a tank is a dead load. The weight of a structure itself is a dead load.

A live load is that which is suddenly applied, or is accompanied with shocks, or impact. Thus the weight of water in waves dashing against a pier, a crowd of people moving on a floor, a railway train crossing a bridge, etc., constitute live loads.

A live load is considered to produce in structures *twice the stress* that equivalent dead loads produce. Thus the quiescent weight of human beings on a floor is about 70 lbs. per square foot, but as allowance must be made for movement, 140 lbs. per square foot is taken as the ordinary basis of calculation.

Value of Factor of Safety.

The factor of safety is often calculated with reference to the load, whether live or dead. The following table gives the factors of safety recommended for various loads and in various materials :—

TABLE I.

Authority.	Nature of Structure.	Nature of Load.	Factor of Safety.	Remarks.
	Timber.			
A.	Tension members	Dead	10	Latest views on this subject.
U.	Generally	,,	7	
U.	,,	Live	10	
A.	Compression with grains ...	Dead	5	
A.	,, across ,,	,,	4	
A.	Shearing	,,	4	
A.	Transverse rupture	,,	6	Extreme fibre stress.
R.	General	Varying & Shorter	10	
S.	Exposed to weather	Dead	10	
S.	Under cover	,,	8	
S.	For temporary purposes ...	,,	4	
W.	Ordinary buildings	,,	4	
W.	Bridges	Live	8	
	Brickwork and Masonry.*			*This depends on a great variety of subjects. If laid with good supervision and care, both as regards materials and workmanship, the margin may be comparatively small
U.	General	Dead	20	
U.	,, ...	Live	30	
R.	,, ...	Dead	4 to 10	
B.	,, ...		av. 8	
S.	Arches	Dead	20	
	Cast Iron.			
U.	General	Dead	4	
U.	,, ...	Live or Varying	6 to 10	
U.	,, ...	Varying with Shocks	15	
S.	Pillars	Dead	6	
S.	,, subject to vibration	Live	8	
S.	,, (if well cast and ends planed) ...	Dead	45	
T.	,, (if not ditto)... ...	,,	15	
	Wrought Iron and Steel.			
U.	General	Dead	3	
U.	,, ...	Live	5	
U.	,, ...	Varying with Shocks	8	
	Many other authorities give practically the same.			
R.	General	Dead	3	
R.	,, ...	Live	4 to 6	

A.—American Commission on Railway Bridges, 1895. U.—Prof. Unwin. R.—Prof. Rankine. W.—Prof. Warren. S.—Prof. Stoney. B.—Prof. Baker. T.—Trautwine.

In determining the factor of safety, an engineer must be guided not only by the nature of the load, whether steady, or varying and jerky, but also by the nature of the material, and in some cases the workmanship. For instance, wrought iron is a material about which our knowledge is fairly definite, and which is little affected by faulty handling, whereas concrete is capable of almost infinite variation, not only in the materials used, but in the manipulation of those materials.

Fatigue of Materials.

Another consideration which affects the value of the factor of safety is the deterioration produced in the fibres of the material by repeated application of the load, or by reversal of stresses. This is known as the *fatigue* of the material. Repeated experiments have been made on this subject by eminent engineers, such as Sir W. Fairbairn, Sir B. Baker, Herr Wöhler, etc. It has been found, for instance, that the chain of a crane will sometimes ultimately fail to carry a load which it has lifted previously with safety.

It is supposed that the effect of frequent shocks or vibration is to change the fibrous structure of wrought iron to a crystalline texture resembling cast iron.

This subject is of importance, especially, as regards structures, in the case of bridges built of open girders, where, owing to changes in the position of the load, certain members are exposed to different stresses at different times. To fix the limit of all the iron or steel in such a bridge at a certain definite amount, as is done in the Board of Trade Rules, is to give excessive strength to some members, and to give a strength which may be dangerously near the limit in others. It is clearly sounder "to make the working stress depend upon the range of stress, or upon the impact, or upon both these causes combined. Thus in the New York Elevated Railroad the flanges of the girders were designed for a stress of 3·4 tons per square inch, the web bracing for 3·4 tons per square inch, and for members subjected to alternating stresses only 2 tons per square inch was allowed."*

Elasticity.

Another basis for fixing the factor of safety in a material is the *elastic limit*, which must now be defined.

* Professor Warren, *Engineering Construction in Iron, Steel, and Timber*, p. 21.

A material is said to be elastic when it has the property of regaining its original shape after having been strained, or, in other words, if the strain disappears after the stress has been removed. A golf or cricket ball when hit "clean" leaves a circular impression almost equal to the diameter of the ball on the face of the club or bat, but if the ball be examined it will be found to have regained its original shape, and not to have become flat at the place where it has received the blow. This regaining of shape is due to its elasticity.

If a bar of elastic material be subjected to a gradually increasing tensile stress, it will be found that the increments of length are in direct proportions to the increments of stress. This is also the case if the bar be subjected to a compressive stress. This principle is known as Hooke's law, "Ut tensio sic vis," as defined by the discoverer in 1676.

But this law is only of limited application. A point is reached in most materials where the strain does not disappear with the removal of the stress. This point is called the *elastic limit* of the material. Some materials have a much greater elasticity than others. Lead, for instance, has no elasticity, while glass is very elastic.* Iron and steel are elastic up to about half their ultimate strength, *i.e.*, in wrought iron about 12 tons, and in mild steel about 18 tons per square inch. It will be at once seen that, as in no structure is a permanent strain (or *permanent set*, as it is technically called) on any account permissible, elasticity is a very important measure of strength.

The term *Modulus of Elasticity* of a material is the standard by which the effects of elasticity in the members formed of that material are measured. It is the ratio of *stress* to *strain*, the former expressed per unit of area, and the latter per unit of length. Hence it may also be defined as the *ideal stress* which would be capable of stretching a perfectly elastic bar of unit area to double its length.

For, if L = original length of a bar in inches.
P = stress applied to it in lbs. or tons per square inch.
l = increment of length arising from P.
E = the modulus of elasticity in lbs. or tons.

$$E = \frac{P}{l} \times L \text{ or } E : L :: P : l.$$

For the exact determination of this, delicate testing machines with automatic registers are employed. These, however, need not

* Anglin, p. 11.

be described. Any irregularity or flaw in the material tested will affect the result, hence no two bars of the same material give exactly the same results.

It follows also that in a framed structure it is not desirable to have the members built up of materials having very different moduli of elasticity, as, for instance, a truss partly made of timber struts and iron tie-rods. Owing to the difference in contraction and extension, it is impossible to calculate with any exactness the amount of the stresses on the various members.

Stiffness in a material is that property which tends to resist being strained. It is a very different thing from strength against breaking. It depends upon the modulus of elasticity, and, as it will be seen when treating of deflection, upon the arrangement of the material.

In all permanent structures it is of the greatest importance.

Resilience is the term used to specify the amount of work done on a bar when the stress just reaches the elastic limit, or the energy stored in the bar during the act of producing a strain. On the removal of the stress the energy will be restored, or visibly reproduced, by effecting the recovery of the figure to its original form.

CHAPTER II.

Theoretical Principles Common to most Designs.

Rules for Centre of Gravity.—Law of Lever.—Parallelogram and Triangle of Forces.—Graphic Methods.—Polar Diagrams.

There are certain leading principles which enter more or less into every problem of engineering design. These may be familiar to most readers of this work, but as a clear understanding on this subject is most essential, it may not, perhaps, be a waste of time to go over these principles before going on to consider other matters.

Professor Rankine calls force "an action between two bodies, either causing or tending to cause change in their relative rest or motion." A force is further defined by an American scientific writer* as "a tendency to change the relative motion of the two bodies between which that tendency exists." Two bodies, at least, must be concerned in the transaction, but we may confine our attention wholly to the motion of one of them, referring its motion either to the other as a fixed point, or to some body different from either. Hence, in speaking of the force, we may consider it as acting on the body under consideration, and call it an external force.

To measure force we take some unit of weight—pounds, cwts., tons, kilogrammes, etc. It may be graphically illustrated by a straight line drawn (1) to a scale, the divisions of which correspond to the units of the force. This straight line may also, on a drawing representing the bodies in question, be drawn (2) in the direction corresponding to the line of direction of the force, and (3) with its initial point at a position corresponding to the point of application of that force. **Hence a line thus drawn will represent graphically the three conditions which we require to know about a force: — (1). Magnitude. (2). Direction. (3). Point of application.**

* De Lanza.

First Mechanical Principle in Structural Design.—Position of Centre of Gravity.

In dealing with a solid substance we may consider the weight of that substance, and the force exerted by that weight, as though the whole were concentrated at the centre of gravity of the substance. This is a principle of the first importance in structural design.

The centre of gravity is the point through which the resultant of the weights of all the particles of the body must pass. It is, in more popular terms, the point at which a force, equal and opposite to the weight of the body, must be applied in order to balance that weight. If a body be freely suspended from a chord, therefore, the prolongation of direction of the chord will pass through the centre of gravity. This is a fact of practical importance, and helps us sometimes to find where the exact centre of gravity is. Thus we may wish to find the centre of gravity of a cross section of the rail shown in *Figs.* 5 and 6.* To do so we cut out in tin or other

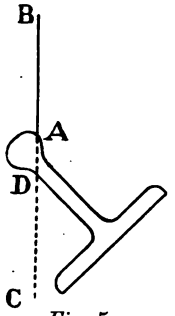


Fig. 5.

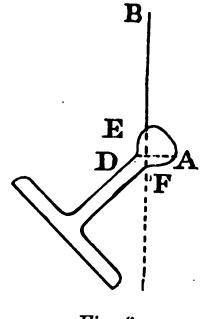


Fig. 6.

substance an exact cross section of the rail and suspend it from any point A so that it is free to adjust itself. The tension in the chord AB is equal and opposite to the resultant weight of the body, hence the centre of gravity must be somewhere in the line AD. We may draw the line AD, and then suspend the body from some other point E (*Fig.* 6). The centre of gravity will again be in prolongation of BE, say in EF. Therefore where the lines AD and EF intersect is the true centre of gravity of the cross section.

The centre of gravity is conveniently expressed by the letters c.g.

* In which the deviation of the lower part of the figure is purposely exaggerated.

The c.g. of a circle is the centre, of a parallelogram the point of intersection of the diagonals, and of a triangle (*Fig.* 7) the point E, found by joining any angle A with the centre D of the opposite side, and then dividing AD so that AE = 2ED or ED = $\frac{1}{3}$AD (*Fig.* 7).

The c.g. of a quadrilateral figure is found as follows:—Let ABCD (*Fig.* 8) be the quadrilateral; join diagonals AD, BC which intersect

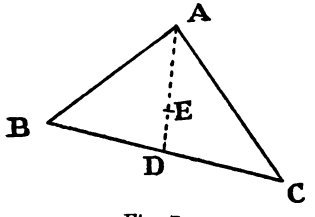


Fig. 7.

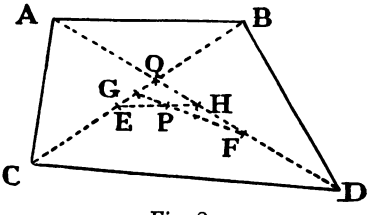


Fig. 8.

at O. Make AF = DO and BE = CO. Bisect EO and FO at G and H, and join FG, EH. The point P where these intersect is the required c.g.

To find the common c.g. of two weights such as W_1W_2 (*Fig.* 9). Draw AB joining the two centres of gravity. Then draw AC perpendicular to AB, and representing on any scale the weight W_2. Similarly from B draw BD on the opposite side of AB, representing on the same scale W_1. Join CD. The point E, where CD cuts AB, is the common c.g., and a force applied there equal to $W_1 + W_2$ in an upward direction would just balance them.

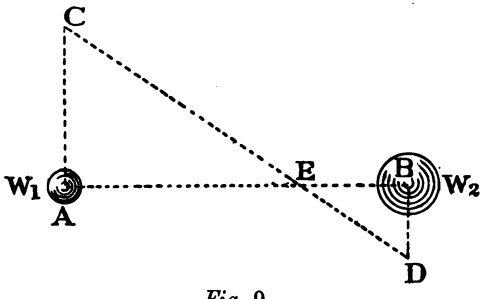


Fig. 9.

For centres of gravity of parabolic segments and other figures, see Appendix to this chapter.

Second Mechanical Principle in Design.

Fig. 9 above leads on to the consideration of the *law of the lever*, which is the second important mechanical principle in structural design. It is otherwise known as the *principle of moments*, the term

moment being used to express the compound action of force multiplied by distance or leverage, and expressed in inch-tons or feet-pounds, etc.

In the figure above it is evident that the line AB is divided so that

$$AE:EB::AC:BD$$
$$::W_2:W_1.$$

Hence
$$AE \times W_1 = EB \times W_2.$$

If the point E be the fulcrum of the lever we have this same principle always holding good, no matter whether E be between the weights, as in *Fig.* 10, where both weights act vertically downwards, or in *Fig.* 11, where the weight (W_2) furthest from E is a force pulling upwards, or in *Fig.* 12, where the weight W_2 is balanced by

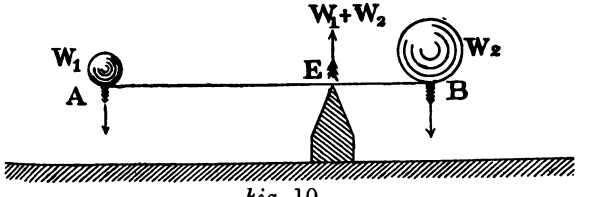


Fig. 10.

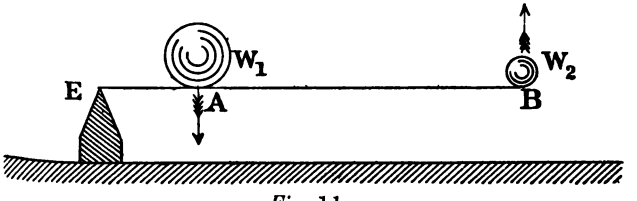


Fig. 11.

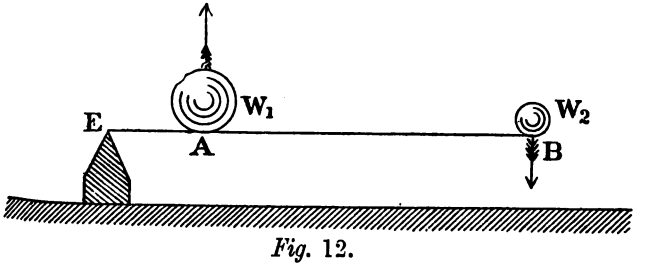


Fig. 12.

a force (W_1) pulling upwards between W_2 and the fulcrum. If in all three cases the length AB be 10 feet, the distance AE = 3 feet, and

the weight W_2 be 20 lbs., then in the first case a weight of $46\frac{2}{3}$ lbs. at A will balance 20 lbs. at B. In the second case a lift of 20 lbs. at B will balance $66\frac{2}{3}$ lbs. at A, and in the third case a force of $66\frac{2}{3}$ lbs. at A will be required to lift 20 lbs. at B.

It must be remembered that the supporting force at the fulcrum E is always equal to the sum of the weights. This may be expressed in the following terms:—If a loaded beam is supported at one or more points, the total reaction at those points is equal to the weights on the beam.

When the lever is not straight, but bent, or where the forces are not parallel, the same principle of the lever holds good, only in these cases the leverage, or distance, is measured by the perpendicular line from the fulcrum to the line of direction of the force.

Thus let AEB (*Fig.* 13) represent an ordinary crank with weights W_1, W_2 at A and B, and pivoted at E. The leverages now are the lines E*a*, E*b* drawn from E perpendicular to the direction of W_1 and W_2, and this is the case whether these directions be parallel to one another or inclined, as in *Fig.* 14.

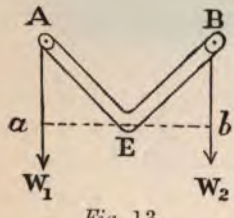


Fig. 13.

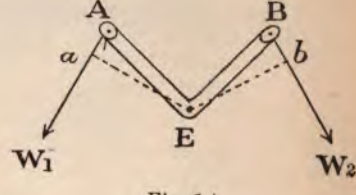


Fig. 14.

The resultant pressure at E in the latter case would be found by the parallelogram of forces, to be now described.

Third Mechanical Principle in Design.

If three forces acting in the same plane on a point are in equilibrium, two of them may be represented in magnitude and direction by the sides of a parallelogram, and the third by the diagonal of the parallelogram.

This—"the parallelogram of forces"—is the third mechanical principle in design. Another way of stating it is:—**If three forces acting at the same point in a plane are in equilibrium, three lines drawn parallel to them will form**

a triangle in which the lengths of the sides are proportional to the magnitude of the forces.

Thus, in *Fig.* 15, if P, R and Q be three forces acting on a particle, and if we draw AB (*Fig.* 16) parallel to P, BC parallel to Q, and make AB, BC on any scale of lbs. or tons, etc., equal to the forces in lbs., cwts., tons, etc., at P (lbs. or tons) and Q respectively, then the value of R will be found by completing the parallelogram ABCD and drawing the diagonal BD. Or the same result would be attained if we draw, in regular sequence, AB = P in magnitude and direction, DA = Q on the same scale, and BD = R (*Fig.* 17).

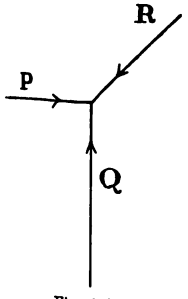


Fig. 15.

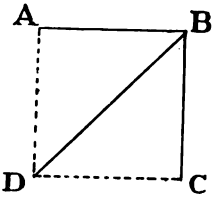


Fig. 16.

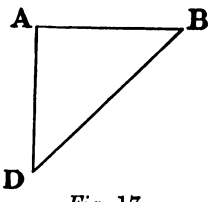


Fig. 17.

It follows also that any simple force acting on a particle may be represented by two others, as long as they represent the sides of the parallelogram, of which the original force is the diagonal. For any one of the forces shown would be balanced by an equal force in an opposite direction. Thus R (*Fig.* 18) might be balanced not by Q and P, but by an equal and opposite force R_1. But R is already balanced by Q and P. Hence Q and P are equal to R_1 or R.

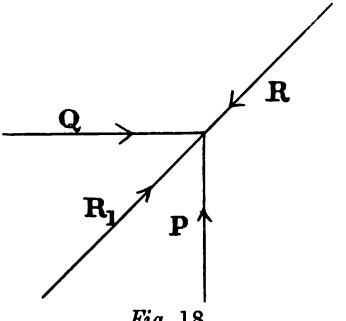


Fig. 18.

Let BD (*Fig.* 19) represent any force acting on a particle. BD may be represented by BC and BA or by BC_1 and BA_1, or by any any other combination of forces so long as these represent the sides of the parallelogram of which BD is the *diagonal.*

Hence any force, as BD, may be *resolved* into two others, BC and

BA forming any angle at the point of application. *Usually* that angle is a right angle. Thus let BD (*Fig.* 20) represent the force in an inclined prop pushing against the ground at B, and against a vertical pillar DA at D. It is required to know how much force is pressing vertically on the ground at B, and how much is pressing parallel to the ground, tending to make the prop slide.

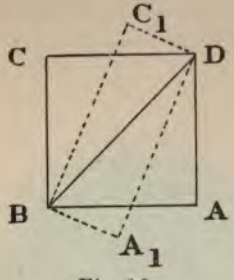


Fig. 19.

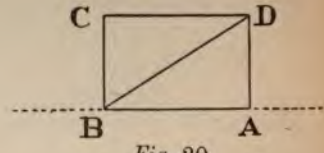


Fig. 20.

This is found by completing the rectangle ABCD to scale and measuring the lengths BC, BA, or, calling the angle DBA = θ.

$$AB = BD \cos \theta; \quad AD \text{ or } BC = BD \sin \theta.$$

This *resolution* of forces is one of the commonest mechanical principles involved in structural design. Where the angle between the resolved forces is a right angle, it is generally easiest to find the values of the components by mathematical analysis, as in the example above. Where this angle is not a right angle it is usual to draw the resolved parts graphically to scale, but it is, of course, possible to represent them analytically.

Thus if R be the force, P and Q the resolved portions, acting at an angle θ (*Fig.* 21)—

$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$ (remembering that $\cos \theta = -\cos (180 - \theta)$),

and if α be the angle which R makes with one of the components—

$$\sin \alpha = \frac{P}{R} \sin \theta, \text{ and } \sin (\theta - \alpha) = \frac{Q}{R} \sin \theta.$$

From the foregoing it will be seen that if any number of forces acting at a point produce equilibrium, those forces may be represented in magnitude and direction by the sides of a polygon drawn parallel to the direction of the forces. Thus let us take the forces P, Q, R

and S (*Fig.* 22). Any two of these forces, say P and Q, may be replaced by P_1, which, with the remaining two, S and R, will still produce equilibrium, and will therefore equally represent the third side of a triangle of which P and Q, or S and R, are the other sides. We may draw the line AD on any scale (*Fig.* 23) equal to the number of units in the force P_1, and draw on one side of it AB, BD parallel to Q and P, and on the other side AC and DC parallel to S and R, and we then get the polygon ABDC, with sides BA, AC, CD, DB corresponding to the forces P, S, R and Q respectively.

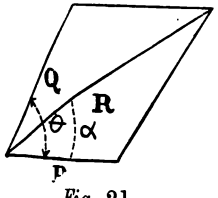


Fig. 21.

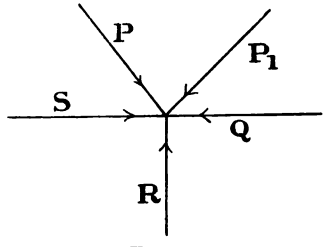


Fig 22.

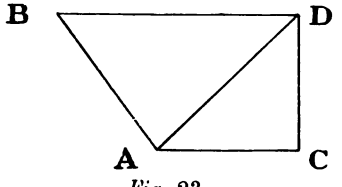


Fig. 23.

The proposition may be thus stated:—If any number of forces be represented in magnitude and direction by the sides of a polygon taken in order, then, if these forces be simultaneously applied at one point, they will balance each other.

Conversely, **if any number of forces simultaneously applied to a point produce equilibrium, they can be represented in magnitude and direction by a polygon, in which the sides taken in order correspond to the forces taken in regular sequence.**

Graphic Representation of Forces.

The above remarks introduce the subject of graphic diagrams, a matter of much importance in connection with the determination of stresses in structures.

The examples given above in *Figs.* 20 and 23 show graphic diagrams of stresses, but the notation there employed is unsuitable for use in large and complicated trusses. The notation devised by Mr. Robert H. Bow, and also recommended by Professor Henrici, is much to be preferred. It is as follows:—The radiating lines in *Fig.* 24, representing forces directed on one point, are represented by capital letters placed in the angles. Thus the force AB is the force on the line between the letters A and B. In the stress diagram (*Fig.* 25) the sides of the polygon are denoted by *small* letters in the ordinary way at the ends of the lines, corresponding to the large letters in the radiating lines of forces. Thus the line *ab* (*Fig.* 25) represents the force AB (*Fig.* 24), and so on. In drawing the

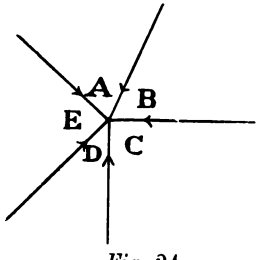


Fig. 24.

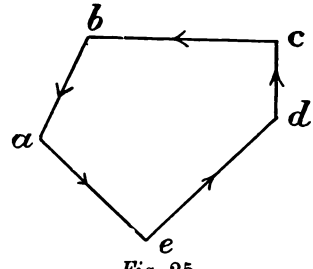


Fig. 25.

polygon of equilibrium, we must always take the forces in regular sequence, working either right-handed or left-handed rotation. By marking the direction of the forces with arrow heads and, in the polygon, making these arrow heads follow one another, we may find the magnitude and nature (whether pushing or pulling) of an unknown force which we know is necessary for equilibrium. In the case above:—Let AB, BC and CD be known in magnitude and direction, while DE and EA are only known in direction, but their magnitude and nature is unknown. Accordingly *ab*, *bc* and *cd* are drawn; *ae* is drawn parallel to AE and *de* parallel to DE. Then, from the polygon, the magnitude of *ae* and *de* can be measured, and from the direction of the arrows it is seen at once that they are pushing, or compressive forces, because the direction of the arrows is *towards* the common point of application.

In order that these may be complete, it is necessary that *not more than two* of the forces at any one point should be unknown. For in the above diagram if AE and CD were unknown, the direction of

only would be known, but its position or magnitude, and, therefore, that of *cd* and *ac*, would be uncertain.

But although it is true that not more than two forces at any point must be unknown, it is quite possible to find all the stresses in a jointed frame where the magnitude of only one force is known, provided the direction of the others is known, and even if the direction of one of the others is unknown, it is still possible to find out all about them. Thus in the jointed frame shown in *Fig.* 26 let the force AB be known both in magnitude and direction, BC, CD, DE and EF be known as regards direction, and the supporting forces EF and FA be unknown.

We begin drawing *ab* (*Fig.* 27) to scale, parallel to AB and equal to it in amount (lbs., etc). From *b* draw *bg* parallel to BG, and from *a* draw *ag* parallel to AG. These lines intersect at a point *g*, and thus we know the magnitude, therefore, of AG and BG. From *b* draw *bc* parallel to BC, and from *g* draw *gc* parallel to CG. This gives the magnitude, therefore, of BC and CG. From *c* draw *cd* parallel to CD, and from *g* draw *gd* parallel to GD. From *d* draw *de* parallel to DE, and from *g* draw *ge* parallel to GE.

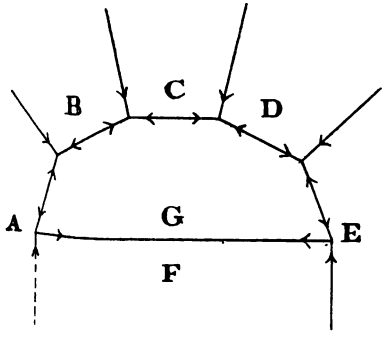


Fig. 26.

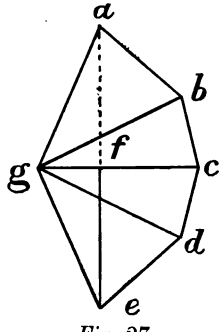


Fig. 27.

From *e* draw *ef* parallel to EF, and from *g* draw *gf* parallel to GF. Then the dotted line *fa* must represent the magnitude and direction of the remaining force AF.

By examination of this figure we see that (1) in every case there is only *one* unknown stress, and (2) by following in each case the direction of the arrows we see that the members GA, GB, GC, GD and GE are all in compression, while GF is in tension. For if we take the diagram for the right-hand lowest corner we begin with the

known force *ge* pushing downwards, then, as the arrows follow on e another, *ef* pushes upwards and *fg* pulls away from the joint (*Fig.* 28).

Graphic diagrams are not only used to represent forces, where th e scale corresponds to units of weights, but they are also used t o represent compound expressions, such as foot-pounds or inch-ton s, *i.e.*, the product of feet multiplied by lbs., etc.

It must be remembered that these diagrams are only **pictoria l representations,** as it were, of forces, etc., which cannot otherwi se be readily manifested. It is easy, comparatively, to arrange for th e material to resist such forces when we see clearly what they ar e. But it does not follow that because a bridge or roof has a particula r stress diagram under a given load, that the external appearance of th e structure when built will have the slightest resemblance to the stre s diagram.

Method of Drawing a Parabola.

As parabolic arcs enter frequently into graphic diagrams, it m y here be advisable to describe one or two methods of drawing the s e in the absence of any special instrument designed for that purpose.

Let A, B, C (*Fig.* 29) be three points through which it is necessa y

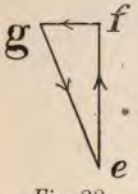


Fig. 28.

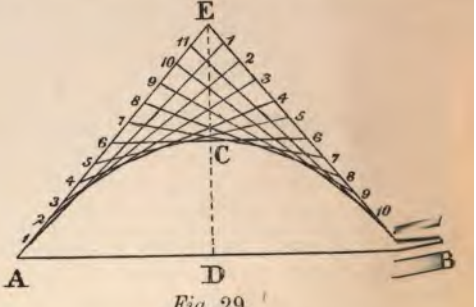


Fig. 29.

to draw a parabola, C being equidistant from A and B. Bisect A B in D, and draw CE = CD. Join AE and EB, and divide them in o any equal number of equal parts, numbering them 1, 2, 3, etc. Jo n 1—1, 2—2, etc. These lines are tangents to the required parabol a, the shape of which is sufficiently indicated by them.

Another method (*Fig.* 30) is to draw from C a line parallel to A B, and divide AD, CD into the same number of equal parts. Draw *aa'*, *bb'*, *cc'* perpendicular to AB. Join C*d*, C*e*, C*f*. Then the intersectio n of C*d* with *aa'*, C*e* with *bb'*, C*f* with *cc'* will give points on the curv e.

If the height of the parabola AD is not greater than $\frac{1}{10}$ AB, the curve may, with sufficient accuracy, be represented by the arc of a circle.

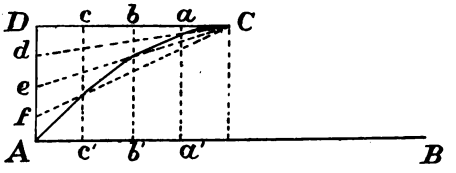


Fig. 30.

The c.g. of a parabola is at G (*Fig.* 31) where DG = $\frac{2}{5}$CD. The c.g. of the half CDB is at H where BK = $\frac{5}{8}$BD. The area of ACB = $\frac{2}{3}$AEFB. The c.g. of the spandril ACE is at a point I where IL = $\frac{3}{10}$EC and IM = $\frac{1}{4}$EA. The area of ECA = $\frac{1}{3}$ADCE. For proofs see Appendix to this chapter.

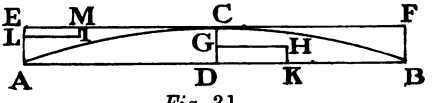


Fig. 31.

Polar Diagrams.

Sometimes in the graphic representation of stresses it is advantageous to make use of what are called *polar diagrams*. In these cases a point is taken outside, and auxiliary lines drawn from it to certain points in the diagram of stresses, which assist in the solving of the problem.

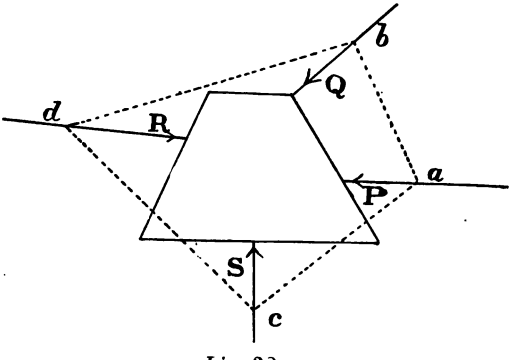


Fig. 32.

To take an example, let there be three known forces PQS and one unknown force R producing equilibrium on a body (*Fig.* 32). Draw the polygon of forces (*Fig.* 33) *pqrs* where *pq* = P, *qr* = Q, *ps* = S. Then *rs* must = R in magnitude and direction. But the point of application of R is still uncertain.

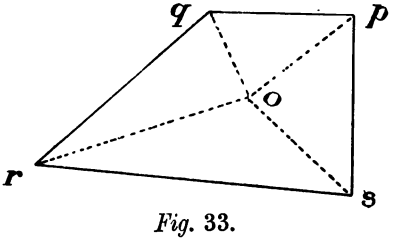


Fig. 33.

To find this, take any point O, called *the pole*, and join O*q*,

Op, Or, Os. From any point a in the direction of P draw ab parallel to Oq, meeting the direction of Q in b, and ac parallel to Op meeting the direction of S in c. From b and c draw lines parallel to Or and Os. These meet in d, which fixes the line of R. From d draw a line parallel to rs. This will give the point of application of R. The figure *abcd* is called the *funicular polygon*. As the pole O may be selected anywhere, it follows that there may be any number of funicular polygons, and as the point a may be selected anywhere, there may be any number of polygons for the same position of the pole. But whatever be the position of the pole, or the starting point a, the point of intersection d will always fall on the line of action of the resultant R.

If the system of forces is in equilibrium, the funicular as well as the force polygon must close.

This method of polar diagrams is most useful in many cases, in the checking of analytical calculations, or in solving problems which otherwise could only be attained by the use of high mathematics.

Further developments of graphic diagrams will be noticed when we come to the application of them.

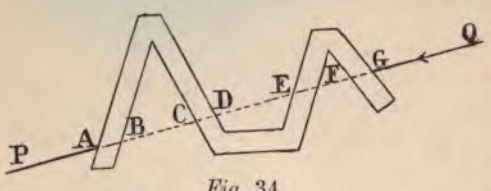


Fig. 34.

In considering the effect of forces of any fairly rigid body as ABCD (*Fig.* 34), if PQ represent the line of direction of the force, the effect on the entire mass is the same, whether imparted as a push at G, F, D or B, or a pull at E. C or A, or applied at any other point in the line PQ.

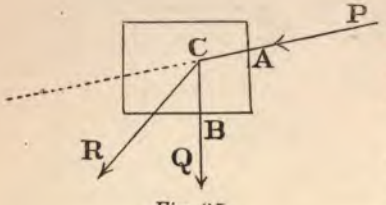


Fig. 35.

Thus by producing the lines of two forces P and Q, acting at points A and B, till they meet at C, we may obtain graphically the resultant force R (*Fig.* 35).

Examples for Practice (*Graphic Diagrams Chiefly*).

1. Find the c.g. of a triangle with sides 3, 4 and 5 units.
2. Find the c.g. of a quadrilateral with sides 3, 3, 6 and 8.

3. Assuming that the distance between the answers to 1 and 2 is 10 units, find the common c.g.

4. With a lever arm of 12 feet hinged at one end, what force applied at 3′ from the hinge would be required to raise 3 lbs. at the other end?

In a right-angled bent lever with arms 5′ and 3′ long, what force making an angle of 60° with the former will be required to lift 30 lbs. at 90° with the latter?

6. In question 5 what would be the pressure on the hinge?

7. A strut presses on the ground at angle of 60° with a pressure of 3 tons. What must be the horizontal force to prevent it from slipping?

8. A force of 80 lbs. is to be resolved into two, making an angle of 60°. Find the values of these components.

9. Five forces, of which two are unknown, meet at a point. The three known are 20, 30, and 40 lbs. respectively, and the angles in all cases are equal. Draw the stress diagram.

10. A triangular roof truss with slope 30° rests on walls 12′ apart. Draw the stress diagram, assuming the weight on each side to be 200 lbs.

11. Draw a parabola where the span = 24 feet and rise = $\frac{1}{4}$ span.

12. Find the c.g. of a parabolic spandril 5′ long, 2′ in maximum depth. What would it weigh at 4 lbs. per square foot?

APPENDIX.*

The centre of gravity of a body, whether plane or solid, is that point through which the resultant of the weights of the several parts into which the body may be supposed to be divided always passes. If we represent the weight per unit of volume of the body by w, and the total weight W, while dV is the unit of volume, and $x_0 y_0 z_0$ are the ordinates of the c.g., we have

$$W = \int w dV,\ x_0 = \frac{\int wx dV}{\int w dV},\ y_0 = \frac{\int wy dV}{\int w dV},\ z_0 = \frac{\int wz dV}{\int w dV};$$

for if R be the resultant in any one plane

$$x_0 R = \int wx dx, \text{ and } R = \int w dx;$$

$$\therefore \quad x_0 = \frac{\int wx dx}{\int w dx},$$

and so for the others.

* Chiefly from de Lanza's *Applied Mechanics*.

If the body is homogeneous, or of the same weight per unit of volume V throughout, w is constant, and $W = w\int dV$.

$$\therefore \qquad x_0 = \frac{\int x dV}{\int dV}, \; y_0 = \frac{\int y dV}{\int dV}, \; z_0 = \frac{\int z dV}{\int dV}.$$

For a flat plate of uniform unit thickness, such as we generally consider in the design of a structure, $dV = t dx dy$, hence

$$x_0 = \frac{\iint x dx dy}{\iint dx dy}, \text{ and } y_0 = \frac{\iint y dx dy}{\iint dx dy}.$$

CASE 1.—*Parabolic half segment OAB* (*Fig.* 36).

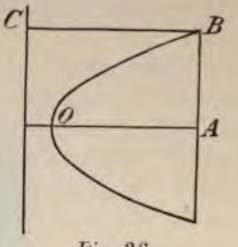


Fig. 36.

Let OA corresponding to CD in *Fig.* 31 $= x_1$, AB $= y_1$. Let O be the origin, $x_0 y_0$ the co-ordinates of the c.g., and, the equation to the curve being $y^2 = 4ax$,

$$x_0 = \frac{\int_0^{2a^{\frac{1}{2}}x_1^{\frac{1}{2}}} \int_0^{x_1} x dx dy}{\int_0^{2a^{\frac{1}{2}}x_1^{\frac{1}{2}}} \int_0^{x_1} dx dy} = \frac{2a^{\frac{1}{2}} \int_0^{x_1} x^{\frac{3}{2}} dy}{2a^{\frac{1}{2}} \int_0^{x_1} x^{\frac{1}{2}} dx} = \frac{\frac{2}{5} x_1^{\frac{5}{2}}}{\frac{2}{3} x_1^{\frac{3}{2}}} = \frac{3}{5} x_1,$$

$y_0 =$ similarly $\frac{3}{8} y_1$, and the area of the segment

$$\int_0^{2a^{\frac{1}{2}}x} \int_0^{x_1} dx dy = \tfrac{4}{3} a^{\frac{1}{2}} x^{\frac{3}{2}} = \tfrac{2}{3} (2a^{\frac{1}{2}} x_1^{\frac{1}{2}}) x_1 = \tfrac{2}{3} x_1 y_1.$$

CASE 2.—*Parabolic spandril OBC.*

$$x_0 = \frac{\int_{2a^{\frac{1}{2}}x^{\frac{1}{2}}}^{y_1} \int_0^{x_1} x dx dy}{\int_{2a^{\frac{1}{2}}x^{\frac{1}{2}}}^{y_1} \int_0^{x_1} dx dy} = \tfrac{3}{10} x_1,$$

and $y = \frac{3}{4} y_1$. Area $= x_1 y_1 - \frac{2}{3} x_1 y_1 = \frac{1}{3} x_1 y_1$.

CASE 3.—*T section.* In this case it is not necessary to resort to integration, as the section is made up of simple figures of which we already know the

centre of gravity. What we practically want to know is the distance of the horizontal line passing through the c.g. from the top or bottom of the section. The area of the figure × the distance of its c.g. from any axis = the area of the component parts of the figure × the distance of the c.g. of each part from the axis. Thus in *Fig.* 37 let the distance of the c.g. of the whole section from the upper edge = x, then, taking moments round that edge, we have

$$x \times \{HB - h(B-b)\} = \tfrac{1}{2}BH^2 - h(B-b) \times \left(H - \frac{h}{2}\right),$$

whence we readily obtain x.

The same reasoning applies to an angle section.

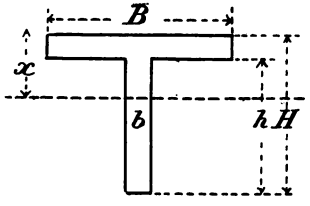


Fig. 37.

CASE 4.—I *section* (*Fig.* 38). Following the same reasoning as before,

$$x\{B(H-h) + b_1(h-h_1) + bh_1\} = \frac{B}{2}(H-h)^2 + bh_1\left(H - \frac{h_1}{2}\right) + b_1(h-h_1)\left(H - h + \frac{h-h_1}{2}\right),$$

whence we can obtain x.

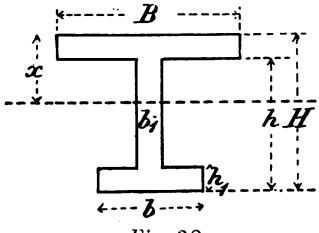


Fig. 38.

CHAPTER III.

Properties of Various Materials Used.

Importance of this Subject.—Various Properties which must be Investigated.—Tests.—Masonry.—Brickwork.—Concrete and Mortar.—Timber.—Iron and Steel.

A KNOWLEDGE of the materials at his disposal, their characteristics, relative cost, and suitability for the work proposed is a matter of absolute necessity to the engineer. It is not proposed here to enter into any dissertation on the characteristics, or method of working, or cost of various materials, as it is considered that such knowledge is outside the scope of this treatise. But the suitability of materials to resist the stresses of various kinds mentioned in a previous chapter must engage attention before we can consider any practical problem.

How to ascertain these properties of resistance is often no easy matter. Nowadays scientific laboratories have been largely established in connection with technical schools. Private establishments, such as the well-known firm of David Kirkaldy & Son, of Southwark, make the subject a speciality, and, being equipped with the best apparatus obtainable, are ready at all times to carry out tests of any sort that the engineer may wish. It may often happen, however, that in out-of-the-way places an engineer may have to carry out work with local material, and time may not admit of his sending it for testing to such reliable authorities as those just alluded to. He is then obliged to fall back upon such rough tests as he may be able to apply himself (and he should invariably do his best to ascertain for himself as much as possible), or he may, by comparing what he can observe of his materials with what has been tested accurately with other similar and similarly situated materials, form a fair opinion as to what he may reasonably expect.

It is proposed here to give a few results of experience in the case

of various building materials as a comparative guide, classifying information under the headings:—(*a*). Masonry. (*b*). Timber, (*c*). Iron and Steel.

(*a*). Masonry.

Under this general heading is included:—(1). Stone. (2). Brickwork. (3). Mortar. (4). Concrete.

The *crushing* strength of stone is generally tested by applying measured force to cubes until they crack or crush. In sending samples to be tested in a laboratory, the cubes should be at least three in number, 6″ sides, truly ground, and with the "quarry-bed" marked. The name of the quarry should be given if possible, and the position of the stone in it.

The point to be elicited by the test is the strength the stone will have when built into a wall. Usually there will be between one stone and another, under such conditions, a cushion of mortar. In testing, therefore, a cushion of some wood, such as Dantzic fir, is usually placed between the stone and the applied force.

The form of fracture in a small cube is usually as shown in *Fig.* 39, the fragments being more or less pyramidal. Some shearing force undoubtedly comes into play, and if the laminations of the bed are very pronounced, fracture will probably more or less coincide with them.

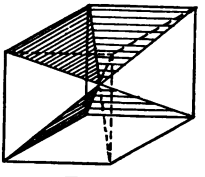


Fig. 39.

If the test specimen is dressed by chisel and mallet the concussion affects the internal conditions, especially with test specimens of small dimensions.

The test specimens are placed either with timber or mill-board between them and the application of the crushing pressure. If lead is used on the face where that pressure is applied there is a considerable reduction of strength—a fact which is both interesting and of practical importance.

The following table has been compiled from various sources, showing the comparative strength of cubes of various stones. It is useful for comparative purposes only, because in a masonry structure the strength depends upon many other factors besides the strength of the stone. The nature of the mortar, the workmanship, the class of work (rubble or ashlar) and various other influences affect the strength of the masonry :—

Table II.

Crushing Strength of Cubes of Stone in Tons per square inch, compiled from various recent Authorities.

Stone.	Maximum.	Minimum.	Authority.
Trap, English	10·1	3·69	Wilkinson.
,, American	10·1	8·9	Prof. Baker.
Granite, Scotch	11·2	8·1	Prof. Unwin.
,, ,, ...	6·0	4·5	Kirkaldy.
,, American... ...	10·0	5·1	Prof. Baker.
,, Bavarian	6·5	4·96	Prof. Bauschinger.
Sandstone, Indian	6·2	1·1	Kirkaldy (for Bombay P.W.D.)
,, Scotch	5·4	·94	Kirkaldy.
,, English ...	4·94	2·2	Unwin.
,, German ..	5·28	1·21	Bauschinger.
,, English ...	4·38	2·34	Royal Commissioners.
,, American ...	6·8	2·24	Prof. Baker.
Marble, Italian	5·4	5·0	Kirkaldy.
,, American	8·9	3·6	Prof. Baker.
,, Tyrolese	5·7	*Nil.*	Prof. Bauschinger.
Limestone, German ...	10·01	2·78	Prof. Bauschinger.
,, American ..	8·9	3·1	Prof. Baker.
,, Portland ...	3·58	1·17	Prof. Unwin.
,, Bath	·65	·54	Royal Commissioners.

The following table gives the ultimate average crushing load in masonry structures :—

Table III.

Crushing Load in Tons per Square Foot on Various Classes of Masonry—

Description.	Authority.			Remarks.
	Rankine.	Trautwine.	Baker.*	
Granite	740	750	300	
Basalt	760	700	*Nil.*	
Limestone, marble	358	625	*Nil.*	* Prof. Baker gives safe load, which is taken here at $\frac{1}{10}$th crushing load.
,, granular ...	273	175	*Nil.*	
,, any ashlar ..	*Nil.*	*Nil.*	250 to 200	
Sandstone, strong	358	350	*Nil.*	
,, ordinary ...	256	200	*Nil.*	
Rubble, masonry	$\frac{4}{10}$ cut stone.	*Nil.*	100 to 150	
Squared Stone (block-in course)	*Nil.*	*Nil.*	150 to 200	

Of the above Professor Baker's are based upon many practical examples. Provided the masonry is the best of its class, his results may be taken as a good practical guide.

Professor Rankine states as follows:—"The resistance of strong sandstone to crushing in a direction parallel to the layers is only $\frac{6}{7}$ of the resistance at right angles to the layers."

As stones generally begin to *crack* at about one-half of their crushing loads, neither stone nor brickwork should be trusted with more than $\frac{1}{6}$th to $\frac{1}{10}$th of their crushing load. As a matter of fact, the greatest pressure on any structure known is about 30 tons per square foot. It cannot, however, be argued that it is unnecessary to investigate the strength of stone, on the ground that its lowest strength, as shown from Table III., is in excess of the greatest load ever brought upon it, because "it is quite impossible in any actual structure to secure a simple condition of crushing stress. Settlement, imperfect bedding, unequal compressibility of different blocks and other causes introduce unforeseen and incalculable straining actions."*

Transverse Strength of Stone.

The cases where stone has to bear pure tension are rare, but it is frequently called upon to resist *transverse* stress, as in lintels, flags over culverts, etc. In this case the value of r, the Modulus of Rupture, *i.e.*, the stress per square inch on the extreme layer which will produce rupture, is shown in the following table :—

TABLE IV.

Value of r, i.e., *Stress on Extreme Layer in lbs. per Square Inch in Stones subjected to Transverse Load.*

Material.	Value of r.	Authority.
Caithness flags...	2,540	Kirkaldy.
"Bluestone" do	2,700	Baker.
Marble, Carrara	1,900	Kirkaldy.
„ American	2,160	Baker.
Slate	5,400	„
Sandstone, American	1,260	„
„ German	2,180 to 625	Bauschinger.
Limestone, American	1,500	Baker.
„ German	3,900 to 1,400	Bauschinger.

* Professor Unwin.

The practice of not allowing much weight to come on lintels is a very sound one. A lintel of stone is a weak form of spanning an opening.

Shearing.

The shearing strength of stone is of value in connection with corbels, etc. The only information obtainable on this subject is from Prof. Bauschinger, who gives the following:—

(1). Granites 100 tons to 34 tons per square foot, say 1,560 to 530 lbs. per square inch.

(2). Limestones 64 tons to 40 tons per square foot, or 1,000 to 625 lbs. per square inch.

(3). Sandstones 68 tons to 12 tons per square foot, or 1,060 to 312 lbs. per square inch.

The shearing in these cases is perpendicular to the bed of the stone. Parallel to the bed it is less, as a rule, but not much less.

Elasticity.

The elasticity of stone is extremely difficult to ascertain. It also varies in various specimens of the same kind of stone, and even from the same quarry. The subject is of considerable importance in connection with many practical operations, *e.g.*, the effect of shock on masonry piers and abutments, of joining new masonry and old, etc.

The following table gives data obtained from the researches of Professor Bauschinger and those of Professor Baker.

TABLE V.

Modulus of Elasticity of Various Stones in lbs. per Square Inch.

Material.	Value of Modulus.	Authority.
Granite, German, average	3,000,000	Prof. Bauschinger.
,, American	9,000,000	Prof. Baker.
Slate	7,000,000	Tredgold, quoted by Baker.
Sandstone, German...	2,300,000	Prof. Bauschinger.
Limestone, Portland	1,530,000	Prof. Baker.
Marble	2,500,000	,,

Weight.

The weight of various classes of stone is a matter about which there is little uncertainty, and this is fortunate, because it is very important.

The weights and specific gravities of various classes of building stones, bricks, sand, concrete, etc., are given in Table VI.

TABLE VI.

Material.	Specific Gravity.	Average Weight of 1 cubic foot in lbs.
Basalt	2·9	181
Bath Stone	2·1	131
Bricks, hard pressed	—	150
,, common	—	125
,, soft inferior	—	100
Brickwork, clamp burnt	—	112
,, kiln	—	130
Cement, Portland, good	3	92 to 100
Granite	2·72	170
Greenstone trap	3	187
Hornblende	3·25	203
Limestone (compact)	2·7	168
Masonry of granite or limestone, well dressed	—	165
,, good rubble in mortar	—	154
,, ,, dry	—	138
,, sandstone	—	120
Mortar (hydraulic lime), 1 to 4 or 1 to 9	1·65	103
Sand	—	90 to 106
Sandstone	2·41	151
Serpentine	2·6	162

Expansion and Contraction Due to Changes of Temperature.

This subject has not received the attention it deserves. It is unquestionably a matter of importance, however; failures have occurred in large masses of masonry due to this cause.

The following extract from a report by Lieutenant (now Major-General) J. C. Ardagh, R.E., on revetments at Newhaven is almost the only information on the subject :—

TABLE VII.*

Co-efficients of Linear Dilatation in Various Substances for a Change of Temperature of 180° *F.*

Brickwork	·0005	
Portland cement	·0010	
Cement mortar, 2 to 1	·0012	
,, concrete	·0014	NOTE.—A fall of temperature of 10° will shorten a stone wall 100′ long by $\frac{1}{16}$″.
Limestone (compact)	·00075	
Marble	·00054	
Granite	·00085	
Slate	·0010	

Absorption of Water.

From experiments made by Professor Unwin, the amount of water absorbed by various classes of stone was on an average as follows :—

Granites about 0·75 per cent., slate 0·28, sandstones from 10·10 to 3·43, and limestones from 12·75 (Bath) to 5·23 (Portland).

The resistance to frost can be inferred from the porosity.

(2). BRICKS.

At meetings of the Royal Institution of British Architects in March and December, 1896, interesting papers were read by Professor Unwin and various other gentlemen reporting results of experiments on the ordinary building bricks used in London under crushing loads. The experiments were on piers 6′ high and 18″ square, two of each kind of brick, one being built in lime mortar (1 to 2), the other in cement (1 to 4). The following was the result :—

* That expansion and contraction, due to changes in temperature, produce the most gigantic stresses on a structure is proved by the observations at the great Vyrnwy Dam in Wales. Here the effect of the sun's heat, even in our temperate climate, is to bend the whole dam perceptibly, causing greater strains than all other causes put together.

TABLE VIII.

Crushing Resistance of Brickwork in Tons per Square Foot.

Description of Brick.	Mortar.		Cement.		Average Strength of Single Brick.
	Began to fail.	Collapsed.	Began to fail.	Collapsed.	
London stocks...	4·18	10·41	7·22	16·03	84·27
Gault...	5·0	21·82	6·98	17·98	182·2
,, ...	6·16	22·03	7·08	17·51	—
Leicester Red	15·20	29·93	17·87	67·36	382·1
,, ,, ...	16·11	31·55	21·82	49·54	—
Staffordshire Blue	22·43	69·22	29·45	84·47	701·1
,, ,, ...	21·42	79·39	16·91	61·14	—

The above give the results at the end of three months.

An examination of the above table shows the important practical fact that the tests of materials taken by themselves do not vary in the same ratio as structures built of these materials. A pier of Staffordshire bricks built in Portland cement mortar is stronger than the same pier built in Leicester red bricks, and the same mortar in the proportion of 701 to 382, and the disproportion is still more variable when the mortar varies. A second series of experiments was tested after 10 months. Stock bricks in mortar commenced to fail at 5 tons per square foot and fell at 10·4 tons. Those in cement cracked at between 7 and 8 tons and collapsed at 14·34 tons. Gault bricks in mortar began to crack at 10·47 tons, crushed at 33·71 tons, and suddenly burst to pieces at 48·10 tons. Gault in cement began to crack at 21·39 tons, crushed at 41·16, collapsed at 51·05. Leicester reds in mortar cracked at 29·28, failed at 86·39. Staffordshire in mortar 74·3. In cement 53·23 cracked; failed at 108·25. Leicester in dry sand failed at 15. The results were affected by form of piers—a half brick under crushing being 18½ per cent. weaker than a whole brick. Professor Unwin considers differences of results were due to the varying kinds of sand used in mortar. Frogs in bricks diminish their resistance to crushing. Hence we see that for practical purposes the testing of materials

by themselves give us only approximate, though, of course, by no means unimportant, imformation.

The following experiments made at Watertown Arsenal, Massachusetts, are reported by Professor Baker. The piers on which the experiments were made were 12″ square, from 1′ 4″ to 10′ high, the mortar was 14½ months old, and the strength of the brick in itself about 15,000 lbs. per square inch, or nearly 970 tons per square foot.

TABLE IX.

Strength of Brick Masonry in lbs. per Square Inch compared with that of the Brick by itself and of the Mortar.

Composition of Mortar.	No. of Experiments.	Ultimate Strength of Pier.	Strength of Mortar 6″ Cubes Crushed.	Strength of Pier in Terms of Strength of the Brick.			Strength of Pier in Terms of Strength of Mortar.
				Min.	Max.	Mean.	
1 lime, 3 sand	15	1,508	124	·06	·18	·10	12
2 mortar (1 lime, 3 sand) 1 Rosendale cement ...	1	1,646	133	—	—	·11	9
2 mortar, 1 Portland cement...	1	1,411	192	—	—	·09	7
1 Rosendale cement, 2 sand	1	1,972	162	—	—	·13	12
1 Portland cement, 2 sand	8	2,544	545	·10	·27	·17	4·7
Clear Rosendale	—	—	521	—	—	—	—
Clear Portland cement...	1	2,375	3,483	—	—	·16	0·7

The piers began to show cracks at one-half to two-thirds their ultimate strength.

From this and similar experiments Professor Baker concludes that "the strength of brick masonry is mainly dependent upon the strength of the mortar." He further states "reasonably good brick laid in good lime mortar should be safe under a pressure of 20 tons per square foot, and the best brick in good Portland cement mortar should be safe under 30 tons per square foot."

Transverse Strength.

Brick masonry is sometimes exposed to transverse stress, as in the case of walls exposed to the overturning action of the wind. This

strength theoretically depends on the adhesion of the mortar to the brick, and on the cohesion of the particles of mortar together; practically, however, the former is the determining factor. This will be discussed later when dealing with the strength of mortars.

As regards shearing there are no satisfactory tests.

Absorption.

Bricks are very absorbent, but vary in different descriptions. Staffordshire blue bricks absorb from 2 to 6 per cent., while Malms absorb about 20 per cent., and common stocks, etc., occupy an intermediate place, absorbing about 8 to 12 per cent.

Hence the great importance of having them wet during building, especially with cement mortar, an importance which increases with the dryness of the climate.

Expansion and Contraction.

The coefficients of linear dilatation for brickwork are about ·0005 for a change of temperature of 100° F.

(3). Mortar.

Intimately connected with brickwork are the properties of mortar. Here the field of investigation has been as extensive as the experiments in finished masonry have been meagre. Here again the experiments have tended rather in the direction of ascertaining the properties of material—lime or cement—which forms the basis of the mortar than of the actual compound itself as used in works.

It would appear therefore better in specifying tests for a particular lime or cement to lay down such tests as will exhibit the behaviour of the article when in combination with the sand, etc., which it is proposed to use with it in actual construction. For it is obviously of little use to have a cement of the best quality with great tensile strength, if the sand it is used with is of an inferior description. A test of a cement by itself is no doubt useful in manifesting the quality of the article itself, but it tells us nothing on which we can base any reliance as to the material as a component part of a structure. Again, it must be remembered that a cement which is strongest neat is not always strongest when mixed with sand.

Quality of Sand.

Experiments have proved beyond a doubt that newly crushed rock gives better results than the best natural sand. In French

tests of mortar, a standard sand is obtained by crushing quartzite; in Germany, specially washed and sifted sand is prepared and sold in Government laboratories. The sand should pass through a mesh of 400 and be rejected by a mesh of 900 to the square inch. In the absence of crushed rock, sharp pit sand carefully washed and sifted should be used for testing.

Lime versus Cement Mortar.

Lime mortar is more plastic, more easy to work, often cheaper, and therefore *pro tanto* better than cement. On the other hand, cement is usually stronger, and is more homogeneous (being an artificially manufactured compound) than natural lime. With care in manipulation hydraulic lime may often be applied in such a way that the strength of the resulting mortar falls little below that made from good Portland cement and the same proportion of sand. This has been conclusively proved by an exhaustive series of experiments made by Mr. G. F. Deacon, M.Inst.C.E. It would appear, therefore, that in the future greater attention will be paid to the proper manipulation of hydraulic lime mortars in masonry, rather than trusting implicitly to the more expensive Portland cement.

Compressive Strength of Mortar.

The following table gives some of the results obtained by various engineers of the compressive and tensile strength of mortar in lbs. per square inch:—

TABLE X.

Strength of Mortars in lbs. per Square Inch.

Description.	Compression.	Tension.	Age.	Authority.
Hydraulic lime, 1 to 1½ sand*.	774	*Nil.*	6 to 11 months	Mr. W. J. B. Clerke.
,,	982	,,	12 to 23 ,,	,,
,,	1,461	,,	Above 23 months	,,
Hydraulic lime (blue lias), 1 to 2½ sand*, slaked and pounded	1,774	161	After 6 months	Mr. G. F. Deacon.
,,	2,892	320	,, 12 ,,	,,
Hydraulic (blue lias) lime, 1 to 3½ sand, ground & pounded	635	103	2 months	,,
,,	2,660	245	6 ,,	,,
Portland cement, 1 to 3 sand.	*Nil.*	80 125	1 week	Committee American Society C.E.
,,	,,	100 200	4 ,,	,,
,,	,,	200 350	1 year	,,
,,	2,270	227	28 days	Baker (from German specification).
,,	*Nil.*	95	1 week	Trautwine.
,,	,,	130	1 month	,,
,,	,,	198	6 months	,,
,,	,,	257	1 year	,,
,,	,,	691†	9 years	Average of 6 briquettes tested by the writer.

* Hydrate of lime, *i.e.*, after slaking.

† The remarkable results here obtained were corroborated by a series of independent tests on briquettes made at the same time and under similar conditions.

A caution, however, is necessary. The strength of the lime mortar above quoted from Mr. Deacon's experiments was the result of exceptionally careful treatment. Ordinary lime mortar may give very poor results if treated in the way it usually is. Mr. Burnell gives only 14 lbs. per square inch in tension and 42 lbs. in compression for ordinary mortar; German engineers allow 1·26 kilogrammes per square centimètre, *i.e.*, 19 lbs. per square inch. Even with Portland cement, the quality and purity, the size and texture of the surface of the sand grains will very largely affect the result.

These uncertainties only point to the conclusion that in any given case the engineer should test his material in the condition in which it is proposed actually to use it.

Adhesion of Mortar.

Experiments have shown that on good ordinary bricks a mortar of Portland cement and sand (1 to 2) has in 28 days an adhesion of 15 to 30 lbs. to the square inch. On glazed surfaces such as Staffordshire blue bricks, or tiles, etc., the cement mortar has comparatively feeble adhesion. Mr. Mann has stated (*Proceedings, Institution of C.E.*, Vol. LXXI.) that Portland cement in 28 days has an adhesion of 80 lbs. with slate, 60 lbs. with Portland stone, and 97 lbs. with granite chiselled. Mr. Mann found that the adhesive strength was from $\frac{1}{8}$ to $\frac{1}{9}$ the tensile strength. In experiments made by Professor Böhme at Berlin it was found that the adhesion of mortar (1 cement to 3 or 4 sand) to common bricks was $\frac{1}{10}$ of the tensile strength. When the cement was neat or mixed with less sand the adhesion was $\frac{1}{6}$ to $\frac{1}{8}$ the tensile strength.

In France experiments have been made by M. Felix de Walgue on bricks of various kinds, and with various classes of mortar. His results are shown on Table XI. Each experiment was repeated four times, and from the mean of the results the figures in the table are obtained. The hard smooth machine-made bricks give much better results than the clamp-burnt bricks, a fact which is contrary to usual ideas.

TABLE XI.—*Table of Adhesion of Cement Mortar to Brickwork.*

MORTAR.		BRICKS.															
		Hand-made Clamp-burnt Stocks.			Hand-made Clamp-burnt Facing Bricks.			Hand-made Clamp-burnt Clinkers, Place Bricks, etc.			Machine-made Re-pressed Hard-burnt Bricks.			Machine-made Facing Bricks.		
Cement.	Sand.	Adherence in lbs. per square inch at			Adherence in lbs. per square inch at			Adherence in lbs. per square inch at			Adherence in lbs. per square inch at			Adherence in lbs. per square inch at		
		8½ Days.	15 Days.	44 Days.	8½ Days.	15 Days.	44 Days.	8½ Days.	15 Days.	44 Days.	8½ Days.	15 Days.	44 Days.	8½ Days	15 Days.	44 Days.
Slow-setting Portland, 1 part	½ Part	17·7	54·5	37·1	17·1	26·3	36·0	14·2	35·2	32·7	36·0	34·6	36·3	34·7	21·3	16·2
	1 ,,	8·2	35·3	42·7	4·5	20·9	20·4	15·3	25·9	...	23·4	27·5	42·7	12·8	16·7	15·2
	2 ,,	10·9	17·8	27·0	7·5	11·2	18·4	10·5	14·2	19·2	13·5	29·7	25·7	11·9	13·4	15·3
Quick-setting Portland, 1 part	½ ,,	16·7	54·3	68·7	13·3	32·8	43·0	19·6	50·0	54·4	49·9	44·2	91·7 above	13·4	17·1	20·4
	1 ,,	11·5	44·4	35·5	11·0	18·1	23·1	14·2	33·6	38·9	32·3	43·8	106·7	9·9	12·8	23·6
	2 ,,	11·0	19·2	21·2	4·5	18·1	24·2	4·9	26·1	37·5	20·3	31·3	33·9	3·3	10·7	13·7
Slow-setting slag cement, 1 part.	½ ,,	7·5	42·0	38·8	10·4	14·0	29·2	14·0	28·6	33·3	20·6	34·1	45·9	10·8	16·7	19·6
	1 ,,	10·4	37·0	29·6	7·3	18·1	27·0	23·0	33·4	26·9	30·0	32·7	45·3	10·3	18·6	13·0
	2 ,,	7·1	19·2	23·4	2·9	18·1	11·8	4·5	26·1	20·1	18·1	19·5	29·2	8·6	10·6	8·7
Medium-setting slag cement, 1 part.	½ ,,	9·7	39·6	39·1	15·5	28·4	29·9	26·3	38·0	47·1	17·9	33·4	41·7	9·4	23·6	16·3
	1 ,,	10·9	33·8	26·6	5·7	12·9	20·3	19·2	25·0	26·7	19·8	26·5	32·0	9·8	16·0	27·5
	2 ,,	4·9	28·9	22·4	8·2	18·3	22·0	10·5	17·8	13·7	21·7	22·0	25·9	6·3	14·4	8·0
Slow-setting Portland, 1 part	½ ,,	20·9	37·7	31·0	16·0	32·4	36·0	17·7	38·4	38·4	18·6	46·9	42·3	11·1	19·0	20·3
	1 ,,	7·3	24·7	19·9	10·3	15·6	21·7	17·8	19·0	21·3	22·8	23·9	28·4	16·3	14·9	21·3
	2 ,,	7·3	11·9	18·9	6·9	12·9	13·3	12·5	15·0	18·8	10·1	8·4	16·7	11·3	6·1	9·4

From these data it is evident that in estimating the transvers strength of brickwork we have to consider the adhesive strength, a that is undoubtedly the weakest link in the chain. This adhesio will be usually greater with a rough surface such as chiselled granite than with a smooth glazed brick, and hence, for any masonry where transverse strength comes into play, the latter should be avoided. The examination of fractured work almost invariably points to the fact that failure has occurred in adhesion.

(4). Concrete.

The same principles which apply to the strength of mortar appl with even greater force to concrete. Variable as the differe qualities of mortar are, they vary even less than the differences i concrete. Here more than in any other work is supervision an care in manipulation necessary. The result in its resistance t crushing or cross breaking is enormously affected by the quality the ingredients, and the manipulation.

One or two broad principles will, however, be evident from th foregoing. One is that the surface of the material forming th aggregate should invariably be rough, so as to secure the greate adhesive strength. Concrete made with rounded pebbles, therefor should not be used for any work where any reliance has to b placed on its strength.

Again, to secure the greatest possible adhesion the interstic should be completely filled with cementitious matter. Hence th concrete should be made, not deluged with quantities of wate after the common slovenly fashion, but with just the lea amount of water necessary to produce chemical action of th cement, and then *rammed* as tightly as possible. The settin action of the cement will probably not begin till the ramming well finished.

Concrete weighs 130 to 160 lbs. per cubic foot dry. If ramme concrete will take 15—20 per cent. more aggregate than unramme It is therefore more expensive, which is, of course, one reason wh contractors prefer to put in the slush which too often goes by th name of concrete.

As far as compressive strength of concrete is concerned, th following summary of results of Portland cement concrete cub made by Mr. G. F. Deacon in the construction of the Vyrnwy Da will be useful :—

Age of block in months.	Strength in tons per square foot.
32 —36	Over 180
28½—29½	„ 162
17 —25½	„ 159
2 — 5	„ 102
1 — 2	„ 114

Mr. Trautwine gives for Portland cement concrete (1 to 5)—

12 months.	100 tons mean.
9 „	85 „ „
6 „	65 „ „
3 „	40 „ „
1 „	15 „ „

The following table gives further information regarding the compressive strength of concrete after about a year :—

Table XII.

Comprehensive Strength of Concrete in Tons per Square Foot.

Lime or Cement.	Proportion to Lime or Cement to Sand and Gravel.			
	1 to 6.	1 to 8.	1 to 10.	1 to 12.
Grey Lime	10·2	4·6	5·2	—
„ Selenitic	18·5	7·6	8·1	—
Lias Lime	11·4	11·1	11·5	—
„ Selenitic	17·2	19·6	10·2	—
Aberthaw Lime, do.	34·1	21·8	15·4	—
Portland Cement...	100·7	76·4	53·5	37·1

Transverse Strength.

The following table gives the transverse strength of various concrete beams as proved by experiment :—

TABLE XIII.

Transverse Strength of Concrete.

No.	Composition.			Age in days.	Modulus of Rupture in lbs. per square inch.	Remark
	Portland Cement.	Sand.	Aggregate.			
1	1	—	1 Coke breeze	7	672	
2	1	—	2 Crushed brick	6	195	
3	1	0	5 Shingle	139	330	A compa of the r of these e
4	1	1	5 ,,	,,	247	ments, e ally Nos.
5	1	2	5 ,,	,,	159	and 9, that not
6	1	3	5 ,,	,,	99	the amou
7	1	2	2 Broken stone, 1½″ 4 ,, ,, 3½″	90	174	cementi material, the gradi
8	1	2	6 Gravel	,,	156	of the si
9	1	2	6 Broken stone, 1½″	,,	136	the aggr so as to fi interstice
10	1	0	9 Shingle	95	142	of import
11	1	1	8 ,,	,,	132	
12	1	2	7 ,,	,,	105	

For coefficients of linear dilatation due to changes of temper see Table VII. The importance of this cannot be over-estim The subject is frequently neglected, and yet there is no doubt the expansion and contraction of large masses is a very fre cause of failure.

B.—TIMBER.

Timber is the material which, more than any other, is us the military engineer, whether in field operations, or in the struction of barracks and other military buildings. It is ther most essential that a knowledge of its properties shoul thoroughly studied. Unfortunately, many of the textbooks been content to accept the data framed by eminent engineers i early years of this century, data which have evidently been on experiments on small homogeneous specimens, but which a applicable to the ordinary material obtained in practice.

The strength and elasticity of timber are much more variable than similar properties in any other material used by the engineer. They are affected by soil, climate, seasoning and various other circumstances. Hence irregularities in the same class of timber, and even in scantlings cut from various parts of the same log, must be expected. A few of these variations have, however, been ascertained by experiment, and are here noted.

(1). Timber felled in winter has greater compressive and tensile strength than that which is felled in summer (northern latitudes).

(2). "Pieces cut near the heart of the tree are much weaker than pieces cut near the periphery" (Unwin).

(3). Timber which is thoroughly seasoned is stronger than green wood. Structures, however, should be designed for the strength of green timber.

(4). Of all defects in timber, *knots* are the most serious in diminishing strength, *even if those knots are tight.* **The old idea that sound and tight knots are not injurious to timber in compression has been proved to be quite a fallacy.**

(5). Top logs are not so strong as butt logs, provided the latter have sound timber.

(6). The coefficient of elasticity varies with the strength and the density.

Compressive Strength.

Tests of the crushing strength of timber are much less difficult than tensile tests. They also "vary less for one species of timber than any other kind; hence if only one kind of test be made, it would seem that a compressive test will furnish the most reliable comparative results."*

Of various published experiments those of Professor Lanza, of the Massachusetts Institute of Technology, appear to be the most exhaustive.† A few of his conclusions are here quoted.

(1). Wooden columns should not be tapered, and should be square rather than round. (The case of endogenous timber is, of course, not considered).

(2). As wood is very weak to resist crushing across the grain, iron caps should be used for pillars in preference to wooden capping pieces.

* Committee appointed by the American International Association of Railway Superintendents of Bridges and Buildings in 1895.

† Lanza's *Applied Mechanics*, pp. 498 *et seq.*

(3). Any cause which operates to distribute the pressure on the ends unevenly, or force the resultant out of the centre, is a source of weakness.*

Green timber is much weaker than seasoned stuff.

The conclusions of the American Railway Committee of 1895 in connection with compression in timber are somewhat as follows:—

(1). Long timber columns generally fail by lateral deflection when the length is more than 20 times the least cross sectional dimension. In practice, when the length is over 15 times the least diameter, the strut should be treated as a "long" column.

(2). Uneven end bearings and eccentric loading of columns produce more serious disturbances than usually is assumed.

The quality of the timber, the influence of seasoning, time of felling and other circumstances of growth affect the compressive strength.

Tension.

Great difficulty has been experienced in making satisfactory tensile tests of timber, because failure generally occurs by the shearing of the wood along the grain. Hence it is considered that where "timber is used as a tie bar in construction it will give way by some means other than direct tension," usually by some failure of the fastenings. The conclusion we arrive at, therefore, is that while the parts of a timber structure which have to bear crushing, cross-breaking and shearing, should be calculated, the size of the members subject to pure tension should rather be estimated from consideration of the least scantling necessary to give secure joints and fastenings, rather than from the tensile resistance of the solid piece.

Transverse Strength of Timber.

The value of the "Modulus of Rupture" or ultimate resistance per square inch exerted by the extreme fibres of a beam under transverse stress is much less (in timber), according to all experiments on the subject, than the resistance to pure tension. This is important, for it proves that although a timber may have a tensile resistance against a direct pull of, say, 10,000 lbs. per square inch, may fail by cross breaking, *i.e.*, the extreme fibres subjected to tensile stress will tear apart, when the resistance per square inch

* This subject will be further examined in Chapter IX.

possibly only 5,000. This fact, proved frequently by experiment, shows that it is unsafe to base calculations for transverse strength on the results of pure tensile experiments.

Again, it has been proved that experiments made on large timbers, say 12″ × 12″ scantling, give results of intensity far below those of small timbers. Professor Warren recommends that 25 per cent. should be deducted from the intensity of resistance to transverse load in the case of timbers 12″ × 12″. Mr. Trautwine suggests that the reduction should be $\frac{1}{4}$ to $\frac{1}{3}$.

For values of transverse strength see Table XIV below.

Shearing Strength.

Timber is much weaker to resist shearing parallel to the fibres than it is across the grain; and inasmuch as the horizontal shearing force in a beam subject to transverse load is equal to the vertical shearing force (as will be subsequently pointed out) failure will in such cases occur along the fibres, if it occur at all. The data for the shearing resistance of timbers are unfortunately meagre, yet the subject is one of too great importance to be ignored.

"Hard" woods are much stronger against shearing than soft or coniferous woods.

Endogenous Trees.

In very few textbooks do we find results of experiments on endogenous trees such as palms, bamboos, and canes. Every engineer who has had to carry out work in tropical countries knows how useful such trees are, and although from the nature of their structure it is not possible to cut them into rectangular scantlings, yet as logs they are often capable of being used even in permanent work. As the layers increase in density from the centre to the circumference, the timber is well adapted for use in columns and piles. It is deficient in rigidity, however, although light and strong.

In British Guiana palm logs are used for bullet-proof stockades.

At Suakin, where palm wood was largely used in the construction of military buildings, it was found that the timber was less inflammable than exogenous wood.

On the other hand, endogenous trees are in certain situations more liable than others to decay and to the attacks of insects.

*Experiments on Timber.**

It is frequently necessary for engineers in out-of-the-way parts of the world to experiment on the timber which they have to use. The following points should be attended to and recorded:—

(1). The age of the tree, as far as can be ascertained.

(2). The locality of growth, the geological nature of the soil, and the aspect.

(3). The part of the tree, whether heart wood or sap wood, butt wood or top wood.

(4). Length of time seasoned, and process, if any, of seasoning.

(5). The deflection of the test piece as well as its breaking weight, scantling, span and method of fixing.

(6). The exact botanical nomenclature. This is most important, and is unfortunately often neglected.

Even with very rough apparatus it is possible to add materially to the sum of scientific knowledge, which in this particular respect is still very slight.†

To use the botanical names may appear pedantic, but it is necessary, to avoid the indefiniteness which follows the use of local terms. If allowed to find a place in a specification these may lead to dispute, and waste of time and money. In a matter where so much is still vague we should at least try and be as accurate as possible.

The following table gives a few results of experiments on timbers of various sorts. It may be mentioned that tropical countries appear to produce the strongest woods, many of which are as yet but little known to commerce or science. There is evidently, therefore, still a vast field for scientific research :—

* In addition to experiments on single pieces, the behaviour of timber in trusses may be experimented on. Experiments of this description are now being carried out at Chatham (1897).

† *Vide* a paper by Mr. F. A. Campbell in *Minutes of Proceedings, Institution of C.E.*, Vol. XCII., p. 413.

TABLE XIV.

Strength and Elasticity in lbs. per Square Inch of Various Timbers used in Engineering, from Experiments by Various Authorities.

Country and Local Name.	Botanical Name.	Average Dry Weight in lbs. per ft. cube.	Tensile Strength.		Transverse Strength.		Compressive Strength of Small Cube.		Shearing Strength.		Modulus of Elasticity.	Authority or Remarks.
			Max.	Min.	Max.	Min.	Max.	Min.	Across Grain.	With Grain.		B=Bauschinger. R=Rankine. P.S=General Percy Smith.
EUROPE GENERALLY.												
Ash	Fraxinus excelsior	48	17,000	4,100	14,000	13,000	9,000	—	—	1,400	1,600,000	R and P.S.
Beech	Fagus sylvatica	48	11,500	4,700	12,000	9,000	9,360	—	—	1,400	1,350,000	R and P.S.
Elm	Ulmus campestris...	35	14,000	6,000	9,700	6,000	10,300	—	—	—	700,000 to 1,340,000	R and P.S.
Spruce fir	Abies excelsa	30	12,000 17,600 B	2,900 3,640 B	12,300 6,400 B	9,900 3,630 B	5,600 5,420 B	3,160 2,260 B	—	600	1,400,000 to 1,800,000	R and P.S.
Scots fir, also red fir, and Northern pine.	Pinus sylvestris	36	14,000 14,300 B	12,000 3,300 B	9,540 R 6,710 B Average	7,100 R 5,040 say 6,000.	6,200 4,600 B	5,375 3,800 B	5,000	500 to 800	1,440,000 to 1,900,000	
Larch	Pinus larix (Linn.)	35	10,000	9,000	10,000 R	5,000 R	—	—	—	970 to 1,700	900,000 to 1,360,000	R
Oak	Quercus pedunculata	55	10,000	19,800	13,600 R	10,000 R	10,000	—	4,300	2,300	1,200,000 to 1,750,000	
N. AMERICA.												
... ...	Quercus alba	61 P.S	10,500	6,740 P.S	7,620 L	5,000 L	6,000 L	2,930 L	4,40 T	(Min.) 752 L	1,292,500 L 1,500,000 T	L=Lanza. T=Trautwine.
pine, also Wey- pine.	Pinus strobus...	32	—	—	7,180 L	3,360 L	—	—	— 2,500 T	267 L 250 to 500 T	— 1,600,000 T	
pine	Pinus Australis*	41 to 58 P.S	9,900 P.S	4,700 P.S	14,000 P.S	—	6,730 P.S	5,000 T	—	—	1,952,000 L	
	,, rigida											
...ed pine	,, resinosa	34	13,400 P.S	2,700 P.S	10,290 P.S	7,100 P.S	4,800 P.S	—	—	—	1,460,000	

* There is much ambiguity about the botanical name of this tree, as, indeed, of all American timbers.

TABLE XIV.—(*continued*).

Strength and Elasticity in lbs. per Square Inch of Various Timbers used in Engineering, from Experiments by Various Authorities.

Country and Local Name.	Botanical Name.	Average Dry Weight in lbs. per ft. cube.	Tensile Strength.		Transverse Strength.		Compressive Strength of Small Cube.		Shearing Strength.		Modulus of Elasticity.	Authority or Remarks.
			Max.	Min.	Max.	Min.	Max.	Min.	Across Grain.	With Grain.		
N. AMERICA—(*cont.*).												
Yellow pine	Pinus mitis	32	10,000 T	2,100 P.S	11,330 L	3,940 L	4,670 L	3,730 L	5,600 T	248 L	1,6.0,000 T	
Spruce	Abies nigra	32	10,000 T	—	8,720 L	2,960 L	—	—	3,250 T	191 L	1,600,000 T	
Oregon pine	Abies Douglasii	40	—	—	—	—	—	—	—	—	—	
INDIA.												
Babul	Acacia arabica	54	16,815 Ro	—	15,900 Ro	15,760	—	—	—	—	1,810,000 Ro	Ro=Roorkee Treatise (adapted).
Bamboo	Bambusa	—	6,300 R	—	17,480	12,340 Ro	—	—	—	—	1,200,000 Ro	
Palmyra palm	Borassus Flabelliformis ...	65	11,898 Ro	—	16,600 Ro	—	—	—	—	—	2,100,000 Ro	
Deodar	Cedrus deodara	40	—	3,500 H	11,800 Ro	2,700 H	—	3,500 H	—	—	1,500, 00 Ro	H=M.W.D. Handbook (adapted to suit notation), with factor of safety of 5.
Cocoanut palm	Cocus nucifera	70	—	9,150	11,000	—	—	—	—	—	1,56),000 Ro	
...on wood	Inga xylocarpa	58	16,657 Ro	—	21,580 Ro	—	—	—	—	—	1,850,900 Ro	
...nir	Pinus longifolia	32	—	3,500	13,200 Ro	2,700 H	—	3,500 H	—	—	1,700,000 Ro	
Sal	Shorea robusta	55	18,243 Ro	11,521 Ro	14,000 Ro	4,950 H	—	6,050 H	—	—	1,850,000 Ro	
Teak	Tectona grandis	45	15,467	14,498 Ro	14,600 Ro	4,230 H	—	6,050 H	—	—	1,720,000 Ro	
Pyinkado	Xylia dolabriformis	66	—	—	14,300	—	11,640	—	—	—	—	Experiments by Forest Department

TABLE XIV.—(*continued*).

Strength and Elasticity in lbs. per Square Inch of Various Timbers used in Engineering, from Experiments by Various Authorities.

Country and Local Name.	Botanical Name.	Average Dry Weight in lbs. per ft. cube.	Tensile Strength.		Transverse Strength.		Compressive Strength of Small Cube.		Shearing Strength.		Modulus of Elasticity.	Authority or Remarks.
			Max.	Min.	Max.	Min.	Max.	Min.	Across Grain	With Grain.		
AUSTRALIA.												
Iron bark	Eucalyptus paniculata ,, leucoxylon ,, crebra ...	73	—	—	18,000	—	11,000	9,500	—	2,000	2,000,000	Professor Warren. Most of the woods are now articles of export.
Spotted gum	,, maculata	53 to 71	—	—	18,000	16,000	10,800	6,000	—	1,500	2,450,000	
Tallow wood	,, microforys ...	71	—	—	10,000	—	8,500	6,000	—	1,100	1,500,000	
Black butt	,, piluiaris	65	—	—	14,000	—	8,500	6,800	—	1,400	2,000,000	
Red gum	,, rostrata	68	—	—	12,000	—	8,600	5,600	—	1,000	1,800,600	
NEW ZEALAND.												
Kauri	Agathis australis	39	4,480	4,543	11,334	8,000	5,000	4,500	—	—	2,880,000	Report by Sir W. B. Perceval, K.C.M.G. The results of experiments are somewhat indefinite.
Rimu	Dacrydium cupressinum ..	37	—	—	—	2,520*	—	—	—	—	—	
Honeysuckle	Knightia excelsa	49	—	—	—	2,800*	—	—	—	—	—	
WEST INDIES. (*Jamaica*).												
Mountain yacca ...	Podocarpus coriacea	33·46	—	—	average	7,200	—	4,350	—	1,200	820,000	From experiments carried out by Professor Unwin, for the Imperial Institute.
St. Ann's yacca ...	,, purdiana	44·67	—	—	11,700	11,300	5,700	5,420	—	775	1,320,000	

* Probably the *safe* transverse strength; the factor of safety, however, is not given.

TABLE XIV.—(*continued*).

Strength and Elasticity in lbs. per Square Inch of Various Timbers used in Engineering, from Experiments by Various Authorities.

Country and Local Name.	Botanical Name.	Average Dry Weight in lbs per ft. cube.	Tensile Strength.		Transverse Strength.		Compressive Strength of Small Cube.		Shearing Strength.		Modulus of Elasticity.	Authority or Remarks.
			Max.	Min.	Max.	Min.	Max.	Min.	Across Grain.	With Grain.		
WEST INDIES—(*cont.*).												
(*Jamaica*)—cont.												
Hog gum	Symphonia globulifera ...	50·1	—	—	11,400	—	6,500	—	—	1,000	1,620,000	From experiments carried out by Professor Unwin for the Imperial Institute
Yellow Sanders ...	Terminalia hilariana	57·29	—	—	10,000	—	9,000	—	—	810	1,610,000	
Mammee Sapota ...	Mammea Americana	59·16	—	—	15,300	—	8,000	—	—	1,220	1,700,000	
Naseberry bullet ...	Sapota sideroxylon	74·27	—	—	20,500	—	9,600	—	—	1,130	2,450,000	
(*Barbados*).												
Bully tree	Swartzia Tomentosa	—	—	—	—	7,000	—	—	—	—	—	C.R.E., Barbados.
Green heart	Nectandra Rodixi...	—	—	—	—	6,600	—	—	—	—	—	,,
White wood	Bignolia leucoxylon	—	—	—	—	3,600	—	—	—	—	—	,,
CEYLON.												
Jak	Artocarpus integrifolia ...	42	—	—	average	11,000	—	—	—	—	1,700,000	C.R.E., Ceylon.
Milila	Vitex altissima	56	—	—	,,	13,800	—	—	—	—	1,360,000	,,
Mendora	Cynometra ramiflora	57	—	—	,,	13,500	—	—	—	—	1,080,000	,,
Halmalila	Berrya ammonilla	48	—	—	,,	15,300	—	—	—	—	650,000	,,
Paloo	Minusops indica	68	—	—	,,	18,800	—	—	—	—	1,090,000	,,

TABLE XIV.—(*continued*).

Strength and Elasticity in lbs. per Square Inch of Various Timbers used in Engineering, from Experiments by Various Authorities.

Country and Local Name.	Botanical Name.	Average Dry Weight in lbs per ft. cube.	Tensile Strength.		Transverse Strength.		Compressive Strength of Small Cube.		Shearing Strength.		Modulus of Elasticity.	Authority or Remarks.
			Max.	Min.	Max.	Min.	Max.	Min.	Across Grain.	With Grain.		
CEYLON—(*cont.*).												
Satin	Chloroxylon Swietenia ...	55	—	—	average	18,600	—	—	—	—	1,750,000	C.R.E., Ceylon.
Palmyra	Borassus Flabelliformis ...	65	—	—	,,	14,600	—	—	—	—	1,880,000	,,
STRAITS SETTLEMENTS.												
Darroo	Apodytes...	—	—	—	,,	9,000	—	—	—	—	—	
Seriah	Hopea	—	—	—	,,	8,700	—	—	—	—	—	
Ballow	?	—	—	—	,,	7,300	—	—	—	—	—	
Miraboo	Afzelia Potembanica	—	—	—	,,	8,100	—	—	—	—	—	
[illegible]issak	Valica?	—	—	—	,,	7,150	—	—	—	—	—	
tanjore	Colophyllum mophyl amum	—	—	—	,,	4.800	—	—	—	--	—	

b.—In many foreign stations, imported timber is used instead of local timber, on account of there being no facilities for felling, cutting into baulks, and seasoning, of the local material which, however, may, in itself, be a superior article.

In the preparation of this table (which is admittedly most incomplete), the following authorities have been consulted :—

(1). For European timbers, Mr. Tredgold, Professor Unwin, and General Percy Smith.

(2). For American timbers, Professor Lanza and Mr. Trautwine. The great difficulty here has been that these authorities use only local and not scientific names. This leads to confusion ; for instance, what is called "yellow pine" in Europe is not known by that name in America, their "yellow pine" being a totally different tree.

(3). For Indian timbers, the Roorkee treatise and the Military Works Handbook. In the latter the factors of safety used appear to be very high.

(4). For Australian timbers, Professor Warren.

(5). For West Indian timbers, Professor Unwin, and reports from C.R.E.'s of foreign stations. The latter, however, give scanty information of the nature here needed.

The compressive strength is supposed to be on "short bars," *i.e.*, those that will not give way by lateral buckling.

For the European builder, of course, the red fir (*pinus sylvestris*) is the most important, and for the Indian engineer the deodar, chir, sal and teak.

Ironbark is the strongest timber so far known. There are, however, certain timbers in tropical forests in Borneo, Madagascar and Central America which may be as strong, but have never been subject to careful investigation.

The following table gives the results of the investigation of the American Railway Committee of 1895 above referred to. Its value is, unfortunately, impaired by local nomenclature :—

TABLE XV.

Average safe allowable Working Unit Stresses in Pounds per Square Inch.

Kind of Timber.	Tension.		Compression.			Transverse Rupture.		Shearing.	
			With Grain.						
	With Grain.	Across Grain.	End bearing.	Columns under 15″ Diameter.	Across Grain.	Extreme Fibre Stress.	Modulus of Elasticity.	With Grain.	Across Grain.
Factor of Safety	10	10	5	5	4	6	2	4	4
White oak	1,000	200	1,400	900	500	1,000	550,000	200	1,000
White pine	700	50	1,100	700	200	700	500,000	100	500
Southern long-leaf or Georgia yellow pine ...	1,200	60	1,600	1,000	350	1,200	850,000	150	1,250
Douglas, Oregon, and Washington fir { yellow fir	1,200	—	1,600	1,200	300	1,100	700,000	150	—
or pine ... ... { red fir ...	1,000	—	—	—	—	800	—	—	—
Northern or short-leaf yellow pine...	900	50	1,200	800	250	1,000	600,000	100	1,000
Red pine	900	50	1,200	800	200	800	600,000	—	—
Norway pine	800	—	1,200	800	200	700	600,000	—	—
Canadian (Ottawa) white pine	1,000	—	—	1,000	—	—	—	100	—
Canadian (Ontario) red pine...	1,000	—	—	1,000	—	800	700,000	100	—
Spruce and Eastern fir	800	50	1,200	800	200	700	600,000	100	750
Hemlock	600	—	—	800	150	600	450,000	100	600
Cypress	600	—	1,200	800	200	800	450,000	—	—
Cedar	800	—	1,200	800	200	800	350,000	—	400
Chestnut	800	—	—	1,000	250	800	500,000	150	400
California redwood	700	—	—	800	200	750	350,000	100	—
California spruce	—	—	—	800	—	800	600,000	—	—

C.—Iron and Steel.

When we pass from timber to consider the properties of iron and steel we leave the region of uncertainty to enter that of very fairly accurate knowledge. Improvements in manufacture, in chemical knowledge, and in testing apparatus have contributed to make this subject, of all those with which an engineer has to deal, the least involved in uncertainty.

Cast Iron.

Formerly cast iron used to be much more largely used in structures than it is now. It is well adapted for compression members, where there is no chance of failure occurring through lateral bending, or buckling, or where there is no necessity to make joints with other members. For pillars or columns of moderate length, therefore, it is suitable, but not for beams or girders under transverse stress. It has several very serious defects: it is liable to hidden flaws, which diminish its strength, it is brittle, it is wanting in ductility, and therefore gives no warning of failure. In addition to these drawbacks, which are serious enough in themselves, there are initial stresses developed frequently in the processes of casting and cooling, due at times to defective arrangements in the foundry, or it may be to the shape of the casting, or the presence in it of iron of different qualities.

Cast iron has been largely used for arched ribs in bridges, but as a material for bridge construction it is not to be recommended. A notable warning against its use in this respect was the case of Rochester bridge, which, built of arched cast-iron ribs, was seriously damaged and rendered almost impassable in February, 1896, by a blow from a barge passing underneath. A failure such as this is sufficient to condemn the material, for any bridge may be called upon to stand blows suddenly applied. Of course, where ornament is desired more than strength, its uses are many.

The ultimate tensile strength of cast iron, as given by Professor Unwin, varies from 15·3 to 6·83 tons per square inch. Mr. Kirkaldy gives from 13·4 to 5·6 tons per square inch. Ten tons may be taken as an average.

The ultimate compressive strength varies, according to the same authorities, from 95 to about 20 tons, the average being about 40 tons where the height is to the width as 1 is to 2.

The coefficient of bending strength is on an average about 18 tons per square inch. This test, being easily applied, is very generally

adopted in practice to test the quality of the iron. Square bars of some specified cross section are cast, placed on supports, and loaded at the centre. The proportions of the section affect the coefficient, and for bars of the same proportions the coefficient is lower as the section is larger. A wide bar gives, generally, a higher, and a deep bar a lower, coefficient. For the ordinary test bar 3 feet span, 2 inches deep and 1 inch wide, good iron ought to carry 30 cwts. Another test commonly specified is that bars 1 inch square, placed on supports 4′ 6″ apart, shall bear a central load of 550 lbs.

The shearing strength of cast iron is about 5 tons per square inch.

It is important in the case of cast-iron columns, pipes, or other similar structures, to specify that they shall be cast vertically.

Wrought Iron.

In wrought iron or steel the determination of the elasticity is of great importance, because one of the advantages of this material is that it will stretch a good deal before giving way if of a soft or ductile quality, but will not stretch much if hard, though having, it may be, great tensile strength. The determination of the yield point of the metal (*i.e.*, that point where there occurs "a large and almost sudden increase of deformation"* in the application of the ordinary tensile test) is a matter of much importance. Beyond the yield point the deformation is plastic, below that point the material tends to regain its original form. "It is obvious that a general plastic yielding of a structure would ruin it for practical purposes, hence the yield point seems to fix a limit of stress independent of that determined from considerations of safety against fracture, which the working stress should not exceed." This yield point is easily determined in testing machines, and by means of autographic apparatus may be recorded. In almost all tables what is given as the elastic limit is the yield point, but the two are not identical.

When a bar or test piece is subjected to tensile stress, the cross section of the piece is at first constant, but when it approaches fracture, there is considerable diminution of the section near the point of rupture, with local extension at that point. The contraction of area of the fractured section should be noted, as well as the load producing rupture, and the elongation in a certain length. As the section is diminishing all through the test, it is evident that the real ultimate intensity of stress is greater than the nominal stress. If

* Professor Unwin.

w is the section when the length is l and w_1, the section when the length is $l+l'$, then $wl = w_1(l+l')$, and the corresponding real stress at the section is $p_1 = \frac{p}{w_1}$.

The *tensile* strength of wrought iron is, on an average, about 24 tons per square inch. The yield point is at about 15 or 16 tons.

The effect of drawing out iron in the form of wire is to increase very largely its tensile strength. If, however, the wire be annealed, this tensile strength is reduced, but this is more the case with steel wire than iron. Annealing is said to have a beneficial effect in the case of iron that has suffered fatigue. The tenacity of iron wire, unannealed, is from 35 to 40 tons per square inch.

The *compressive* strength of wrought iron is from 16 to 20 tons per square inch. It begins to yield at about 12 tons.

The *shearing* strength of wrought iron is from 17 to 23 tons, average about 20 tons.

The coefficient of linear expansion for 1° F. is ·0000064.

The modulus of elasticity is, on an average, 29,000,000 lbs. per square inch, or 13,000 tons.

Steel.

The milder forms of steel now used for structures differ but little from wrought iron, but the average strength of steel is much greater than that of iron, and the cost is not now much greater. The steel that is suitable for bridges, roofs, girders, etc., is that which gives tensile resistance of from 24 to 40 tons per square inch with elongation of about 20 per cent. and about 30 per cent. contraction of area. The elongations are measured, usually, on lengths of 8 inches, which is a convenient standard for comparison with foreign results, where the usual length is generally 200 mm. (7·87 inches), and width 30 mm. (1·2 inches). Sometimes a length of 10 inches is adopted by English engineers.

The Admiralty tests for mild steel plates, bars, etc., are:—Strips cut lengthways or crossways to have an ultimate tensile strength of not less than 26 and not more than 32 tons per square inch, with an elongation of 20 per cent. on 8 inches. Strips $1\frac{1}{2}$ inches wide, heated uniformly to a low cherry red, and cooled in water at 82 F., must stand bending in a press to a curve of which the inner radius is $1\frac{1}{2}$ times the thickness of the steel tested.

The steel used in the tension members of the Forth Bridge was specified to have a strength of from 30 to 33 tons, with 20 per cent. elongation, measured on 8 inches. The Board of Trade working

limit for steel in railway bridges is 6 tons in all members, a rule which is open to the disadvantage that some members are subject to fatigue more than others, and therefore some variation in the strength should be made.

The ultimate strength in *tension* of the mild steel suitable for construction may therefore be taken at 30 tons per square inch.

As regards *compressive strength*, it has been found that the resistance to direct thrust is about the same as to tension; experiments made by Mr. Kirkaldy, however, show that the elastic limit was reached at about 16 tons per square inch. For purposes of calculation 6 tons may be taken; this will give an ample margin of safety.

The *transverse strength* of steel has recently also been the subject of experiment, made by Mr. Kirkaldy on rolled steel joists. The results show great variety and an average modulus of rupture higher than the tensile strength, the difference, however, not being sufficiently great to justify any higher value being taken in practice. It is noteworthy that the variations mentioned did not appear to be in any way affected by the dimensions of the specimen, and the inference is that they were wholly due to variations in the chemical constituents of the steel. The joists were manufactured by Messrs. Dorman, Long & Co. by the Siemens-Martin process. Mr. Kirkaldy observes, with reference to these tests, that it appears unnecessary to use so mild a steel, and that much harder steel can be used with uniformity and safety. Messrs. Dorman, Long & Co., in their published section book, guarantee 28 to 32 tons tensile stress, 20 per cent. elongation in a test length of 8 inches; they suggest three different factors of safety, viz.—3, 4, and 5, or 10·6, 8, and 6·4 tons per square inch. The first is suggested for permanent dead loads not liable to variation, the second for ordinary live loads applied gradually, and the third for live loads rapidly applied, and for machinery in motion.

The modulus of elasticity of steel is about 30,000,000 lbs. per square inch.

The shearing strength of mild steel is approximately equal to $\frac{3}{4}$ of the tensile strength, say 24 tons per square inch.

Steel wire is relatively much stronger than steel bars. That used for the construction of steel wire ropes has a tensile strength of about 90 tons.

The coefficient of linear expansion in steel is ·0000115 for 1° Centigrade.

Tests of Iron and Steel.

The following are the tests proposed by a committe appointed by the American Institution of C.E. for considering the subject of testing iron and steel:—

For cast iron the committee recommend the following transverse test:—That a specimen 2 inches by 1 inch section, and 27 inches long, resting flat on dull knife edge supports 24 inches apart with an applied concentrated weight of 2,000 lbs., should have a centre deflection before breaking of at least 0·3 inch, and indicate a good quality of grey iron.

As regards wrought iron, the committee recommend that the yield point should be noted, as well as the contraction at the point of rupture. A minimum gauged length of 8 inches is recommended for the general practice of testing specimens, with a distance of not less than 12 inches between shoulders of the specimen, or between the jaws of the testing machine. Under these conditions, however, it is understood that the gauged length shall in no case be less than four times the greatest diameter of the test piece, and that the minimum thickness shall be $\frac{1}{4}$ inch; also that the sectional area of the test piece shall not be less than $\frac{1}{2}$ square inch. In obtaining the final elongation the curves of reduction on each side of the point of fracture should be included in the included length. In testing it is recommended that the load be applied continuously, and the rate as nearly as possible of 4,000 to 5,000 lbs. per square inch per minute below the yield point, and of 7,000 to 8,000 lbs. above the yield point. The ordinary bending test should be made rapidly with pressure, rather than with the blows of a hammer, as the results are more comparable. The nicked bending test is not recommended. The committee give the following test loads:—For bars, tension rods, compression members, angles, or any other shape, the yield point should be 26,000 lbs., with an ultimate strength of 48,000 to 50,000 lbs., the elongation in 8 inches being from 20 per cent. in bars to 12 per cent. in compression members, and reduction from 25 per cent. in bars to 16 per cent. in compression members. For web plates the above figures would be 26,000 lbs., 46,000 lbs., 8 per cent., and 12 per cent. respectively.

With rolled steel, a quenching test is recommended, by which a specimen heated to cherry red is to be quenched in water at 82° F., and then tested as usual. A mean ultimate resistance should be specified for each grade of steel, and the ultimate resis-

ance of the test specimen should not vary more than 4,000 lbs. per square inch either way from the resistance specified. The following are recommended for the various grades of steel:—

Ultimate Strength per Square Inch.

Low steel	...	...	60,000 lbs. + or – 4,000.
Medium do.	...	...	65,000 lbs. + or – 4,000.
High do.	...	...	70,000 lbs. + or – 4,000.

Yield point = 55 per cent. of the ultimate resistance of specimen.

Per cent. elongation in 8 inches = $\left\{\frac{1{,}500{,}000.}{\text{ultimate.}}\right.$

Per cent. reduction of area = $\left\{\frac{2{,}800{,}000.}{\text{ultimate.}}\right.$

Rivet steel, when heated to a low cherry red and quenched in water at 82° F. must bend to close contact without sign of fracture. Specimens of low steel, when treated and tested in the same manner, must stand bending 180 degrees to a curve whose inner radius = the thickness of the specimen without sign of fracture. Specimens of medium steel, as cut from bars or plates, and without quenching, must stand bending 180 degrees to an inner radius of $1\frac{1}{2}$ times the thickness of the specimen without sign of fracture; while those of high steel, also without quenching, must stand bending 180 degrees to a radius of twice the specimen without sign of fracture.

In steel castings, the tension test is recommended, with the following requirements:—Yield point, 35,000, and ultimate strength 65,000 lbs. per square inch; elongation in 8 inches = 15 per cent., and contraction = 25 per cent.

* * * * *

The following table of the strength of iron and steel is given, as a summary of the facts in the foregoing notes:—

TABLE XVI.

Strength of Iron and Steel.

Material.	Weight, lbs. per inch cube.	Resistance (ultimate), Tons per Square Inch.				Coefficient of Linear Expansion.	Elasticity, Tons per Square Inch.		Remarks.
		Compression.	Tension.	Transverse.	Shearing.		Limit.	Modulus.	
Cast Iron Bars... ...	·25	40 (av.)	10 (av.)	18	5	·0000066	6·85	8,200	
Mallable Cast Iron...	·25	—	16	30	—	·0000066	4·4	—	
Wrought Iron	·26	18 (av.)	24	20	20	·0000064	15	13,000	
Mild Steel, suitable for construction ...	·288	30 (av.)	30	30	24	·0000064	16	13,400	
Steel Wire	—	30 to 50 when annealed, but 50 to 90 unannealed (Trautwine)	—	—	—	—	—	—	

Each 12° to 15° of heat produce in wrought iron an expansion equal to that produced by a tension of 1 ton per square inch.

CHAPTER IV.

External Influences Tending to Produce Rupture in Beams.

General Principles. — Cantilever Moments. — Supported Beams, various Cases. — Graphic Representations of Moments of Flexure, various Cases.—Beams Strained over one or more Supports.—Examples.

HAVING discussed the general principles, which apply, more or less, to all structural problems, and having investigated the properties of the various materials employed, we are now in a position to examine how these principles and practical facts should be applied to particular cases.

The case of beams under transverse stress is first investigated, because the results of such investigation apply to many other structural problems, which will subsequently be examined.

Beams under transverse loads, acting at right angles to their longitudinal axes, are divided into two classes, viz.: *Cantilevers*, where one end of the beam is fixed, the other being free, and *Beams*, where both ends of the structure are either supported or rigidly fixed.

The method of investigation in both cases is similar.

The effect of the loads is to produce certain *moments*, which excite in the particles of the beam stresses which produce within the beam other moments, tending to resist rupture. The moments which tend to cause the beam to break are called *Moments of Flexure*, sometimes *bending moments*, while the moments produced in the beam are called *Moments of Resistance*. The former are also called *active*, and the latter *passive*. Obviously, for equilibrium, the passive forces or moments must be equal to the active at all parts of the structure.

In this chapter we shall only consider the active or external influences on beams and cantilevers.

The Moment of Flexure at any point in a beam is usually represented by the symbol M_b and the maximum M_f by the symbol M_{ff}; the problem then is usually to find the value of the M_f at various

points along the beam, and especially to find the value and position of M_g.

The M_f will vary with the following:—

1. The *amount* of the load on the beam.
2. The *mode of applying* the load.
3. The *place* of its application.
4. The *length* of the beam.
5. The *method of fixing*, or *supporting*, the ends.

It is entirely independent of the material of the beam, and of the form of its cross section. It is important for beginners to bear these facts in mind, for some are apt to suppose that the load alone tends to break a beam, whereas the load, although important, is only one out of several factors which have to be considered.

Cantilevers.

In cantilevers the rule for finding the M_f at any section is to take *the sum of the moments produced at that section by the forces on the unsupported side of that section.*

The M_g in a cantilever will always be at the point of fixing.

CASE 1.—*Cantilever loaded at the outer end, or at any point, with a single concentrated load.*

In *Fig.* 40 AB is a cantilever, built into a wall at B, and loaded at A with a single load W. If x be the distance of any section CC_1 from A, it is evident that the M_f at that point $= Wx$, and the M_f at $B = Wl$, or in other words, the M_f varies directly with the distance from W.

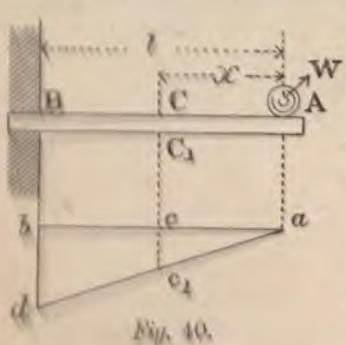


Fig. 40.

The values of the Moments of Flexure at various points may therefore be represented graphically by the values of the ordinates drawn from the base *ab* of a triangle, of a length representing the length of the cantilever, the perpendicular *bd* being of a length corresponding to the M_g. Thus in *Fig.* 40 let *ab* represent the length AB, and *bd* represent on any scale of inch-tons, etc., the value of the M_g at B, then, if we join *ad*, the triangle will graphically

represent the moments in AB, so that if we wish to know the M_f at any point C, we find its corresponding position *c* on *ab*, and, drawing cc_1 parallel to *bd*, find, from the measurement of that line on the same scale of moments as *bd*, the value of the M_f without calculation.

In such a simple case as this the graphic method presents no great advantage, as compared with direct calculation, but in many cases, such as beams with many loads of varying nature, the graphic methods of ascertaining the M_f are invaluable, and in any case are useful, in that they present to the eye a diagram, readily apprehended, of the variations in the moments at different points. Graphic diagrams have other uses, which, however, need not here be described.

It is well to note that the ordinates in diagrams of M_f represent compound expressions, such as inch-tons or foot-pounds, whereas the ordinates in some other graphic diagrams represent simple expressions, as lbs. or tons. This subject will be alluded to when treating of shearing stress diagrams, in due course.

CASE 2.—*Cantilever with several loads at certain points.*

In *Fig.* 41 let W_1, W_2, W_3 be the loads at distances L_1, L_2, L_3 from the end of the cantilever. The M_{ff}, at the point of fixing, will be the sum of the moments, viz.—$W_1 \times L_1 + W_2L_2 + W_3L_3$. The M_f at any point CC′ distant *x* from B will be found by taking the sum of the moments produced at C by the weights between C and A.

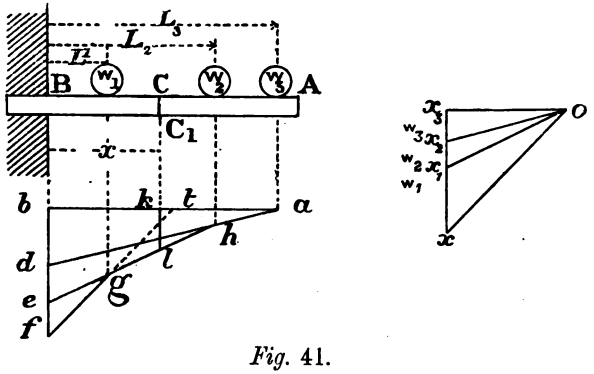


Fig. 41.

Graphically, the moments can be represented as follows:—

Let *ab* represent the cantilever, draw *bd* at right angles to *ab* to represent on any scale of moments the moment of W_3 at B, then produce *bd* to *f*, making *de* equal to the moment of W_2 at B, and, similarly, *ef* to represent the moment of W_1 at the same point, and

on the same scale of moments. Then if h be a point on ad vertical below W_2, and g a point on eh vertically below W_1, the polygon fgh will represent the diagram of moments for the whole beam, and the M_f at any point C may be found by drawing kl from k, which corresponds in position with C, to l on the polygon. If the line fg be produced to meet ab at t, then t will mark the centre of gravity of the weights W_1, W_2, W_3.

In this graphic solution some calculation has to be done, in order to fix the first ordinates bd, dc, and ef. A purely graphic solution of the problem may be arrived at, however, by using a polar diagram. If the length ab represent on any convenient scale the length of the beam, a line x_3O may be taken to represent on the same scale any integral number, such as 10. On the vertical line xx_3 lay off on a scale of weights $W_1 = xx_1$, $W_2 = x_1x_2$, $W_3 = x_2x_3$. Join the polar point O with x, x_1, x_2, x_3, and from a draw ad parallel to Ox_2, meeting the perpendicular from b at d. From the point h in ad, corresponding to the position of W_2, draw he parallel to ox_1, and in like manner draw gf parallel to ox. This construction gives the polygon of moments, and the value of any ordinate can be ascertained by measuring it and applying a scale one-tenth that of the vertical scale adopted.

For example, let the length of the cantilever be 5 feet, and the weights be 1, 2, and 3 tons placed at the end, and at 2 feet interval respectively along the length AB. To find the M_f at 2 feet from the point of fixing, $M_f = 1$ ton $\times (60 - 24)$ inches $+ 2$ tons $\times (36 - 24$ inches $= 60$ inch-tons, $M_B = 1 \times 60 + 2 \times 36 + 3 \times 12 = 168$ inch-tons.

CASE 3.—*Cantilever with uniform load throughout its length.*

In this case let w be the load per unit of length, so that $wl = W$ total weight. The M_f at any point c distant x from the end is $wx \times \frac{1}{2}x = \frac{1}{2}wx^2$.

As the value of this M_f varies with the square of the distance, the successive moments may be represented by the ordinates of a parabola, of which the extremity of the cantilever is the vertex. Make bd, therefore, in *Fig.* 42 on any scale of moments, equal to $\frac{wl^2}{2} = \frac{W}{2}$ *i.e.*, the M_B where x is a maximum $= l$. From d draw a parabola passing through d and touching ab at a. The M_f at any point can be found by measuring the corresponding ordinate to the curve. A tangent to the curve at d will mark the position of the c.g. at t.

If the cantilever be loaded only partially with a distributed load as in *Fig.* 43, the simplest method is to consider the load as collected

at its c.g. and lay off the M_{ff} *bd* as for a single load W at *c*. The ordinate *ee'* will then $= \frac{wa^2}{2}$. Draw the parabolic curve *e'a*, *de'a* will then be the curve of moments.

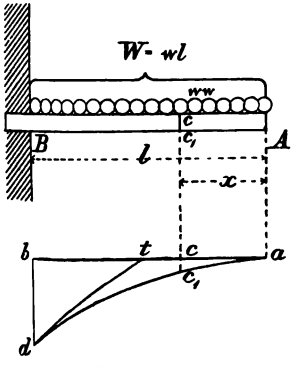


Fig. 42.

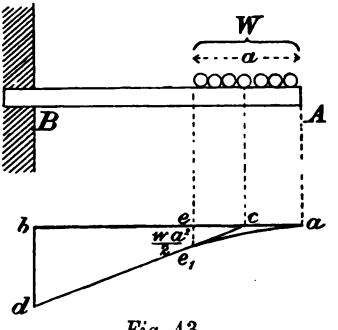


Fig. 43.

CASE 4.—*Cantilever loaded both at outer end and uniformly throughout.* This is simply Cases 1 and 3 combined. All that is necessary is to add the parabola to the triangle, as in *Fig.* 44.

Sometimes this case is represented by the triangle being on one side of the beam line and the parabola on the other, but it is better to fix a general rule to draw the diagrams always on that side of the beam line on which compression would come, *i.e.*, below in cantilevers, where the curve is convex upwards, and above in supported beams, where the curve is sagging or concave.

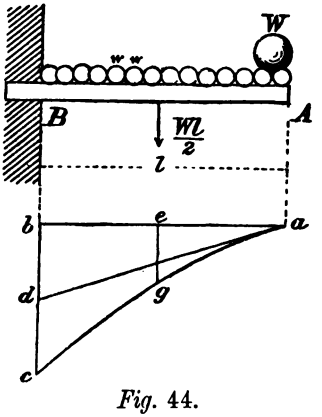


Fig. 44.

Before leaving the subject of cantilevers, it may be well to note that, for equilibrium, there must be, beyond the point of fixing, a force which will produce a moment = that on the cantilever itself. Thus if, instead of the cantilever being built into a wall, it is kept in equilibrium by an anchorage, as at C, *Fig.* 45, the force acting at C must be such that its moment at A is equal to the moment of

W × AB. This may be represented graphically as shown in the lower part of the figure, or it may be represented by considering the moment W × the distance BC. Let *ce* represent this moment, on the same scale as before, then this moment will manifestly be equal to the moment at C of the reaction R, and the triangle *ecd* will be the diagram of moments for the reaction R, the ordinates *hg*, etc., being measured upwards from the line *ed*, and the difference in the ordinates *fh*, *hg* being *fg*, the actual moment at the section *tt'*.

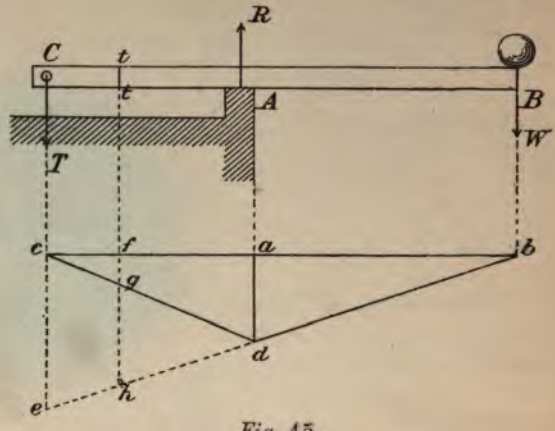


Fig. 45.

Supported Beams.

The rule for finding the M_t at any section in supported beams is:—*Take the algebraic sum of the moments due to the reaction of either support and to any loads between that support and the section in question.*

The first thing to be done is to find out the reactions. In symmetrically loaded beams each reaction is evidently = half the total weight, but in other cases the reaction is obtained by taking moments round the opposite support. The reason for this will at once be evident if we consider the supporting abutment replaced by a chain lifting the end of the beam. For equilibrium, the tension on such a chain will be found by the law of the lever, the fulcrum being the opposite abutment.

The reactions can also be found graphically, as follows:—

In *Fig.* 46 let the beam AB carry any load W. At B lay off on any convenient scale B*a* = W. Join A*a*, cutting the vertical C*b*,

through the c.g. of W, in *b*. Draw *bc* parallel to AB. Then Bc will represent the reaction at B, and *ca* will = reaction at A.

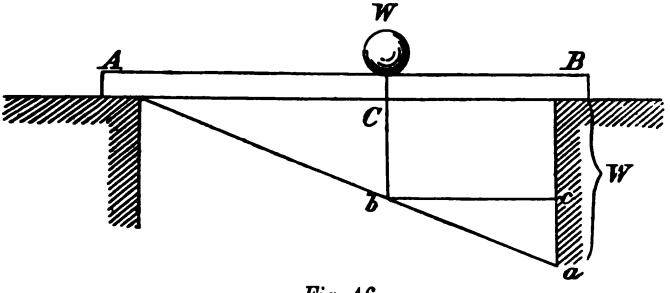


Fig. 46.

Where there are several loads, lay them off one below the other, as in *Fig.* 47, where $Bc = W_3$, $cb = W_2$, $ba = W_1$. From A draw A*a*, A*b*, A*c*, and also draw verticals through the centres of gravity of W_1, W_2, W_3. From *d*, where the vertical through W_1 cuts A*a*, draw *de* parallel to A*b*, and from *e* draw *ef* parallel to A*c*, and from *f*, *fg* parallel to AB. This line will cut B*a* at a point *g* such that B*g* and *ga* are the required reactions.

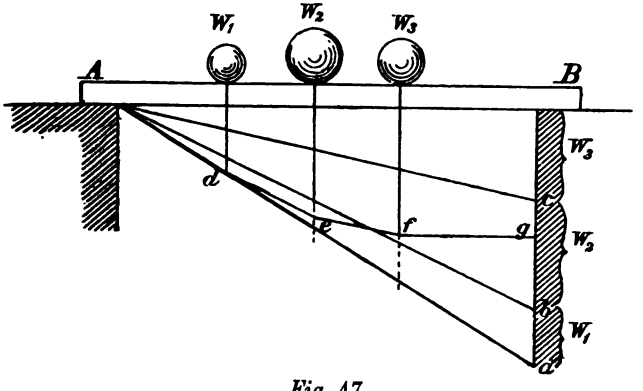


Fig. 47.

CASE 5.—*Beam supported at both ends, and loaded with a concentrated load.*

This case is exactly the converse of the cantilever shown in *Fig.* 45

Let BA (*Fig.* 48) be the beam with the load W at C. Let AB = l, BC = a.

Then reaction at B = $W\frac{l-a}{l}$, and reaction at A = $W\frac{a}{l}$.

Hence M_f at C $= Wa \times \frac{l-a}{l} = Wa - \frac{Wa^2}{l}$ .....(1).

This will be the maximum M_f when the load is at C, for if we consider the M_f at any other section, as F, distant x from B, then the M_f at x will be

$$= Wx\,\frac{(l-a)}{l} - W(x-a) = -\frac{Wax}{l} + Wa \quad .....(2).$$

That is, considering the algebraic sum of the moments between B and F. If we consider the moments between A and F we get the same result, *i.e.*, $= Wa - \frac{Wax}{l}$. So that it does not matter which of the two supports we consider in finding the M_f at any point. Comparing (1) with (2) we see that x is greater than a,

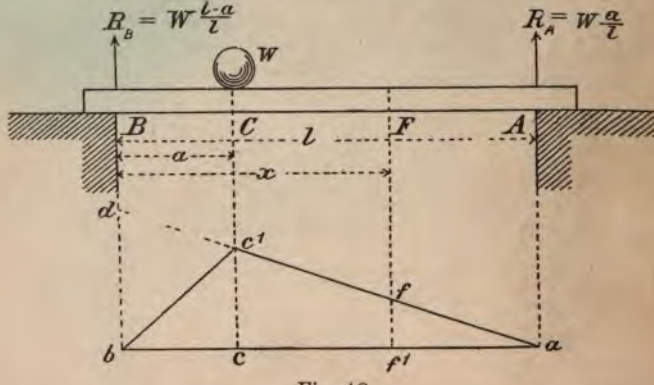


Fig. 48.

Wax is greater than Wa^2, so that for all positions of F between A and C the M_f gets less in proportion as F approaches A, being 0 at the supports, and a maximum under the load. The same is true for all positions of F between B and C.

The case may be graphically represented as shown in the lower part of the figure, where $ac'b$ is the diagram of moments, cc' representing on any scale the moment $Wa\,\frac{(l-a)}{l}$, and the moment at any point F being found from the ordinate ff'.

As in *Fig.* 45, *bd* represents the moment of the reaction at A round B, and also the moment of W at B, these moments being equal. The triangle *bc'd* represents the moments which would be produced if BC were a cantilever, loaded with W at C.

CASE 6.—*Beam supported at both ends and loaded with a single load at the centre* (*Fig.* 49).

This case, which is a common and therefore important one, follows from the preceding one. The reactions here are $=\frac{W}{2}$, and the M_f at the centre will be $\frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$. This may at once be seen from the diagram, where *bd* represents the moment of the reaction at A round $B = \frac{Wl}{2}$. Then, completing the triangle as in the preceding case, we see that as $cc' = \frac{1}{2}bd$, the $M_{ff} = \frac{Wl}{4}$.

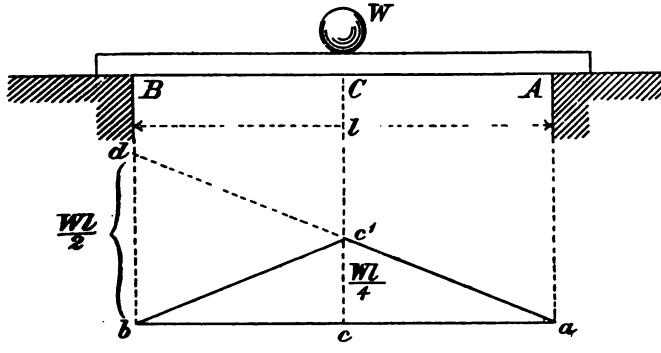


Fig. 49.

CASE 7.—*Beam supported and loaded with any number of loads at certain points.*

The reactions having been found as before described, the M_f at any point is found by taking the sum of the moments between that point and either support, Graphically, there are various ways of representing this case. One way is shown in *Fig.* 50. Here *ab* is the beam line, *cc'*, *dd'*, *ee'* represent the moments produced by W_1, W_2, W_3, considered by themselves alone. Then make $c'k = cf + cg$. Similarly make $d'm = do + dp$, and $e'n = eq + er$. Then *bkmna* is the polygon of moments.

Another method, which is purely graphic, is to use a polar diagram. In *Fig.* 51 lay off on any scale $x_1x_2 = W_1$, $x_2x_3 = W_2$, $x_3x_4 = W_3$, and take any pole point O, joining Ox_1, Ox_2, Ox_3, Ox_4. Next, drawing perpendiculars through *b*, *c*, *d*, *e*, and *a*, take any point *b'* in the perpendicular through *b* and from *b'* draw *b'c'* parallel to Ox_1, from *c'* draw *c'd'* parallel to Ox_2, then *d'e'* and *e'a'* in like manner. Join *b'a'* and from O draw OR parallel to *b'a'*. Then x_1R and Rx_4 will give the reactions, a graphic method of ascertaining them similar to that described above, on p. 67.

To find the Moments of Flexure, draw from R RO' horizontal, equal to some integral number on the horizontal scale of the beam line. Join $O'x_1$, $O'x_2$, $O'x_3$, $O'x_4$, and draw bc_2, c_2d_2, d_2e_2, e_2a_2 parallel to $O'x_1$, $O'x_2$, $O'x_3$, $O'x_4$. This will give the polygon of moments, and the amount can be ascertained by measuring the ordinate on the scale for the weights and multiplying the result by the polar distance O'R.

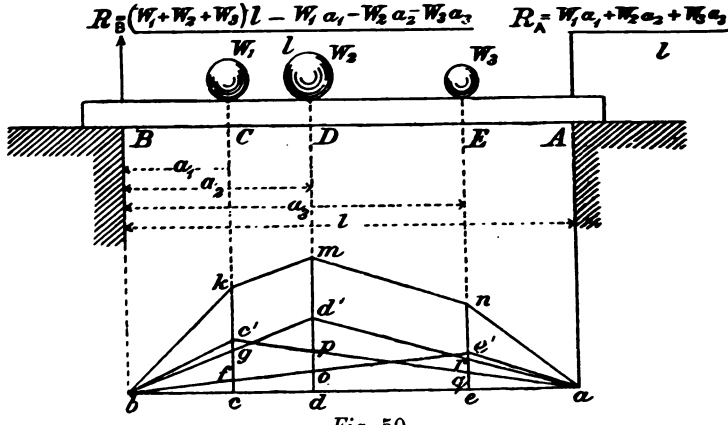


Fig. 50.

An important expression for the value of the M_f from the polygon *b'c'd'e'a'* and the polar diagram $Ox_1x_2x_3x_4$ can be obtained, which is of much use in the investigation of arched ribs and suspension bridges. Draw Ox_1 from O, at right angles to the load line $x_1x_2x_3x_4$, and let the scale for length be the same as that for forces, then to find the M_f at any point, say C, we know that it

$$= R_B \times BC = x_1R \times BC.$$

But from similar triangles

$$x_1R : OR :: c'g : b'g \text{ and } x_1R : Ox :: c'g : BC.$$

Hence M_f at C, which

$$= x_1R \times BC = Ox \times c'g,$$

or the M_f at any point is equal to the portion of the vertical through th

point, enclosed in the polygon $b'c'd'e'a'$, and multiplied by the perpendicular distance OX.

This method is known as Culmann's method, from Professor Culmann, of Zurich, who introduced it.

For example, let the beam be 10 feet long, with weights of 100, 200, and 300 lbs. at 3, 6, and 8 feet from the left abutment. Find the M_f under each of the weights and at the centre of the beam.

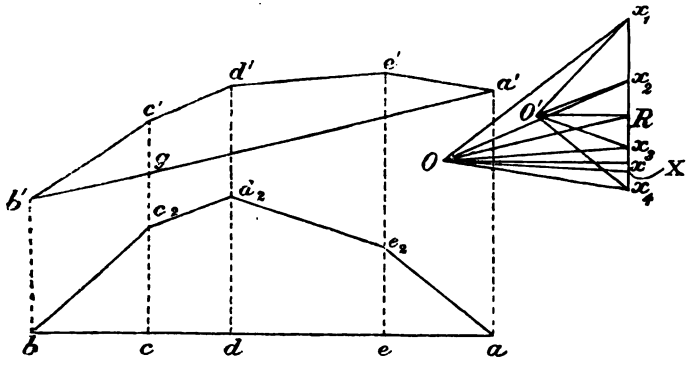


Fig. 51.

Here $w_1 = 100$, $w_2 = 200$, $w_3 = 300$. Draw these to scale, take any point O, and proceeding as above described, we find the reactions to be 210 lbs. and 390 lbs. respectively. Then drawing $O'R = \frac{1}{48}AB$, and joining the points x_1, x_2, x_3, x_4 with O', and thus obtaining the polygon of moments $bc_2d_2e_2a$, we measure each ordinate on the scale of weights and multiply by the constant $10 \times \frac{1}{4}$. The M_f at c measures 250, which multiplied by the constant = 627 inch-lbs. The calculated moment = 630, and is no doubt more accurate. The other moments are 960, 780, and 850 inch-lbs., under d, e, and the centre respectively.

If the diagram of moments is correctly drawn, the last line, drawn through e_2 parallel to $O'x_4$, should pass exactly through a. This is a useful check on the accuracy of the work.

Still another graphic solution of this problem is possible. The principle is that previously illustrated in *Figs.* 45 and 49, *i.e.*, where the moment produced by the reaction at one support is considered at the other and the moments of the weights are subtracted.

Thus, in the case already considered, in *Fig.* 52 let ba represent the beam. Lay off from b the ordinate bx = the moment of the

reaction at a multiplied by the distance ab, and xy = weight at $e \times$ distance eb, yz = weight at $d \times db$, and zb = weight at $c \times cb$. Then join ax, from the point e' where the vertical from e cuts ax, draw $e'y$, cutting the vertical through d at d'. From d' draw $d'z$, cutting the vertical through c at c'. Then $bc'd'e'a$ is the diagram of moments, and it will be observed that it is the same as that for a cantilever, as in *Fig.* 41, only the ordinates in this case are subtracted from, instead of being added to the first ordinate.

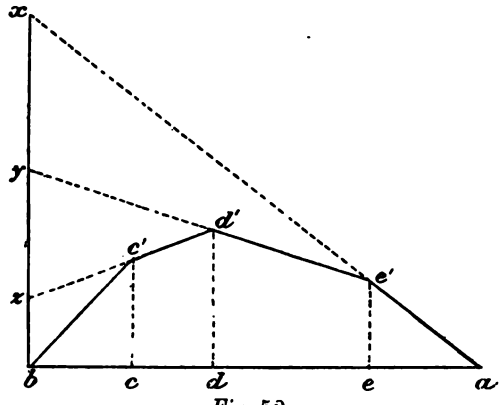


Fig. 52.

These methods will repay careful study. Any graphic method is useful, but each of the above has its own merits.

CASE 8.—*Beam supported at both ends and loaded with two equal weights at equal distances from the centre.*

This is a particular form of the previous case. In *Fig.* 53 let AB

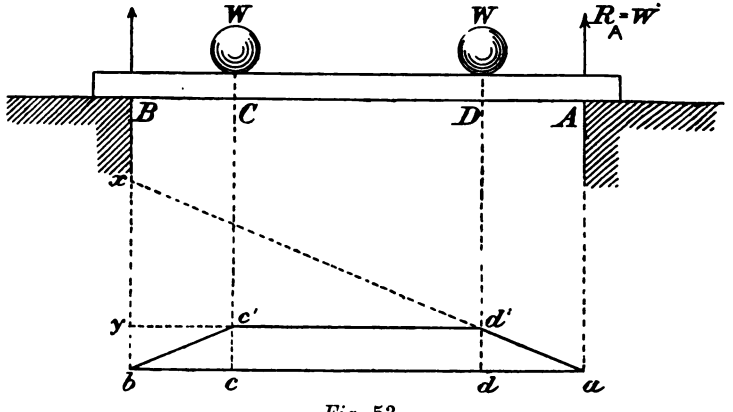


Fig. 53.

represent the beam loaded with weights W, W, at C and D equi-

distant from the centre. Then the M_f at any point between C and D distant x from B is

$$Wx - W(x-a) = Wa.$$

In other words, the M_f between the weights is a constant. Graphically, this can be shown by the same method as the last graphical solution in the last case. Let *ba* be the beam line, with weights at *c* and *d*. Draw bx = reaction at A (this is obviously = W) × *ab* and make $xy = W \times bd$, and $yb = W \times bc$. Join *ax*, *d'y*, *bc'*, then *bc'd'a* is the diagram of moments, in which *c'd'* is evidently parallel to *cd*, indicating that the M_f there is a constant.

This case is common; the cross girders in a railway bridge, the main girders, sometimes, in a framed floor, the "distance pieces" in a double-lock spar bridge, are familiar instances.

CASE 9.—*Beam supported, and loaded throughout its length with a uniformly distributed load.*

This is the commonest case of all, and therefore the most important. It should be most carefully studied.

In *Fig.* 54 let AB represent the beam, and let the unit of load be *w*, with length *l*, so that if W be the total weight, $W = wl$. Then the reaction at each support will evidently $= \frac{1}{2}W = \frac{1}{2}wl$.

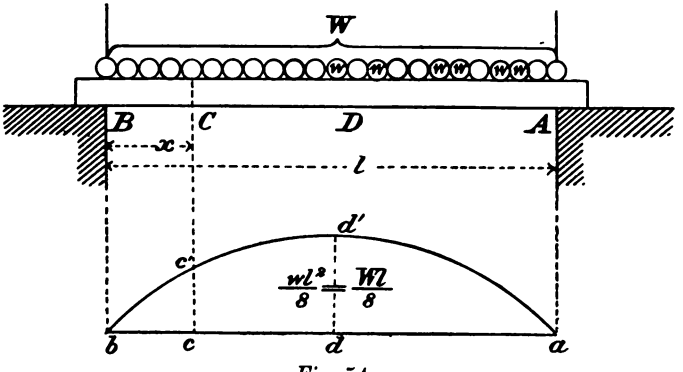


Fig. 54.

To find the M_f at any point C distant x from B we have the reaction × the distance BC or x – the weight on BC × ½BC.

Or $$M_f = \frac{1}{2}wlx - \frac{1}{2}wx^2 = \frac{1}{2}wx\,(l-x) \ldots\ldots\ldots\ldots \ldots(a).$$

The M_{ff} is evidently at the centre where $x = \frac{1}{2}l$.

Substituting this value, we have

$$M_{ff} = \frac{wl^2}{8} = \frac{Wl}{8} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta).$$

Now equation (α) is that of a parabola where the origin of the co-ordinates is at a point on the curve distant $\frac{1}{2}l$ from the axis; hence to draw the diagram of moments we lay off from the centre $dd' = \frac{1}{8}Wl$, and draw a parabola passing through $bd'a$.

For further proof see Appendix I. to this chapter.

If the load, instead of being uniformly distributed throughout the whole length, be equally divided among any number of cross pieces fixed at uniform intervals (as often happens), the same parabolic curve will enable us to ascertain the moments, for all that is necessary is to draw ordinates from the fixed points to the curve, as in *Fig.* 55. For this reason we consider the rafters in a roof, the joists in a floor, etc., as uniformly loaded beams. In a bridge there is no object in dividing the span into very wide sections or panels, as there is no great reduction in the M_f.

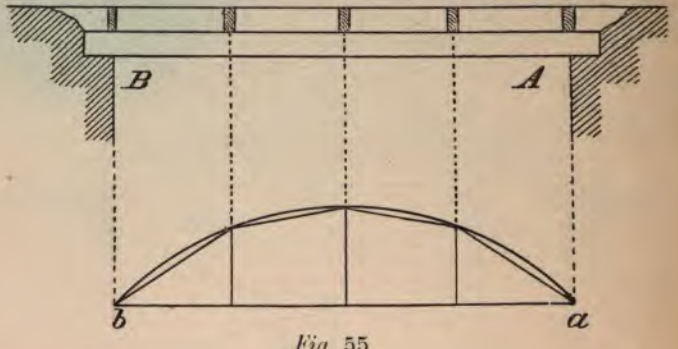


Fig. 55.

CASE 10.—*Beam supported, and loaded with a distributed load over a part of its length.*

To follow the purely analytical method in this case would involve the calculus; it is, therefore, given in Appendix II. to this chapter.

Graphically, let the beam BA (*Fig.* 56) be partially loaded with the distributed load from C to D. First consider the weight of the whole load as if it were concentrated at E, the central point of CD. Draw the diagram for such a load as in Case 5. Then consider the length CD as if it were a beam supported at C and D, and with its

uniform load. Draw the diagram *cfd* for such a load as in Case 9. From *c* and *d* draw *cc'* and *dd'* meeting the first diagram at *c'* and *d'*. Join *c'd'*, and draw ordinates above *c'd'* equal to the ordinates of *cfd*. Then *bc'f'd'a* is the required diagram of moments.

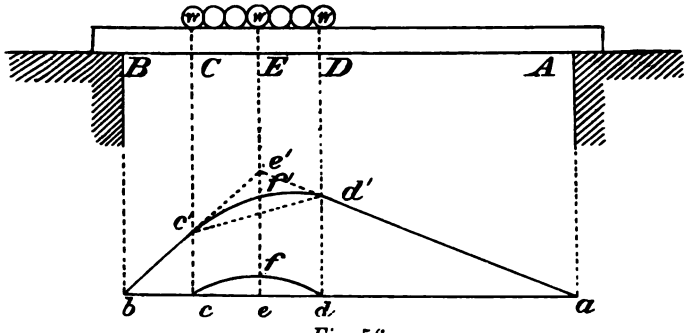


Fig. 56.

These are the most ordinary cases of beams; any others are compounds of two or more of the above, as when a beam is uniformly loaded, and has one or more weights at fixed points as well. In such compound cases the sum of the moments gives the required compound moment.

It will now be evident how the variations in the M_f do not depend on the weight alone, but on the other considerations mentioned at the beginning of this chapter. It is useful to notice the different values in the M_{ff} for a given length and a given load in the various cases. If the length be l and the weight be W, then—

(1). In a cantilever, the weight being all at the end, $M_{ff} = Wl$.

(2). In a cantilever, the load being distributed, $M_{ff} = \frac{1}{2} Wl$.

(3). In a beam, the load being central and concentrated, $M_{ff} = \frac{1}{4} Wl$.

(4). In a beam, the load being uniformly distributed, $M_{ff} = \frac{1}{8} Wl$.

It will subsequently be seen, when treating of fixed beams,* that in a beam fixed at both ends and loaded in the middle, the M_{ff} is at the centre *and* at the points of fixing, and $= \frac{1}{8} Wl$. Also in the same beam with the load uniformly distributed, the M_{ff} is *at the points of fixing* and $= \frac{1}{12} Wl$, the M_f at the centre being $= \frac{1}{24} Wl$.

From the above we see:—(1). That in all cases a beam or cantilever is twice as strong with a distributed load as it is with a concentrated

* In Part II.

load in the centre. (2). A cantilever requires four times the strength of a supported beam of the same length and similar load. (3). Fixing a beam increases its strength, as compared with a supported beam, by 2 if the load is concentrated in the centre, and by 1·5 if uniformly loaded.

Before leaving the subject of the Moments of Flexure of beams, it may be well here to add something about beams which are strained over one or two piers, and loaded in various ways.*

CASE 11.—*Beam hinged at one end and strained over a pier, loaded at the end and at the middle.*

Let the beam BA be hinged at B and strained over the pier C, loaded with the weight W at A and W_1 at D between B and C (*Fig.* 57).

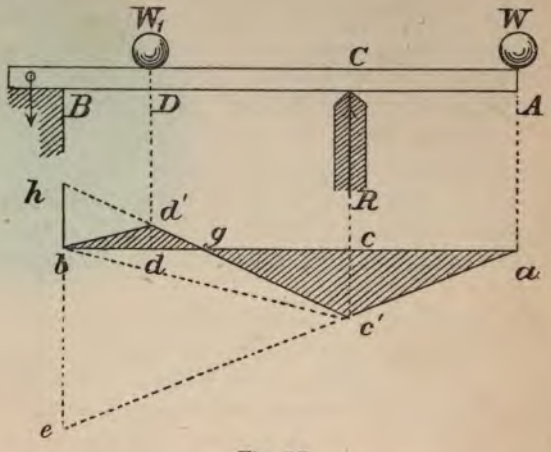


Fig. 57.

Now if there were no weight except that at A, the case would be that of Case 5 above reversed, and the diagram of moments would be *ac'b* where *be* is the moment $W \times AB = R \times BC$. This is simply the same as *Fig.* 45.

With a weight W_1 at D, the diagram for the span BC will be the triangle *bd'c'*, and the bending moments for the whole beam will be the shaded portion *bd'gc'a*. In this case the value of the reaction R must be obtained from the equation

$$R \times BC = W \times AB + W_1 \times DB.$$

In the diagram the ordinate *be* represents $W \times AB$, *bh* represents

* These are taken chiefly from Mr. Claxton Fidler's *Practical Treatise on Bridge Construction.*

$W_1 \times BD$. Hence *eh* represents $R \times BC$. Joining *hc'* and *bd'*, we get the diagram of moments. As shown in *Fig.* 57, the point *d'* is above *ba*, indicating that the beam is concave upwards or sagging from *b* to *g*, and convex or hogging from *g* to *a*. It might have happened that the weight W_1 was small compared with W, in which case the point *d* might have come, as in *Fig.* 58, below the line, indicating that the curvature of the beam was hogging throughout. If the force at the hinged end B is upward, the curvature will be hogging, and *vice versâ*.

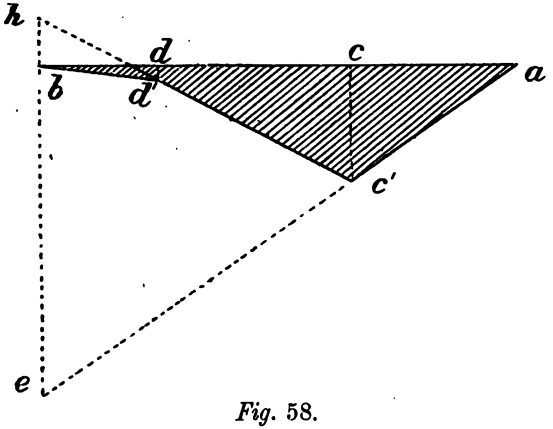


Fig. 58.

The point where the curvature changes from convexity to concavity, as *g* (*Fig.* 57), is called the *point of contra flexure*, or *contrary flexure*. This point is of much importance in the investigation of fixed and continuous beams.

CASE 12.—*Beam strained over two piers.*

In this case let BA (*Fig.* 59) be the beam strained over the piers C and D and loaded with weights at the extremities and at some point E between the piers. If the weights at B and A are equal, and the lengths BC and AD also equal, and if there be no other weight on the beam, then the diagram will be *bc'd'a*, and will be the converse of Case 8 above (*Fig.* 53). When, however, there is an intermediate weight between C and D, the reaction at one or other of the piers must be determined by equating the moments at the other. Thus the reaction at D is found by taking moments at C:—

$$W_1 \times BC = W_3 \times AC - R_2 \times DC + W_2 \times EC.$$

From this we obtain the value of R_2. Without obtaining this value, however, we can draw the diagram of moments by drawing $cc' = W_1 \times BC$, and $cg = W_3 \times AC$, and $c'f = W_2 \times CE$. The shaded figure represents the diagram of moments for the whole beam. $R_2 \times DC = gf$.

The points h and k are the points of contra flexure.

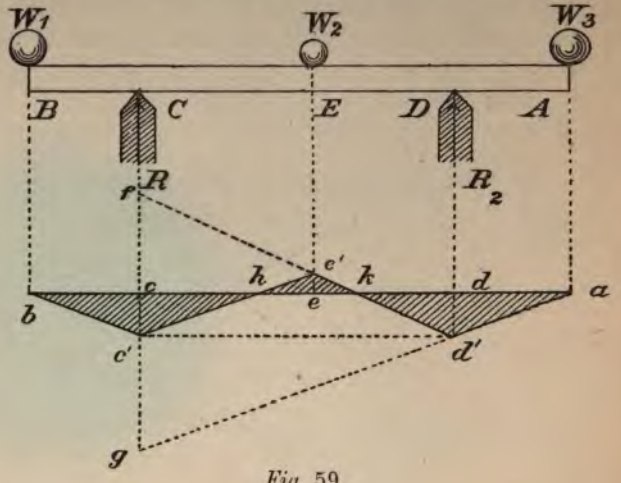


Fig. 59.

As in the former case, the relative value of the weights determines whether the beam is in sagging or hogging curvature, and where the points of contra flexure are situated.

The cases of (1) a beam hinged at one and strained over one pier while loaded throughout with a uniform load, and (2) a beam strained over two piers and loaded with a uniform load, are shown on *Figs.* 60 and 61. No comment is necessary on these cases, as the principles will be quite clear to those who have followed the investigation in the previous cases. In *Fig.* 60 the line *cd* represents the moment of the unknown vertical force at the hinge A. This vertical force can, of course, be ascertained algebraically.

In *Fig.* 61 the diagram can be obtained either by first finding the values of cc' and dd', joining $c'd'$ and drawing on that line the parabola for the load on CD, or by making cg equal to the moment of AD at C, and $c'f = \frac{1}{2}$ the load on CD $\times$ CD, and drawing the parabolic arc by ordinates measured below $d'f$ and passing through c'.

As far as the positions of the points of contra flexure are concerned, this obviously depends on the relative length of AD and C as compared with CD.

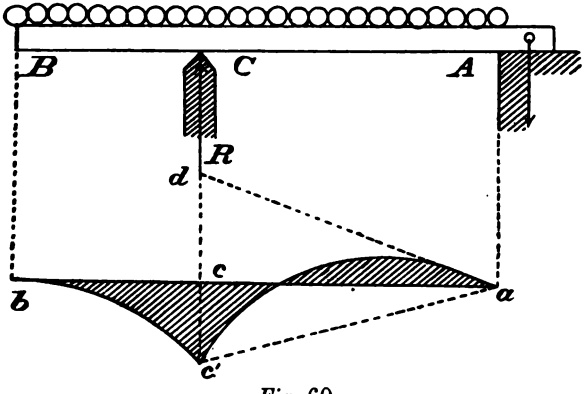


Fig. 60.

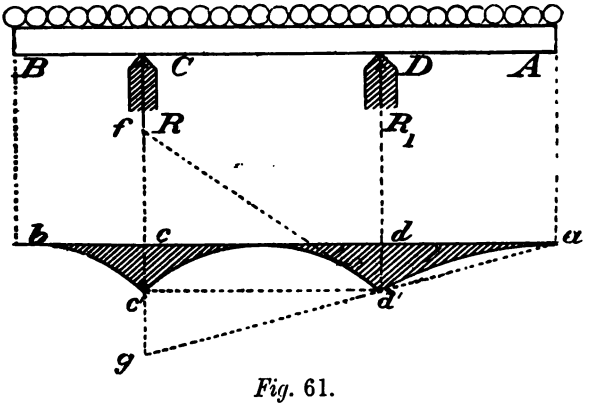


Fig. 61.

A few examples are now given of various cases.

EXAMPLE 1.—*A beam* 10′ *span has a weight of* 200 *lbs.* 3 *feet from one end and* 300 *lbs.* 3 *feet from the other end. Find the reactions.*

In *Fig.* 62— $R_A \times 200 \times 7 + 300 \times 3 \div 10 = 230$ lbs.

$R_B = 300 \times 7 + 200 \times 3 \div 10 = 270$ lbs.

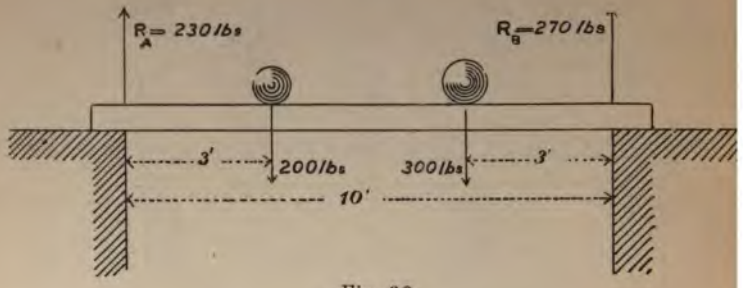


Fig. 62.

EXAMPLE 2.—*A cantilever* 6′ *long may be called upon to bear at its extremity a weight of* 14 *cwts. Its fixed end is built into* 2 *feet of brickwork* 5′ *in width. What must the least height of the brickwork be?*

In *Fig.* 63 AB = 6′, CB = 2′, and let x = the height of the brickwork in feet.

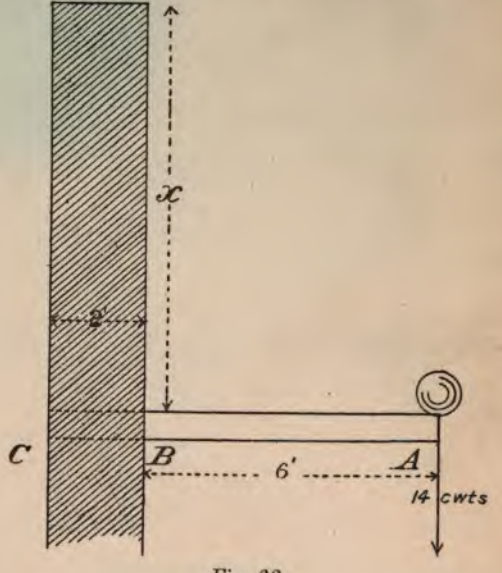


Fig. 63.

Then taking moments round B—

$14 \times 6 = 2 \times 5 \times x \times 1$ (weight of 1 cubic foot of brickwork) $\times 1$ ($\frac{1}{2}$ BC)

$84 = 10x$.

$x = 8{\cdot}4$ feet.

EXAMPLE 3.—*A* 40′ *girder has a uniform load of* 2 *tons per foot run, and a weight of* 8 *tons in the centre. Find the* M_f 8′ *from one abutment.*

From *Fig.* 64 R = 44 tons. Hence the M_f at 8′ from either abutment = 44 × 8 − 16 × 4 = 298 foot-tons.

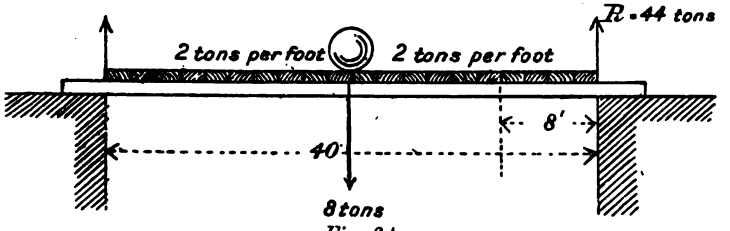


Fig. 64.

EXAMPLE 4.—*A locomotive with* 8 *tons on the leading,* 15 *tons on the driving, and* 8 *tons on the trailing wheel, is standing on a* 60′ *bridge with the trailing wheel* 20′ *from one abutment. The distances of the centres of the axles is* 6′ 6″. *Find graphically the reactions and the diagram of moments.*

In *Fig.* 65 let *ba* represent to scale the bridge, and *m*, *n*, *o* be the

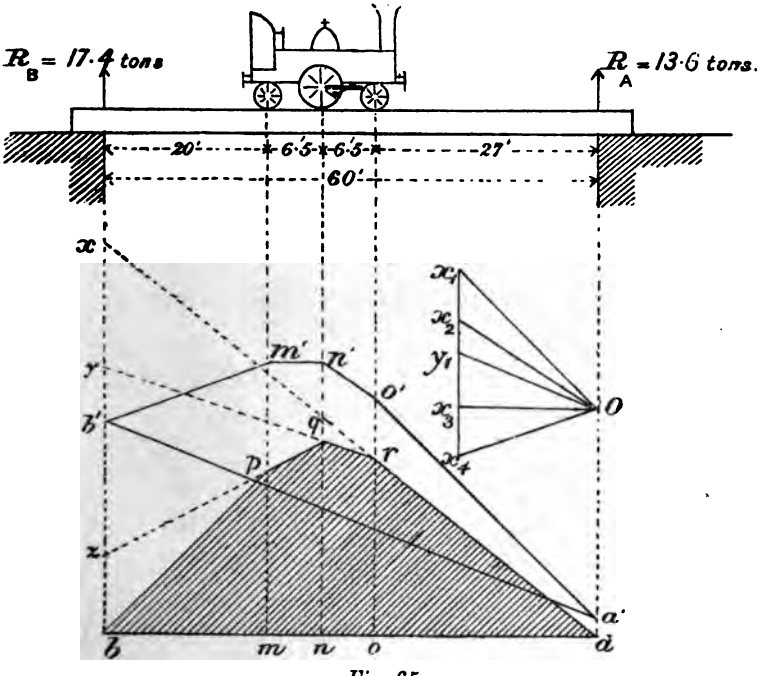


Fig. 65.

positions of the three wheels. On the polar diagram draw x_1x_2, x_2x_3, x_3x_4 representing the weights on these wheels, take any point O and join Ox_1, Ox_2, Ox_3, Ox_4, and, as in *Fig.* 51, draw $b'm'$, $m'n'$, $n'o'$, $o'a'$ parallel to the lines on the polar diagram. Join $b'a'$, and draw through O, Oy_1 parallel to $b'a'$. Then x_1y_1 and y_1x_4 represent the reactions.

We might go on to ascertain the moments as before in a former example, but, for variety, let us take the third method of finding the moments.

The reaction at one abutment a being found to be 13·6 tons, lay off bx = the moment of that reaction at b, *i.e.*, $13{\cdot}6 \times 60 = 816$ foot-tons. Then make xy on the same scale = the moment of the weight at $o \times$ the distance ob, *i.e.*, $8 \times 33 = 264$ foot-tons, and similarly, yz = the weight of $n \times nb$, *i.e.*, $15 \times 26{\cdot}5 = 396$ foot-tons. Join ax, cutting the perpendicular through o in r, join yr, cutting the perpendicular through n at q, and so on. Then $bpqru$ is the diagram.

If the work is correct, then bz will scale 160, *i.e.*, the moment of the weight at m into the distance bm, or 8×20 foot-tons.

By measurement we find the M_{ff} to be 405 foot-tons. By calculation it is 404.

EXAMPLE 5.—*A girder supporting a floor* 24′ *span has* 3 *binders resting on it dividing the span equally into four parts. Each binder carries* 30 *cwt. Find the* M_f *under each.*

This is most easily worked out by calculation, though the graphic method might be used. *Fig.* 66 shows the loads on the girder.

The M_f at the centre is

$$2{\cdot}25 \text{ tons} \times 12 \text{ feet} - 1\tfrac{1}{2} \text{ tons} \times 6 \text{ feet} = 18 \text{ foot-tons.}$$

The M_f at B $= 2{\cdot}25 \text{ tons} \times 6 \text{ feet} = 13{\cdot}5$ foot-tons.

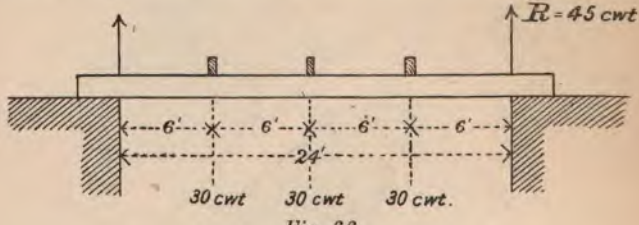


Fig. 66.

In practice it would usually be necessary to reduce these to inch-tons, by multiplying them by 12, so as to compare them with the

moments of resistance, which are usually expressed in inch-tons or inch-lbs.

EXAMPLE 6.—*A beam 20′ span is uniformly loaded for half its distance with 40 lbs. per foot run. Draw diagram of moments and find the M_f in the centre.*

In *Fig.* 67 half the beam is loaded with $10 \times 40 = 400$ lbs., of which the reactions produced at the abutments are 300 at A and 100 at B. Draw Aa = reaction at $B \times BA$, *i.e.*, 2,000 foot-lbs. From C, the centre of the load, draw Cc, meeting Ba at c. Then, if the whole load had been concentrated at C, AcB would be the diagram of moments.

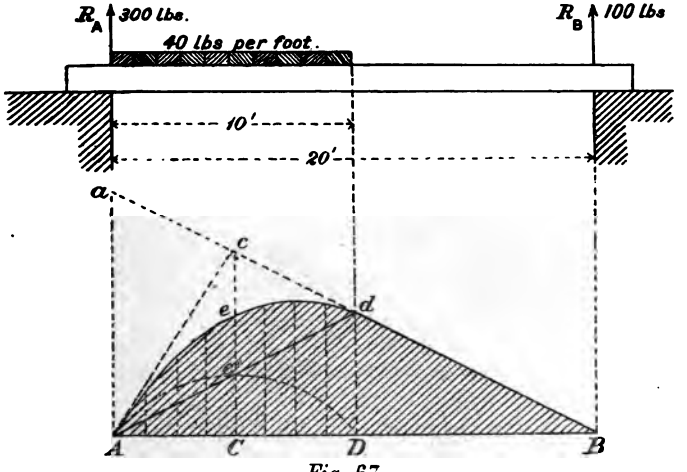


Fig. 67.

To find out, however, the true diagram, consider AD by itself, and draw its diagram as though it were supporting at A and D. Make $c'C = \frac{Wl}{8}$, *i.e.*, $400 \times 10 \div 8 = 500$ foot-lbs., on the same scale as Aa, and draw the parabolic arc $Ac'D$. Join Ad and draw ordinates from Ad = those of the parabolic arc. Then AedB is the true diagram of moment. The M_f in the centre = 1,000 foot-lbs.

APPENDIX I.

To prove that the curve of moments for a uniformly loaded supported beam is a parabola (*Fig.* 68).

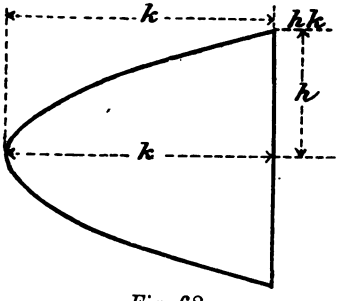


Fig. 68.

The equation to a parabola is

$$y^2 = 4ax.$$

This may be written, to suit notation—

$$x^2 = 4ay.$$

Transfer origin to a point hk on the curve, then

$$(h-x)^2 = 4a\,(k-y) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a).$$

But in the case of a beam, supported at both ends and loaded with a uniform load wl—

$$h = \frac{l}{2} \quad \text{and} \quad k = \mathrm{M}_{ff} = -\frac{wl^2}{8},$$

$$h^2 = \left(\frac{l}{2}\right)^2 = 4ak = -4a\,\frac{wl^2}{8};$$

$$\therefore \qquad 4a = -\frac{l^2}{4} \times \frac{8}{wl^2} = -\frac{2}{w}.$$

(a) becomes

$$\left(\frac{l}{2} - x\right)^2 = 4ak - 4ay = +\frac{wl^2}{8} \times -\frac{2}{w} + \frac{2}{w}y;$$

$$\therefore \qquad -x\,(l-x) = \frac{2}{w}y;$$

$$\therefore \qquad y = -\frac{wx}{2}\,(l-x).$$

But this is the value of M_f at any point x, hence the value of M_f varies with y, the ordinate of a parabola.

APPENDIX II.

To find M_x in a supported beam partially loaded with a distributed load (*Fig.* 69).

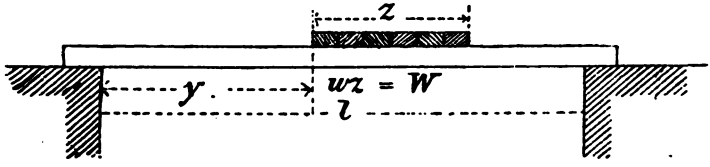


Fig. 69.

The reaction at left support

$$= -wz\left\{\frac{l-(\frac{1}{2}z+y)}{l}\right\}.$$

The moment at any section under the load distant x from the support is

$$-wxz\left\{\frac{l-(\frac{1}{2}z+y)}{l}\right\}+\frac{w(x-y)^2}{2}.$$

For a maximum, differentiating, and equating with 0—

$$x=z+y-\frac{(\frac{1}{2}z+y)z}{l}.$$

i.e., the section divides the load, so that the portion at either side = the adjoining reaction.

CHAPTER V.

Internal Influences Tending to Resist Rupture in Be

Internal Moments Excited in Beams. — Considerations Affecting t Moments.—Discrepancy between Experiments and Theory.—Rectan Beams.—General Investigation of Moments of Resistance.—Remark Modulus of Rupture.—Rules for Obtaining Value of I.—Example Rolled Steel Joist.—Moments of Resistance of Unsymmetrical Bean Examples.

In the preceding chapter we have considered the effects prod by external forces acting transversely to beams in various ways; next step is to calculate the nature of the Moments of Resist excited in the fibres of the beam by the external influences. clear that these Moments of Resistance must be equal to Moments of Flexure at every point throughout its length, expressed algebraically—

$$M_r = M_f,$$

and for safety the M_r which the beam is *capable* of affor must be greater than M_f. As has already been pointed out previous chapter, in practice the M_r is arranged to be greater the M_f by a definite ratio called the factor of safety, which v with various materials and the nature of the loads coming on th

Considerations Affecting M_r.

The Moments of Resistance are affected by the following consi ations:—(1). The material of the beam. (2). The shape of cross section. (3). The arrangement of that cross section.

With regard to (1), it is evident that the natural properties of material play a most prominent part in the resistance afforde any given case. But this is not the only consideration, for (2) shape of the cross section affects the result, *e.g.*, a circular log afford a different resistance from a rectangular beam of the s sectional area, and even this rectangular beam will afford a diffe resistance according to (3) the arrangement of its section, b

stronger when its longer side is perpendicular than when laid flat on its longer side.

Discrepancy between Experiments and Theory.

In the investigation of this subject we are confronted with the difficulty that the deductions from the best theories on the question do not always accord with the results of experiment. The inference is that the theories are in fault, and, therefore, unreliable. But although it is admitted that theory has as yet not entirely mastered all the details of the subject,* yet the results, when applied in the light of recorded experiment, are sufficiently accurate for all practical purposes, and are, at all events, fairly true within the limits of elasticity of the material concerned.

Theoretical Principles.

When a beam is bent under the action of tranverse forces, we see, without much reasoning on the subject, that the fibres on the one side of the beam are stretched, and those on the other side are compressed. If the beam be supported at both ends, the *lower* fibres will be stretched; if the beam be a cantilever, then the *upper* fibres will be stretched. As the stress is proportional, or nearly so, to the strain, or the visible change of form that any fibre undergoes, we may, by observing the *strain*, find the relative intensity of the *stress* at any fibre. This will be true as long as the elastic limit of the material is not exceeded, but when that limit is passed, the rule no longer holds good, and this is one reason why the theories on this subject do not correspond with the results of experiment. We also see that there will be some layer in the beam where there is neither extension nor compression in the fibres; this layer is called the *neutral layer*, and the line where the neutral layer intersects any cross section of the beam is called the *neutral axis* of that cross section.

In *Fig.* 70 let the beam, when unloaded, have a number of vertical lines drawn on one vertical side, at fixed intervals. When a load is brought upon the beam, if the theory be correct, then these vertical lines will radiate towards a common centre. Any section as *wxyz* will no longer be a rectangle, but a trapezoid, as in *Fig.* 71, the side *wx* being shortened to *w'x'*, and the side *yz* being elongated

* Theory has mastered a great deal more of the subject than is usually supposed. French writers—Navier, Saint-Venant, and others—have gone much more fully into the subject than English ones.

to $y'z'$. The increment zz' = the diminution xx'. At the neutral layer nn' there is no change. Inasmuch as the strain corresponds to the stress, the stress in the fibres of the beam will increase in regular proportion from the neutral layer to the outer fibres in the beam, or if the stress at 1 inch from the neutral layer be 2 lbs. per square inch, the stress at 2 inches from the neutral layer will be 4 lbs. per square inch. Hence the stresses may, as regards the whole cross section of the beam, be represented by a wedge-shaped figure in *Fig.* 72, where NN′ or WW′ represents the width of the beam, WY its depth, and the triangles NWw and NYy the stresses above and below the neutral axis NN′. The resultant pressure of the stresses will be clearly represented by the area of the triangle NWw ($\times$ breadth NN′) for stresses above, and NYy ($\times$ breadth NN′) for those below, the neutral layer, and the point of application of these resultants will be the centres of gravity of the triangles, while the

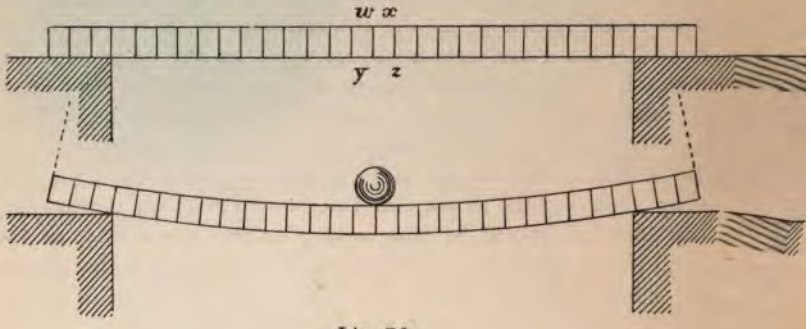


Fig. 70.

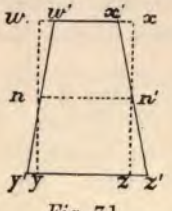


Fig. 71.

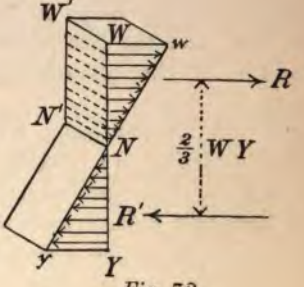


Fig. 72.

directions of these resultants will be at right angles to WY. We thus have the stresses reduced to two equal and opposite forces of

which we can ascertain the magnitude, direction, and point of application. To find the Moment of Resistance of these forces we may either consider one of them acting with a leverage equal to the distance between the two, or, which comes to the same thing, we may consider the effect of both round the central point or fulcrum N. In either case the expression for the value of the moment is as follows:—

Let r be the value of the intensity, *i.e.*, the pressure per square unit, at the extreme fibre, *i.e.*, let $r = \mathrm{W}w$ or $\mathrm{Y}y$. Let $\mathrm{WY} = d$, the depth of the beam, and $\mathrm{NN}' = b$, the breadth. Then R, the resultant stress above or below the neutral axis = area of triangle × breadth $= \frac{1}{2} r \times \frac{1}{2} d \times b$. The leverage or distance between R and R′ $= \frac{2}{3} d$. Hence the whole Moment of Resistance for the cross section, when it is rectangular, is

$$\frac{r}{2} \times \frac{d}{2} \times \frac{2}{3} d \times b = \frac{1}{6} rbd^2.$$

Balancing of M_f by M_r.

It may help in the consideration of this subject if, at this stage, we consider how the Moment of Flexure is balanced by the Moment of Resistance. To take a simple instance, let ABEF (*Fig.* 73) represent a beam loaded with a central weight W at the section CD. The pressure of the weight gives rise to reactions $= \frac{1}{2}$ W at the supports,

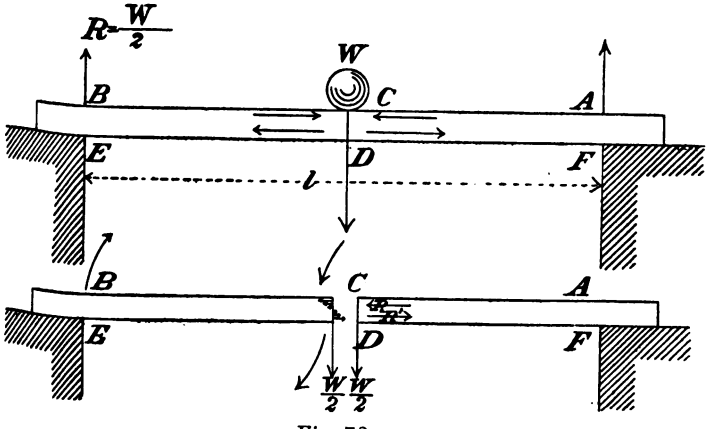


Fig. 73.

and to stresses in the beam indicated by arrows in the upper part of *Fig.* 73. Let us conceive the beam to be divided in two at CD, but

acted upon by the same forces that maintained equilibirum before. We then have $\frac{1}{2}$ W and the reaction tending to cause the portion BEDC to rotate in the direction of the hands of a watch, the moment of the couple being $\frac{1}{2}$ W $\times \frac{1}{2} l = \frac{1}{4}$ Wl, and we have the resistances R and R′ in the right half tending to turn BEDC in the contrary direction, and in a rectangular beam the moment of these resistances $= \frac{1}{6} rbd^2$. Hence

$$\frac{Wl}{4} = \frac{rbd^2}{6}.$$

This is the equation of equilibrium for a rectangular beam, supported at both ends and loaded in the middle. We may apply it to any given case. For instance, let the beam be 10 feet long between supports and 4″ by 8″ in section, and let the modulus of tranverse strength, or modulus of rupture as it is sometimes called (about which more presently), be 6,000 lbs. per square inch. The value of r may then be taken at 6,000 ÷ 5 (factor of safety) = 1,200 lbs. It is required to find which is the greatest weight that the beam can safely bear (1) when placed with its longest dimension vertical, and (2) when laid flat on that side.

Substituting values, we get in the first case—

$$W \times \tfrac{1}{4} (10' \times 12'') = 1{,}200\ (r) \times 4\ (b) \times 8 \times 8(d^2) \div 6.$$

Hence $W = 1{,}706$ lbs.

In the second case—

$$W \times \tfrac{1}{4}\ (120) = 200 \times 8 \times 4 \times 4.$$

Hence $W = 853$ lbs.

The difference illustrates the advantage of putting a beam with its longest side vertical.

Extravagance of Beams which are Rectangular in Section.

As the extreme fibres in rectangular beams, of whatever material they may be composed of, are the only parts of the beam that are exerting their full working resistance, the rest of the beam might, as far as its resisting effect is concerned, be proportionally reduced in section. In a beam of rectangular and uniform cross section throughout its length, only a very small portion indeed of the beam is doing its full amount of resistance. In timber rafters, joists, etc., it would not be worth while to carry out the reduction of the section to suit theory only, because the labour on the beam would be expensive, and the portions cut off would not be of much further use; but in iron or

steel beams this principle is carried into practical effect, so that beams of these metals are rolled in the shape of the letter I, or else built up of plates or angle irons, in order that the resistance to rupture may be borne by the horizontal *flanges* at the extremities, and the vertical part or *web* may act as a connecting piece between the flanges, and may also take the shearing stress, as will be pointed out in a subsequent chapter.

It may be, however, here pointed out that the rectangular form of cross section, ordinarily given to timber structures, has an important bearing on the question of carpenter's joints. It is evidently possible so to arrange these that they shall come at the parts of the beam subjected to least stress, and thus that the strength of the beam shall be interfered with as little as possible. For instance, if we have two beams at right angles to one another, as the trimmers and joists in a floor, the proper way to join them is by a mortice and tenon joint, such as that in *Fig.* 74, where the mortice is cut out of that part where there is least stress, and not by notching, as in *Fig.* 75.

Fig. 74.

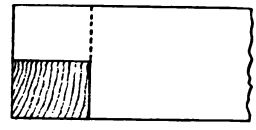


Fig. 75.

Graphic Representation in Rectangular Beams.

If we take a cross section of a rectangular beam and join the diagonals AC, BD, we have a *graphic representation* of the stresses from the neutral axis to the edges, and the shaded areas AOB, DOC represent the total tension and compression. For if in *Fig.* 76 AB represent on any scale of weights the intensity of stress at AB, say 1,000 lbs. per square inch, at a distance 4″ from the neutral axis, then we know that, on the reasoning we have investigated above, the stress intensity 2″ from the neutral axis will be $1{,}000 \times \frac{2}{4}$, or

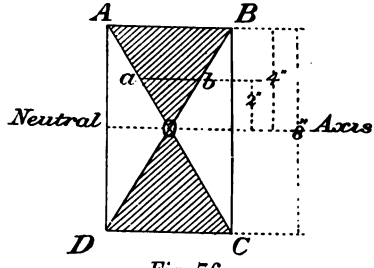


Fig. 76.

500 lbs. per square inch. But ab is $\frac{1}{2}$ AB, and, therefore, on the same scale represents 500 lbs. Hence the triangle AOB and the triangle DOC represent *areas of equal resistance* above and below the neutral axis, and graphically represent the quantity and distribution of the resistance. The moment of resistance might be obtained from these triangles in the same way as in *Fig.* 72. If AB $= b$, BC $= d$, the area of the triangle AOB $= \frac{1}{4} bd$. The leverage of the M_r is $\frac{2}{3} d$.

Hence $$M_r = r \times \tfrac{1}{4} bd \times \tfrac{2}{3} d = \tfrac{1}{6} rbd^2,$$

as before.

General Question of Resistance.

We have spent some time over the consideration of rectangular beams, because they are common and because the application of the principles of the Moments of Resistance can be more readily understood in their case than in beams in general; it is, however, now necessary to consider the general question of resistance.

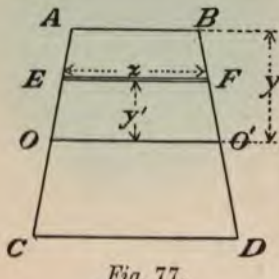


Fig. 77.

In *Fig.* 77 let ABCD be the cross section of a beam, OO′ its neutral axis, then if EF be any strip distant y' from the neutral axis, with breadth $= z$ and depth $= dy$, the intensity of stress on that strip will vary as the distance y', on the principles hitherto considered, and the Moment of Resistance of that strip will vary as zy'^2.

If r be the intensity of pressure at the strip AB, and r' that at EF,

then $$r : r' :: y : y',$$

or $$r' = \frac{r}{y} \times y'.$$

Hence the stress on the whole strip EF

$$= z \times dy \times r' = zdy \times y' \times \frac{r}{y}.$$

Hence the moment of resistance of the whole strip is the stress multiplied by the leverage with which it acts, *i.e.*—

$$\frac{r}{y} y'^2 z dy,$$

ıd the Moment of Resistance of the *whole* cross section ABCD is × the sum of all the particles × the square of their distances from he neutral axis, *i.e.*—

$$M_r = \frac{r}{y} \Sigma y^2 z dy$$
$$= \frac{rI}{y}.$$

Moment of Inertia.

This expression $\Sigma y^2 z dy$ is called the *Moment of Inertia* of the cross section. This is usually expressed by the symbol I, and values for various forms of section are given in tabular form below. As it is of great importance to have a clear idea of what is intended by this symbol, it should be remembered that it means **the limit of sum of the products of the areas into which the surface is divided, multiplied by the square of their distances from the neutral axis.**

General Equation for Moment of Resistance.

The equation $M_r = \frac{rI}{y}$ is a most important one. It is the general expression for the Moment of Resistance of any cross section, of whatever shape, in terms of: (1), the intensity of stress on the fibres furthest from the neutral axis; (2), the Moment of Inertia of the section; and (3) the distance of the extreme fibre from the neutral axis.

It will be observed that the whole of the reasoning in the above investigation depends on one assumption, viz., that the stress on any fibre, whether in tension or compression, varies directly with its distance from the neutral axis. **This assumption is correct, or nearly so, for all stresses within the elastic limit of the material,** and as we may take the value of r in the equation $\frac{rI}{y}$ at any value within the elastic limit of the material which we are dealing with, we may ascertain with fair accuracy the resistance of the section *when the extreme fibre is undergoing a stress of that value of* r. This, in practice, is what is required, for it is not desired that the beam shall be strained permanently, but that it shall be strained within its elastic limit, and recover its form after the load is removed. Hence the equation $M_r = \frac{rI}{y}$ is true for all practical purposes, though

it is not in accordance with the results of experiments of beams loaded to breaking, because, among other reasons, the beam when broken has been strained beyond its elastic limit.

Another matter on which the solution of this equation depends is the position of the neutral axis. This is ascertained by finding the centre of gravity of the cross section, and drawing a horizontal line through it. For it is evident that as the sum of the compressive forces in the section of the beam are equal to the sum of the tensile forces in the section, these will balance one another. Hence, from the principles of statics, the axis about which equilibrium takes place passes through the centre of gravity of the cross section.* This, again, is true only within the limits of elasticity, for when the strains in a beam are not proportional to the stresses producing them, there is no longer equality of stress, and the position of the neutral axis is altered. It is supposed that in bending, approaching rupture, the extreme fibres are partially relieved of stress, which is transferred to the layers of fibres nearer to the neutral axis. The centre of gravity tends to move further away from the extreme fibre until the point of rupture is reached.

To ascertain the safe value which we can give to r, it is not sound to take the tensile or compressive strength. What is practically done is to ascertain the bending moment which produces fracture, applying the fundamental formula

$$M_f = M_r = \frac{rI}{y},$$

and from the known and observed values of the other terms find the value of r. This value of r is called the *modulus of rupture*, or *coefficient of transverse strength*. It is not the same as the tensile or the compressive strength of the material, as may be seen on examination of Table XIV. in Chapter III. As the subject has already been considered in that chapter, it is here only necessary to point out that the modulus of rupture is less than the tensile strength in the case of timber, but greater in the case of iron and steel.

Remarks on the Modulus of Rupture.

The values of the modulus of rupture are, as a rule, found from small experimental beams, of the material in question, 1 inch square,

* Pointed out by French scientists in the 18th century.

12 inches long between supports, and loaded in the centre with a weight W. The bending moment in such a case is $\frac{1}{4}Wl$, and as the M_r is $\frac{rbd^2}{6}$, we have $r = \frac{1}{4}Wl \times \frac{6}{bd^2}$.

The weight W is generally increased till the beam breaks.

Hence we see that r is equal to 18 times the weight that will break the bar across, with the dimensions above mentioned.

There is, however, this objection to this procedure—that in timber the specimens experimented on were, as a rule, perfectly dry, free from defects, and hence the results are too favourable. This has already been pointed out in Chapter III. But this method has the advantage that it is readily applied in the case of unknown timbers. It is easy for the engineer to test specimens of materials in this way, and by applying the formula for M_r with a factor of safety of about 5, to be within safe and practical limits, whereas it is not easy to observe the amount of compression or extension of the materials under different stresses.

It must, however, be remembered that, in using the modulus of rupture thus obtained by experiment, it is only applicable to the calculation of beams of section similar to that experimented on. Thus a modulus derived from a rectangular beam would, if applied to a beam of circular section, give very different results. This shows the difficulty of applying theoretical rules to field structures.

The ratio of strength of a round pole to a square beam with side equal to the diameter of the round pole is nearly 6:10. This is nearly the ratio of the moments of inertia, *i.e.*, $\cdot 7854R^4$ and $\frac{1}{12}d^4$, where $d = 2R$. The rule is, of course, only approximate, for the reasons just stated. Field structures, therefore, need a liberal factor of safety.

Where, however, the elastic limit of the material is not exceeded, the formula $M_r = \frac{rI}{y}$ is true, and the coincidence of the neutral axis with the axis through the centre of gravity is also true.

Rules for Obtaining the Value of I.

We shall now consider some rules for obtaining the value of I. This may be obtained by integrating the expression y^2zdy for any section between the limits y and $-y$. A few examples of such integration are given in Appendix I. to this chapter. We can find it for a rectangular section, however, by equating the expression for

the M_r of a rectangular beam, already found, with the general formula, *i.e.*, $\frac{1}{6}rbd^2 = \frac{rI}{y}$. As $y = \frac{1}{2}d$ in a rectangular beam, we see that $I = \frac{1}{12}bd^3$. From this value of I we can obtain the value of any figures of rectangular section. For instance, take the hollow rectangle represented in *Fig.* 78. Here the value of I is $\frac{1}{12}\{bd^3 - b'd'^3\}$.

If, however, we take a form such as that represented in *Fig.* 79 the ordinary form for girders, the value of I may be obtained either as in *Fig.* 79, where the I for the two side rectangles is deducted from the whole figure, *i.e.*—

$$I = \tfrac{1}{12}\{bd^3 - 2kh^3\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1),$$

or as in *Fig.* 80, where the value of the I for the web is taken first, and to it is added the value of the I for the flanges, *i.e.*—

$$I = \tfrac{1}{12}\{ch^3 \times b\,(d^3 - h^3)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

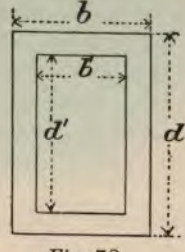


Fig. 78.

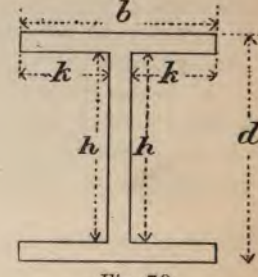


Fig. 79.

It will be evident that algebraically these values are equal, for, substituting in (2) for c its value $b - 2k$, we have the two expressions identical.

This illustrates two methods for finding the values of the Moment of Inertia for a rolled joist or beam of iron or steel, of the form shown in *Fig.* 81, so largely used in the present day. By dividing the area into equivalent rectangles, as in *Plate* I., and taking the values of the I as above in (2), we may obtain the I for the whole with sufficient accuracy for all practical purposes. Or we may divide the area into two rectangles, and four triangles, as in the lower part of *Plate* I., the area of these being equal to the difference between the whole rectangle and the section of the rolled beam, and by the method illustrated in (1) obtain the value of the I for the beam.

In this last case, however, the neutral axis of the whole does not coincide with that of the four triangles. In order to ascertain the value of the Moment of Inertia of the triangles AGF, KHC round the axis MN, we first find the value of the I for each triangle about its own neutral axis, and add to this the area of the triangle multiplied by the square of the distance of its c.g. from the axis MN. The general rule for such cases is as follows:—*The Moment of Inertia of a plane figure about an axis not passing through its c.g. is equal to its Moment of Inertia about a parallel axis passing through its c.g. plus the area of the figure multiplied by the square of the distance between the two axes.*

A proof of this is given in Appendix II. of this chapter.

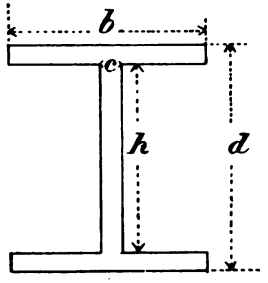


Fig. 80.

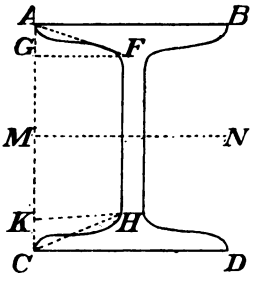


Fig. 81.

An application of this to an ordinary iron joist is as follows:—

EXAMPLE 7.—*An iron joist of the section shown in Plate* I. *is to be used in the construction of a floor. It is required to find*—

(1). The Moment of Inertia by two methods as described above.

(2). The value of the Moment of Resistance that it is capable of exerting with safety.

To find the value of I by the first method we divide the flange into a number of equivalent rectangles, as shown in the upper part of the figure. The following measurements result:—

(1). $h = \cdot 66''$, $a = 9 \cdot 14''$, $k = \cdot 84''$, $b = 10 \cdot 24''$, $l = 3 \cdot 78''$, $c = 10 \cdot 80''$, $m = 6 \cdot 0''$, $d = 12 \cdot 0''$. Then

$$\begin{aligned} I &= \tfrac{1}{12}\{ha^3 + k(b^3 - a^3) + l(c^3 - b^3) + m(d^3 - c^3)\} \\ &= \tfrac{1}{12}\{\cdot 66 \times 9 \cdot 14^3 + \cdot 84(10 \cdot 24^3 - 9 \cdot 14^3) + 3 \cdot 78(10 \cdot 8^3 - 10 \cdot 24^3) \\ &\qquad + 6(12^3 - 10 \cdot 80^3) \\ &= \tfrac{1}{12}(503 \cdot 94 + 260 \cdot 56 + 627 \cdot 37 + 2929 \cdot 73) \\ &= \tfrac{1}{12}\, 4321 \cdot 60 \\ &= 360 \cdot 13. \end{aligned}$$

To find the value of I by the other method we divide the of the containing rectangle, not occupied by the beam, in rectangles and four triangles. Let b_1 be the breadth of one o rectangles, h_x the depth, while h_0 is the depth pq of one triangles pqr, and h_1 the distance from its centre of gravity neutral axis.

Then the Moment of Inertia of the beam is found by tak Moment of Inertia of the whole containing rectangle and de therefrom the Moments of Inertia of the component parts outs limits of the beam.

That is to say—

$$I = \tfrac{1}{12}\, md^3 - 2\,\frac{b_1 h_x^3}{12} - 4\,(\tfrac{1}{36}\, b_1 h_0^3 + \tfrac{1}{2} b_1 \times h_0 \times h_1^2).$$

From measurement,

$$m = 6'', \; d = 12'', \; b_1 = 2{\cdot}66'', \; h_0 = {\cdot}34'', \; h_1 = 5{\cdot}2'', \text{ and } h_x = 10$$

Substituting these values—

$$\begin{aligned} I &= \tfrac{1}{12} \times 6 \times 12^3 - \frac{2 \times 2{\cdot}66 \times (10{\cdot}12)^3}{12} \\ &\qquad - 4\,\{\tfrac{1}{36}\, 2{\cdot}66 \times ({\cdot}34)^3 + \tfrac{1}{2}\, 2{\cdot}66 \times {\cdot}34 \times \\ &= 864 - 459{\cdot}484 - 4 \times 12{\cdot}312 \\ &= 864 - 508{\cdot}73 \\ &= 355{\cdot}29. \end{aligned}$$

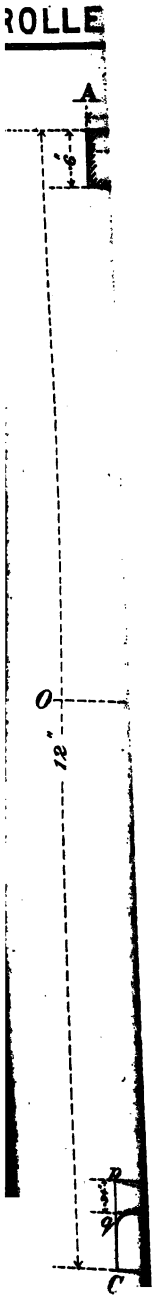
The value obtained by the other method was 360·13. The ence may be due to slight inaccuracies of drawing.

The lowest value 355 may be taken as being on the safety.

(2). To find the value of the greatest Moment of Resistanc a beam of this section is capable of exerting with safety, w of course, to consider the material. If wrought iron b probably a modulus of rupture of 20 tons would be taken, with a factor of safety of 4, gives 5 tons per square inch as value of r in the equation $\frac{r}{y}$ I, which represents the Moment of ance. In this case, taking r as 5 tons, I as 355, and y or we get the value of $M_r = 296$ inch-tons, very nearly.

If the material of the beam be steel, with a modulus of ru 30 tons, with the same factor of safety, we get

$$M_r = 7{\cdot}5 \times 355 \div 6 = 443 \text{ inch-tons.}$$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There is another method of finding the M_r less accurately than the above, it is true, yet sufficiently correct for all practical purposes, based on the principle mentioned in a former part of this chapter, viz.—that of equal areas of resistance. In this case, the joist is considered as composed of a rectangle (*i.e.*, the whole of the *web*, measured from top to bottom of the beam section), and two half flanges at each end. If A be the area of the web, *i.e.*, $h \times d$, then the M_r of the web alone $= \frac{1}{6} rhd^2$, or $\frac{1}{6} rAd$. And if a' be the area of the flange on either side (*i.e.*, above or below) of the web (*Fig.* 82), then, on the assumption that all the flange is under the same intensity of stress, the M_r of the flange is $ra'd$ where d is *theoretically* the distance between the centres of gravity of the two flanges, but is *practically* taken as the total depth of the beam, as before. Hence the total M_r of the beam $= rd\,(\frac{1}{6}A + A') = rad$ where a = area of flange $+ \frac{1}{6}$ area of the web (*Fig.* 82).

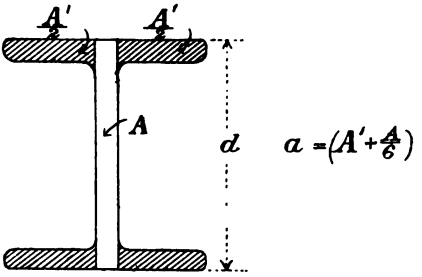

Fig. 82.

This expression $M_r = rad$ is a very useful one, as it enables us to ascertain without much calculation, and with sufficient accuracy for practical purposes, the resistance of girders of all the ordinary types.

To apply it to the case in *Plate* I.—

$$A = (6'' \times \cdot 6'' + 3 \cdot 78'' \times \cdot 28'' + \cdot 55 \times \cdot 84'') - \cdot 66 \times (\cdot 55 + \cdot 28 + 60) + \cdot 66 \times 12'' \div 6 = 5 \cdot 5 \text{ inches.}$$

Hence rad = for wrought iron $5 \times 5 \cdot 5 \times 12 = 330$ inch-tons

By the more accurate formula $\frac{rI}{y}$ the value is 296 inch-tons, as we have seen above.

Moments of Resistance of Unsymmetrical Beams.

The beam in the above example is taken as symmetrical about its centre. It will now be necessary to examine the case of a beam, such as a flat-bottomed rail, that is not symmetrical.*

* Taken from General Wray's *Application of Theory to Practice of Construction.*

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The first thing to be done is to find the position of the c.g., and draw a horizontal axis through it. This will be the neutral axis. Then divide the whole cross section into equivalent rectangles, on the same principles as in the former example. Then in *Fig.* 83 the I of any given rectangle such as *ha* with respect to the axis OO′ is the I of the rectangle about its own neutral axis plus the area of the rectangle × the square of the distances between the axes, *i.e.*—

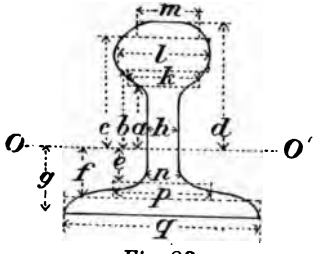


Fig. 83.

$$I = \tfrac{1}{12} ha^3 + ha \times (\tfrac{1}{2}a)^2 = \tfrac{1}{3} ha^3.$$

This applies to all the component rectangles. Hence

$$I = \tfrac{1}{3} \{ha^3 + k(b^3 - a^3) + l(c^3 - b^3) + m(d^3 - c^3) + ne^3 + p(f^3 - e^3) + q(g^3 - f^3)\}.$$

The following table shows the values of Moments of Inertia of various forms :—

TABLE XVII.

Value of Moments of Inertia of Various Sections ordinarily in use.

Section.	Value of I.	Terms.	Remarks.
Rectangle ...	$\frac{1}{12}bd^3$.	b = breadth of base. d = height of side.	
Triangle	$\frac{1}{36}bd^3$.	b = breadth of base. d = perpendicular from apex to base.	
T Section... ..	$\frac{1}{3}\{bd^3 + b_1d_1^3 - (b_1 - b)d_2^3\}$.	See *Fig.* 84.	d and d_1 represent distances of neutral axis from extremities.
H Section, with equal flanges.	$\frac{1}{12}\{bd^3 - (b - b_1)d_1^3\}$.	See *Fig.* 85.	
+ Section, with equal arms.	$\frac{1}{12}(bd^3 + 2b_1d_1^3)$.	See *Fig.* 86.	
Solid semi-circle	$\cdot 11R^4$.	R = radius of circle.	Useful for calculating half-round logs.
Circle (solid) ...	$\cdot 7854R^4$.		
Hollow ring ...	$\cdot 7854\,(R^4 - R'^4)$.	R and R′ are radii of outer and inner circles.	
Hollow rectangular section of equal flanges, connected at intervals.	$\frac{1}{12}b\,(d^3 \quad d_1^3)$.	See *Fig.* 87.	Useful in struts formed of two bars, connected by distance pieces.
Angle iron ...	$\frac{1}{3}\{b'd^3 + bd_1^3 - (b - b')(d_1 - d_2)^3\}$.	See *Fig.* 88.	This may be computed also as the I Section above.

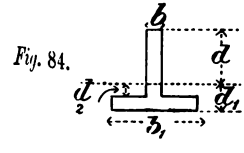


Fig. 84.

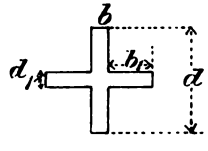


Fig. 86.

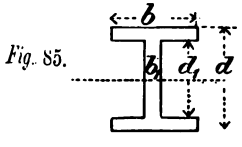


Fig. 85.

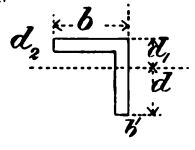


Fig. 88.

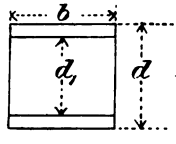


Fig. 87.

The following table gives the values of I for angle and T i
usually obtained in the market:—

TABLE XVIII.

Values of I (the Moment of Inertia) and y *(the distance of the axis from the extreme fibre) in various market sections of* T *and* L

Greatest breadth (b_1, *Fig.* 84).	Greatest depth ($d+d_1$, *Fig.* 84).	Thickness (b, *Fig.* 84).	Value of I.	Value o (*Fig*
Inches.	Inches.	Inch.		
4	4	$\frac{1}{2}$	5·5614	1·1
3	4	$\frac{1}{2}$	5·0485	1·3
4	3	$\frac{1}{2}$	2·4234	·8
$3\frac{1}{2}$	$3\frac{1}{2}$	$\frac{1}{2}$	3·6350	1·0
$3\frac{1}{2}$	$3\frac{1}{2}$	$\frac{3}{8}$	2·8651	1·0
$3\frac{1}{2}$	3	$\frac{1}{2}$	2·3281	0·8
$3\frac{1}{2}$	3	$\frac{3}{8}$	1·8463	0·8
3	3	$\frac{3}{8}$	1·7597	0·8
4	5	$\frac{1}{2}$	10·4562	1·5
$4\frac{1}{2}$	$3\frac{1}{2}$	$\frac{7}{16}$	3·5202	0·9

The values of I have been worked out from the precedin
the position of the neutral axis having been ascertained first
principles explained in Chapter II., Appendix, Case 3. The
sections are those of Messrs. Dorman, Long & Co., of Middlesb
Similar tables have been published by Col. Broadbent, R.I
Molesworth, p. 148, Ed. 1893). His results agree very close
those above.

This table will repay careful examination. It shows how
increase in depth gives great increase in strength.

Graphic Representation of Areas of Equal Resistance.

We have seen above that in a *rectangle* the areas of equa
ance are shown by joining the diagonals.

In the case of a *circle* (*Fig.* 89), lines are drawn across, a
and perpendiculars let fall from their extremities, such as *bd*
on a horizontal line drawn at the top of the section, *c* an
joined to O, the centre of the circle, intersecting *ab* in *a'b'*, *a'*

give points on the outline of the area of equal resistance. So for all other points. In the same way the symmetrical figures (*Figs.* 90 and 91) can be found.

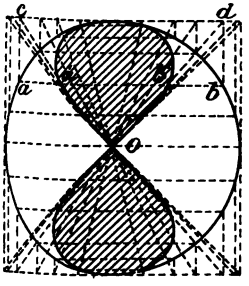


Fig. 89

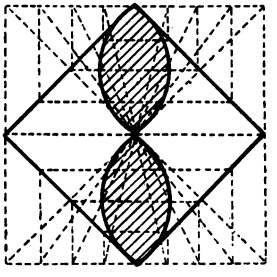


Fig. 90.

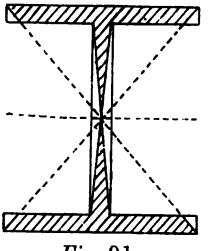


Fig. 91.

With unsymmetrical figures, the horizontal line through the c.g. is first drawn, and the line at the top of the figure parallel to it. Another line at equal distance is drawn at the bottom, and measurements for the lower half of the figure referred to it (*Figs.* 92 and 93).

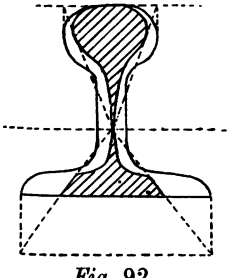


Fig. 92.

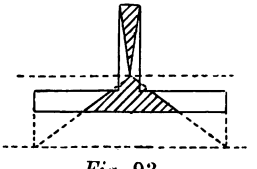


Fig. 93.

These graphic methods may be turned to practical account as follows:—If we draw the resistance diagram on a thick piece of

Bristol board, or any material of uniform thickness, weigh the c[illegible]ut out diagram, and also one square inch of the same material, then t[illegible]he first weight divided by the second will give the area required in square inches, that is, the true value of a in the equation $M_r = rad$[illegible].

Examples.

A few miscellaneous examples of the foregoing principles m[illegible]ay help to illustrate the text.

Example 8.—*Of two beams, one of red fir* 6″ × 4″, *and the other of larch* 7″ × 3″, *which is the stronger?*

From Table XIV., Chapter III., the minimum values of the modulus of rupture for red fir and larch are 4,500 and 5,000 lbs. respectively. Placing the beams in the most favourable positions, *i.e.*, with the longer side vertical, we get the following values for the moment of resistance:—

For the red fir—

$$\frac{rbd^2}{6} = 4{,}500 \times 4 \times 36 \div 6 = 108{,}000 \text{ inch-lbs.}$$

For the larch—

$$\frac{rbd^2}{6} = 5{,}000 \times 3 \times 49 \div 6 = 122{,}500 \text{ inch-lbs.}$$

So that the latter is the stronger. If, however, we take the average minimum for red fir, as given in Table XIV., *i.e.*, 6,000, we get the first moment of resistance = 144,000. There is, therefore, little difference in the strength of the two beams.

Example 9.—*A floor joist at* 12″ *central interval from the next one has to carry* 160 lbs. *per square foot. With a span of* 12′ *and width of* 2″, *what should the depth be?*

As the fixing of the ends may be imperfect, it is better to assume the joist as supported only.* The greatest Moment of Flexure in a uniformly loaded beam supported at both ends is $Wl \div 8$. The moment of resistance of a rectangular beam is $\frac{rbd^2}{6}$.

For equilibrium, $M_f = M_r$.

* It will be afterwards shown (in Part II.) that a beam which is half fix[illegible] is really as strong as one which is perfectly fixed. For present purpose[illegible], however, this may be disregarded. It is in accordance with usual custom t[illegible] consider floor joists as *supported* only.

Hence, substituting values—

$$W = 160 \times 1 \text{ foot} \times 12 \text{ feet} = 1{,}920 \text{ lbs.}$$

$$Wl \div 8 = 1{,}920 \div 144 \div 8 = 34{,}560 \text{ inch-lbs.}$$

The average minimum modulus of rupture for red fir, as in the last example, being 6,000, we may take the value of r, with a factor of safety of 5, as 1,200.

Hence $$rbd^2 \div 6 = 1{,}200 \times 2 \times d^2 \div 6 = 400d^2.$$

Hence $$400d^2 = 34{,}560 \text{ inch-lbs.},$$

$$d^2 = 86{\cdot}4,$$

$$d = 9{\cdot}4 \text{ inches.}$$

This, of course, takes no account of stiffness, which will be considered in a later chapter.

For such a span Hurst (Handbook, p. 34) gives $2'' \times 9\frac{1}{2}''$, which corresponds with the above calculation.

EXAMPLE 10.—*A cantilever of oak 6′ long and 4″ square is fixed into a wall. Find the greatest load that is safe to put at its extremity.*

Here the $M_{ff} = Wl = W \times 72$ inch-lbs.

The modulus of rupture of English oak is 10,000lbs., which, with with a factor of safety of 5, gives 2,000 as the value of r.

$$M_r = \frac{rbd^2}{6} = 2{,}000 \times 4 \times 16 \div 6 = 21{,}333 \text{ inch-lbs.},$$

$$M_f = M_r,$$

$$W \times 72 = 21{,}333,$$

hence $$W = 300 \text{ lbs.}$$

EXAMPLE 11.—*In a double lock spar bridge, the distance between fork transoms is 15′, the distance to the frame transoms 3′, and the diameter of the distance pieces 8″, the timber being larch. What load per square foot can two distance pieces carry safely, with a 10′ roadway?*

In *Fig.* 94 the elevation of the bridge is shown, 45′ total span. The distance piece AB is supported on the frame transoms at A and B, and is weighted at the fork transoms C and D, which carry the roadway from E to F, and from F to G. It is required to calculate the weights that would be safe at C and D, with the above conditions, and thence to deduce the safe weight per square foot.

The M_f is a constant between C and D (see Chapter IV. Case 8), and is equal to $W \times AC = W \times 36$ inch-lbs.

The M_r is found from the general equation $rI \div y$, where I is
Moment of Inertia for a circle $= \cdot 7854R^4 =$ in this case $\cdot 7854 \times 4^4 =$ I
The modulus of rupture of larch is 5,000, which, with a factor
safety of 5, gives a value to r of 1,000. The value of y, *i.e.*, the
tance of the extreme fibre from the neutral axis, is half the diame
of the beam, 8″, hence $y = 4$″.

Thus $rI \div y = 1{,}000 \times 194 \div 4 = 48{,}500$ inch-lbs.

Equating M_f with M_r we have

$$W \times 36 = 48{,}500,$$

hence $$W = 1{,}348 \text{ lbs.}$$

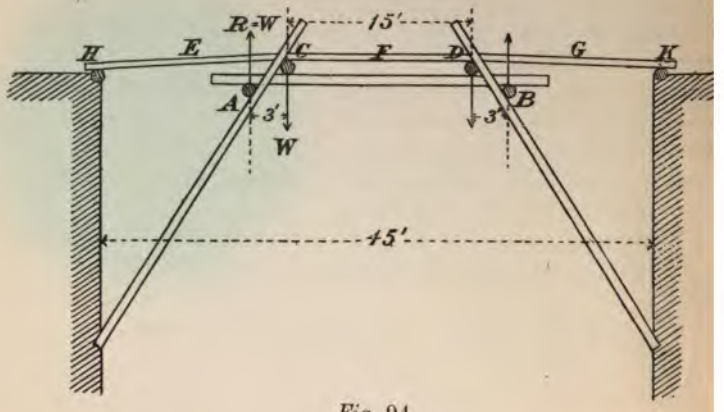


Fig. 94.

This is the greatest weight that each distance piece can saf
bear at C and D. Each of the two pieces has to bear half
weight of the roadway between E and F, *i.e.*, if w be the weight
square foot, then

$$15 \times 5 \times w = 1{,}348 \text{ lbs.},$$

or $$w = 18 \text{ lbs.}$$

From this it appears that two distance pieces of this diam
would not be sufficient for the ordinary purposes of a bridge,
weights on which are generally about 70 lbs. per square foot,
load. It would be necessary, for safety, either to increase
number of the pieces or increase the diameter very considerably.

EXAMPLE 12.—*In the rolled joist shown on Plate I., the material b*

steel, a span is assumed of 15 *feet. What is the greatest central load it will bear safely?*

The span is 15′, or 180 inches. The M_{ff} will be at the centre, and is equal to $Wl \div 4 = W \times 45$ inch-tons.

From the example worked out above for investigating the resistance of this joist, we found that the value of I was 355, r was 7·5, with a factor of safety of 4, and $y = 6$.

Hence $rI \div y = 463$ inch-tons.

Equating M_f with M_r— $W \times 45 = 463$,

whence $W = 10{\cdot}03$ tons.

If wrought iron were used with a safe value of r of 5 tons, the value of W would be 6·58 tons.

EXAMPLE 13.—*A square bar of cast iron* 3″ × 3″ *rests on two supports* 5′ *apart. A weight of* 3 *tons is suspended from the centre. What additional weight suspended at* 1′ *from the centre will break the beam?*

Let W be the weight required. Then in *Fig.* 95 the reaction nearest to that weight $= (42 \div 60W + 1\frac{1}{2})$ tons.

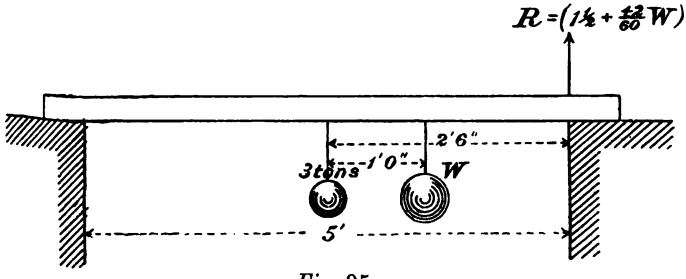


Fig. 95.

The M_{ff} is in the centre, and is equal to

$$(42 \div 60W + 1\tfrac{1}{2})\ 30 - 12W \text{ inch-tons.}$$

The M_r is

$$rbd^2 \div 6 = 18 \times 3 \times 9 \div 6 = 81 \text{ inch-tons,}$$

taking the modulus of rupture of cast iron at 18 tons.

Equating M_{ff} with M_r—

$$(42 \div 60W + 1{\cdot}5)\ 30 - 12W = 81,$$

$$9W = 36, \text{ and } W = 4 \text{ tons.}$$

EXAMPLE 14.—*A hollow tube of steel* 8″ *diameter,* ½″ *thick, is placed on supports* 10′ *apart. What weight in its centre will break it?*

Here the M_{ff} is $Wl \div 4$, and as the span is 10 feet, $M_{ff} = 30W$ inch-tons.

The value of the M_r is found from the general equation $rI \div y$. Here r for mild steel (ultimate strength) is about 30 tons. I for a circular ring $= \cdot 7854\ (R^4 - R'^4)$. In this case $R = 4$ inches and $R' = 3 \cdot 5$ inches. Hence

$$I = \cdot 7854\ (196 - 150) = 36.$$

Hence $rI \div y$ becomes $30 \times 36 \div 4 = 270$ inch-tons.

Equating M_{ff} with M_r— $30W = 270$,

whence $W = 9$ tons.

APPENDIX I.

To find the Moments of Inertia of a rectangle, triangle, circle, etc.

The definition of the Moment of Inertia of a plane surface about an axis being "the limit of the sum of the product of the elementary areas into which the surface may be conceived to be divided, by the square of their distances from the axis in question," it follows that if the surface be referred to a pair of axes in its own plane.

$$I = \iint x^2 dx dy.$$

(*a*). Rectangle (*Fig.* 96).

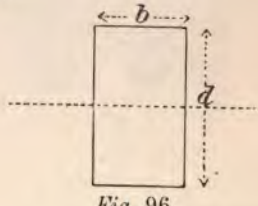


Fig. 96.

$$I = \int_o^b \int_{-\frac{d}{2}}^{\frac{d}{2}} x^2 dx dy$$

$$= b \int_{-\frac{d}{2}}^{\frac{d}{2}} x^2 dx$$

$$= \tfrac{1}{12} b d^3.$$

(*b*). Triangle* (*Fig* 97).

The Moment of Inertia of a slice of the triangle about a line KBM, drawn

* Proof kindly furnished by Captain C. F Close, R.E.

through one apex parallel to the opposite side $=x^2dm$, dm being the area of the slice.

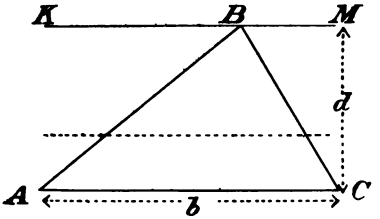


Fig. 97.

Now $$dm=\frac{bx}{d}.$$

Hence for the whole triangle—

$$I=\frac{b}{d}\int_o^d x^3dx=\frac{bd^3}{4}.$$

Now let the Moment of Inertia about a line parallel to AC through the c.g. be A, then $\frac{bd^3}{4}=A+$**area of triangle** $\times$ **square of the distance of its c.g. from KBM,** ***i.e.*—**

$$\frac{bd^3}{4}=A+\frac{bd}{2}\times(\tfrac{2}{3}d)^2=A+\tfrac{4}{18}bd^3.$$

$$\therefore \quad A=\tfrac{1}{36}bd^3.$$

(*c*). Circle (*Fig.* 98).*

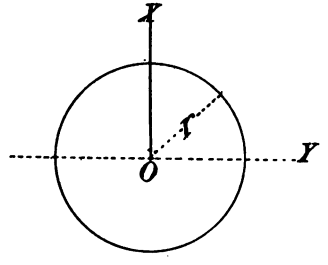


Fig. 98.

$$I=\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}}\int_{-r}^{r}x^2dxdy=2\int_{-r}^{r}x^2\sqrt{r^2-x^2}dx$$

$$=2\left\{-\tfrac{1}{4}x\,(r^2-x^2)^{\frac{3}{2}}+\frac{r^2}{4}\int\sqrt{r^2-x^2}dx\right\}_{-r}^{r}$$

$$=\frac{\pi r^4}{4}.$$

* From De Lanza's *Applied Mechanics.*

APPENDIX II.

THE Moment of Inertia of a plane figure about an axis not passing through the centre of gravity = Moment of Inertia about a parallel axis which *does* pass through the c.g. + the product of the area × the square of the distance between the axes, *i.e.*—

$$I = I' + a^2A.$$

Let ABCD (*Fig.* 99) be the surface; let OY be the axis not passing through the c.g.; let P be an elementary area $\triangle x \triangle y$, whose co-ordinates are OR and RP = y; and let $OO_1 = a$ the distance between the axes. Let $O_1R = x_1$.

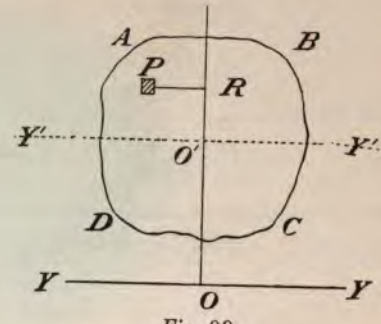


Fig. 99.

Then
$$x = a + x_1,$$
$$x^2 = x_1^2 + 2ax_1 + a_1^2,$$
$$x^2 \triangle x \triangle y = x_1^2 \triangle x \triangle y + 2ax_1 \triangle x \triangle y + a_1^2 \triangle x \triangle y.$$

Hence, summing and passing to the limit—

$$\iint x^2 dxdy = \iint x_1^2 dxdy + 2a\iint x_1 dxdy + a^2\iint dxdy,$$

but if we were seeking the abscissa of the c.g. when the surface is referred to Y_1OY_1, and if this abscissa be denoted by x_0, we should have

$$x_0 = \frac{\iint x_1 dxdy}{\iint dxdy};$$

and since
$$x_0 = 0, \iint x_1 dxdy = 0,$$

$\therefore$
$$\iint x^2 dxdy = \iint x_1^2 dxdy + a^2\iint dxdy,$$

i.e.,
$$I = I + a^2A.$$

CHAPTER VI.

Shearing Stresses in Beams.

nvestigation.—Horizontal and Vertical Shearing Stress.—Rules for ing Stress.—Various Ordinary Cases.—Distribution of the Shearing s.—Appendix.

ion to the direct stresses which we have been considering in ious chapters, there are shearing stresses produced in a the action of any force external to it. These shearing are related to the direct stresses, inasmuch as they supply necessary to maintain equilibrium in the varying conditions irect stresses.

stance, in *Fig.* 100 let ABCD represent a cantilever forme led iron beam, ith a weight W d. The diagram ents will be ted (as we have Chapter IV. by the triangle the M_f at any as YY′ will be nted by the e *yy′* in the Now if we section ZZ′ near the M_f at that vill be different

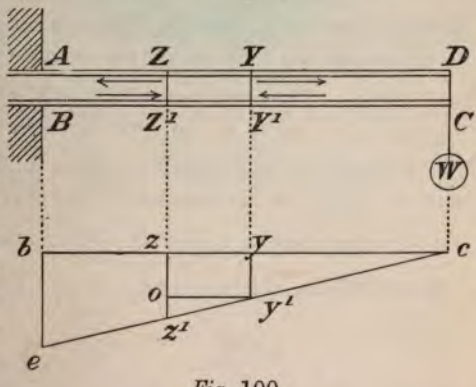


Fig. 100.

nite amount, represented on the diagram by the amount *oz′*. ill be at Z a greater tension than at Y, and at Z′ a corres- y greater compression that at Y′. In order to preserve um, therefore, there must be in the fibres of the beam an al force, besides that at Y, pulling away from Z and pushing Z′. The tendency of these forces is to shear away the rom the web of the beam.

orce is called the horizontal shearing force. It will presently

be proved that there is an equal vertical shearing force, acting at all points. Meantime it is well to observe that the horizontal shearing force is measured by the moment $z'o$ divided by the distance zy,* or (which comes to the same thing) by the tangent of the angle of inclination of the line ce in the diagram of moments. As this is a straight line in the case before us, it follows that the shearing stress is a constant for this case. In most cases the line ce is curved, so that the ratio varies from point to point, but whatever be its curvature, the inclination of the tangent to the diagram of moments at any point will be a measure of the shearing force exerted per unit of length at that point.

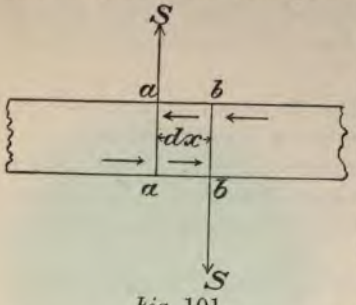


Fig. 101.

That the *vertical* shearing force is also connected with the direct stresses may be proved as follows:—

Let bb, aa (*Fig.* 101) be two adjacent sections of a beam at a distance from each other dx. The left couple represents the smaller M_r at aa, and the right couple represents the greater M_r at bb.

$$\text{The couple at } aa = M_r = M_f.$$
$$\text{,, ,, ,, } bb = M_r + dM_r = M_f + dM_f.$$

The difference of the opposite couples is dM_f, which for equilibrium must be equal to the moment of the only other existing force, *i.e.*, the shearing stress couple, Sdx. Hence $Sdx = dM_f$ and $S = \frac{dM_f}{dx}$.

If in a beam the continuity of the fibres be interrupted by a number of vertical slices cut in it, as in *Fig.* 102, it is evident that these slices will slip past each other when the beam is loaded. In the uncut beam the same *tendency* to slip exists, although actual sliding is prevented by the cohesion of the beam.

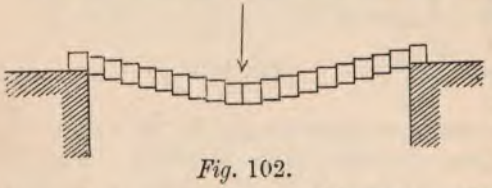


Fig. 102.

* $\frac{z'o}{zy}$ is the shearing force, *i.e.*, if $z'o$ be in inch-tons, and zy in inches, $\frac{z'o}{zy}$ will be in tons, and will represent a *force*, not a *moment*.

he beam be cut into horizontal slices the layers will slide on ›ther as in *Fig.* 103, and in the uncut beam there is the endency to slide. In *Fig.* 100, if we imagine the web severed ntally from the upper flange, the beam will no longer be ' as a whole, but only the upper flange can be considered as a

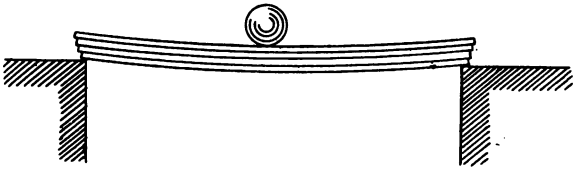


Fig. 103.

n this it will be understood how important a function the web n a girder, uniting the flanges into one beam, transmitting the s, and transforming them from vertical into horizontal ones.

t the horizontal shearing stress is equal to the vertical at every nay be proved as follows :—

small square be drawn on the neutral layer of an unloaded gular beam, as in 04, and then a load plied, it will be seen he square is distorted rhombus, as in the part of the figure. ve know that at the l layer there are no stresses. Hence the ion must be pro- by the vertical g stresses as shown › arrows VV and by rizontal arrows HH. ›re is equilibrium the

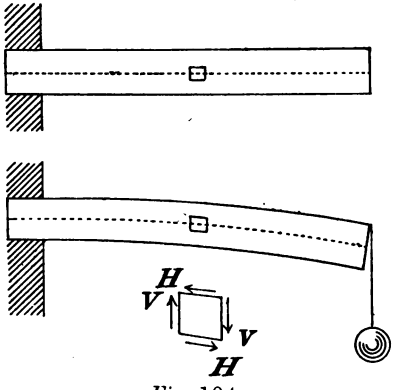


Fig. 104.

t produced by the action of each is equal, and the forces lves equal.

Rules for Finding Shearing Stress.

vertical shearing stress at any section of a beam is the l resultant of all the forces, acting on either side of that . Hence—

1. For cantilevers, to find the shearing stress at any section, take *the total weight* lying between that section and the free end.

2. For beams, whether supported or fixed, to find the shearing stress at any section take the *reaction at either support, minus any loads* between that support and the section in question.

As it is usually necessary to ascertain the maximum shearing force, it is generally sufficient to find the shearing stress at the supports, where it is equal to the reaction.

Various Ordinary Cases.

The shearing stress for various cases may be represented graphically.

Following the same order of cases as in Chapter IV. we have—

CASE 1.—*Cantilever with load at the end.*

In this case the shearing stress at every section is equal to the weight at the end, and is therefore a constant, and may be represented by the ordinates to a rectangle, as in *Fig.* 105.

Here it may be well to compare the diagram of moments with that of shearing stresses. *Fig.* 106 represents the former, and should be remembered that the ordinates in this case represent compound expressions, such as inch-tons, foot-lbs., etc. In the case

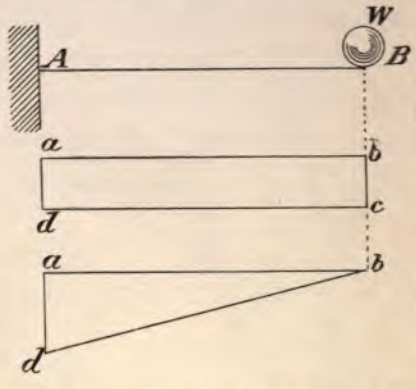


Fig. 105.

Fig. 106.

the shearing stress diagram, the ordinates represent forces—tons, lbs., etc. The diagram of shearing stress will, however, give the value of the Moments of Flexure, by taking the *area* of the figure, *e.g.*, in *Fig.* 105 the area of the rectangle *abcd* gives the value of the M_f at A, for that M_f is Wl, and in the figure *ab* represents l, and *bc* represents W. This rule applies in all cases.

CASE 2.—*Cantilever with several loads.*

Fig. 107 gives the diagram of shearing stresses.

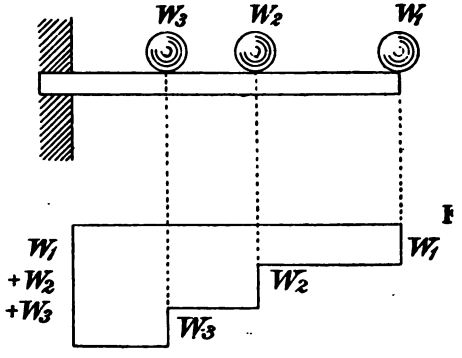


Fig. 107.

CASE 3.—*Cantilever with uniform load.*

Fig. 108 gives the diagram.

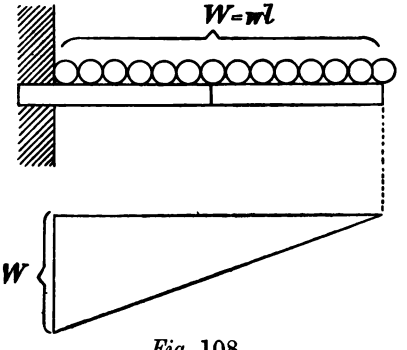


Fig. 108.

CASE 4.—*Cantilever loaded both at outer end and uniformly.*

Fig. 109 gives the diagram. It is simply the addition of Cases 1 and 3.

CASE 5.—*Beam supported at both ends, and loaded at some point with a single load.*

First find the reactions *ac*, *bd*. Then the shearing stress will be a constant between the supports and the load, and may, therefore, be

represented by two rectangles, as in *Fig.* 110. Here it is desirable to show the rectangles on either side of the beam line, to indicate the difference of the action of the shear, as shown further by the directions of the arrows. The difference of direction is sometimes referred to as positive and negative shearing stress. In this case the area of the rectangles is equal, being the value of the M_f under the load.

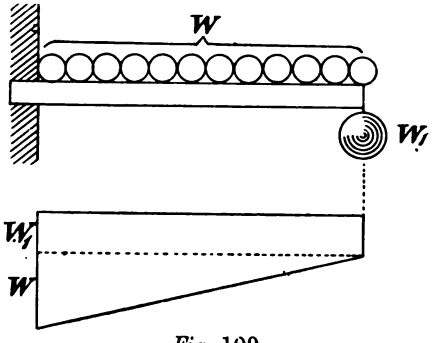


Fig. 109.

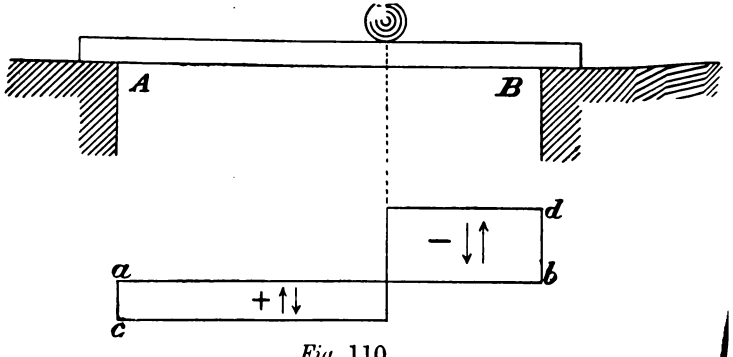


Fig. 110.

CASE 6.—*Beam supported and with a single load in the centre.*

Fig. 111 shows the diagram.

CASE 7.—*Beam supported and loaded with any number of loads at certain points* (*Fig.* 112).

Finding first the reactions, we know that the shear will be a constant up to the position of the next load, which will have to be deducted from the reaction. The point where the shearing stress

ıges sign will vary with the amount of the weights, but will ılly be under one of the weights near the centre.

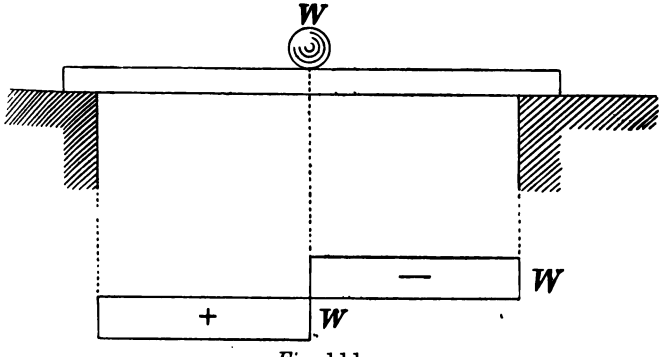


Fig. 111.

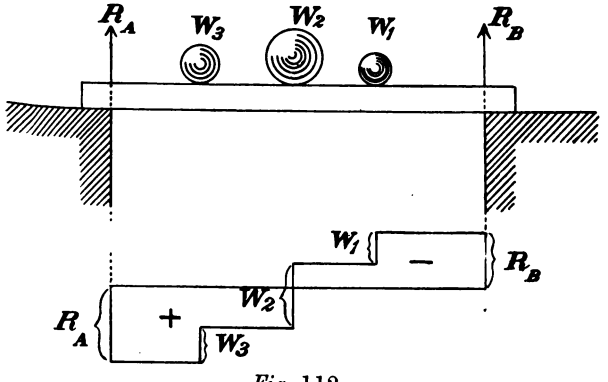


Fig. 112.

Case 8.—*Supported beam with weights equal and equally placed with ference to the centre.*

Here there is no shearing stress at all in the central portion ig. 113). The reason of this is that, in that central portion, all the ce is pressing downwards; there is no reaction pushing upwards. e effect of the equal weights is to shear the entire portion away at points C and D.

Case 9.—*Beam supported and uniformly loaded.*

Fig. 114 shows the diagram. The sign changes in the centre.

CASE 10.—*Beam supported and partially loaded with a uniform load.* From the points of support to the limits of the load the shearing stress will be a constant. Hence we first find the reactions, and draw rectangles as far as the limits of the load. Joining the opposite angles *cd* of these rectangles, we have the shearing stress in the beam beneath the load represented by the triangles, and the point O where the line cuts the beam line shows where the stress changes sign (*Fig.* 115). This point also shows us where the Moment of Flexure is a maximum, for that will occur where the shearing stress = 0. Hence this method is a simple way for finding the exact point of maximum M_f which could not otherwise be found except by the use of the calculus.

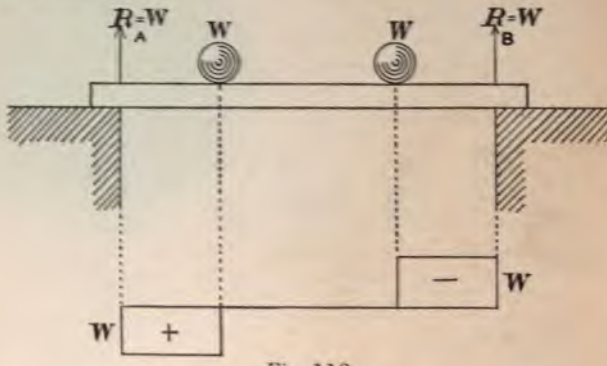


Fig. 113.

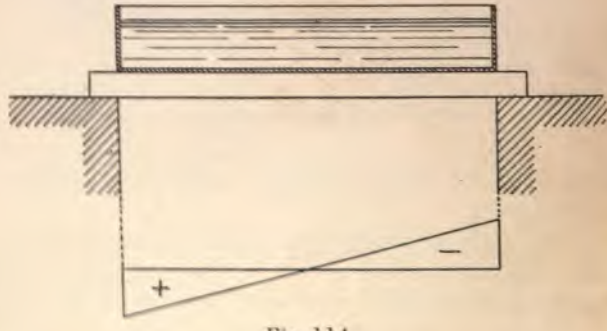


Fig. 114.

It is, however, seldom worth while to draw a shearing stress diagram at any particular section, for it is generally found easily

culation. As a rule, the maximum shearing stress is all that uired, and that is equal, invariably, to the reaction.

At the same time shearing stress diagrams are valuable, in showing a glance the variations in this stress for different cases, in showing relation between the Moments of Flexure and the shearing stress, l occasionally (as in Case 10 above) they afford a simple means for ling out the position of the maximum M_f. In cantilevers, the ition of the maximum shearing stress coincides with the maxi- um M_f, whereas in supported beams the reverse occurs, and in ed beams, the maximum shear occurs at the point of fixing, the nimum under the greatest weight.

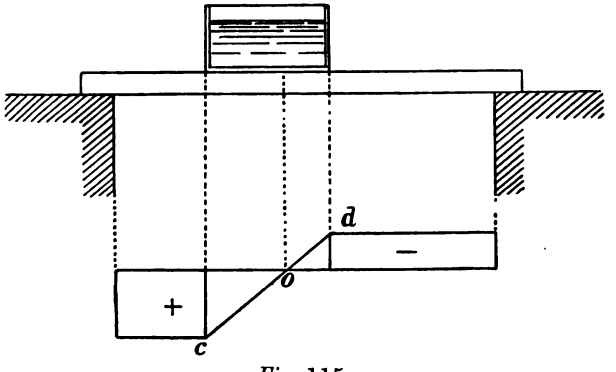


Fig. 115.

*Distribution of the Shearing Stress.**

The shearing stress is not equally distributed over the cross section a beam. For we have seen above that, in any two adjacent layers a beam which is loaded in any way, the shearing stress supplies e difference between the Moments of Flexure or the direct stresses. or instance, in *Fig.* 116, let HFD be a projection of a transverse ction of a beam of uniform section, and GEB another section very ose to it.

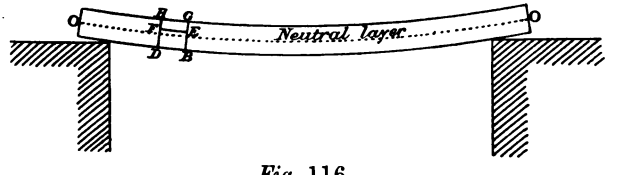


Fig. 116.

* Chiefly taken from General Wray's *Instruction in Construction*

If the M_f at HFD differs from that at GEB, there must be a corresponding difference in the amount of direct stress on two corresponding parts of the section planes, such as GE and HF. The difference of these direct stresses is a horizontal force acting on the solid HFEG tending to force it away from the centre, and in order to maintain equilibrium, the amount of the shearing stress on the plane EF must be equal and opposite to this horizontal force. The amount of the difference of the direct stresses at GE and HF, divided by the area of the plane FE, gives the *intensity* of the shearing stress, *i.e.*, in lbs. per square inch, or similar quantities.

Now the longer GE and HF are, the greater will be the direct stresses on them, and the greater the difference of those stresses, hence the greater will be the shearing stress on FE as it approaches the neutral axis OO'.

If we take a beam, of any material, and cut it into slices, horizontally, we get ocular proof of this, for it will be seen that the overlap of the layers will be greater if we cut the beam in two slices down the middle, than if we cut it into several slices.

From this we get the broad principle that the shearing stress is a maximum at the neutral axis and a minimum at the edges of the beam. This is the converse of the direct stresses.

In a rectangular beam the shearing stress varies as the ordinates of a parabola, where the greatest is at the neutral axis, and is equal to the following expression, viz., $\frac{3R}{2db}$, where R = the total shearing stress, and d = the depth of the beam in inches, or any other unit. For proof, see Appendix to this Chapter).

This may be expressed by saying that the *average* value of the shearing stress throughout the depth of the beam is two-thirds of the maximum intensity.

This is important in connection with timber beams, for, as has been pointed out in Chapter III., the shearing resistance along the fibres is, as a rule, much less than the shearing resistance transverse to the fibres, and also it has been found from experiment that beams frequently begin to give way, when under transverse stress, by shearing along the neutral layer.

This investigation is also of practical importance as showing what ratio of depth to length in timber beams should not be exceeded, if failure by shearing longitudinally is to be avoided.

For if we take a beam of rectangular section, supported at both ends and uniformly loaded, we know that

$$\frac{Wl}{8} = \frac{rbd^2}{6} \text{ (see Chapter V.).}$$

i.e.,
$$W = \frac{4rbd^2}{3l},$$

and from consideration of shearing, if S = total amount of shearing force, s = average intensity of shear, $sbd = S = \frac{W}{2}$, and the maximum intensity of resistance, $s' = \frac{3}{2}s$.

$$\therefore \quad W = \frac{4bds'}{3}.$$

Hence
$$\tfrac{4}{3}\frac{rbd^2}{l} = \tfrac{4}{3}bds',$$

i.e.,
$$d : l :: s' : r.$$

If the load is at the centre $d : l :: 2s' : r$.

From experiments made by Professor Lanza with American timbers, the average values of r are

Spruce (*Abies nigra*)	...	...	...	4451 lbs.	per square inch.
Yellow pine (*Pinus mitis*)	...	...	...	7486	„ „

and the values of s' *along* the grain are

Spruce	...	...	...	...	...	191	„ „
Yellow pine	...	...	...	...	...	248	„ „

"These values for s' are less than those obtained by direct experiments on shearing resistance, as in these cases the specimen is compelled to shear along a particular plane, whereas in testing an ordinary beam, failure naturally occurs along the line of least resistance. In the experiments referred to, failure occurred as often by shearing along the grain as by direct tensile stress developed at the extreme fibre."

We see then that $d : l :: 191 : 4451 = 1 : 23$ for spruce, and $\frac{1}{30}$ for yellow pine, load distributed in each case. With central loads the proportions would be 1 : 11·5 and 1 : 15.

If the depth of the beam *exceeds* the proportion of the span thus arrived at, the beam may fail by longitudinal shearing along the grain; if it is less there will be no danger of shearing.

With the ordinary timber used in England, viz., Scots fir (*pinus sylvestris*), the proportion would be, for a distributed load, about 1 to 10. This proportion is seldom likely to be exceeded in practice. Hence in investigating ordinary cases it is quite sufficient to consider the transverse stress alone, and neglect the shearing stress.

It is true that for ordinary purposes the distribution of the shearing stress is not considered. In built-up iron and steel girders, the stress is taken as uniformly distributed. This assumption is on the side of safety, because the point to be determined is the stress where the flanges and web join. At this point the actual stress is less than the average, and hence, if taken as equal to the average, we are on the side of safety.

APPENDIX.

The proof of the parabolic distribution of shearing forces in a rectangular beam is as follows:—In Fig. 117 let MN, QR be two adjacent vertical sections of a rectangular beam. Let the triangle AOM represent the direct stresses at MN, and the triangle BOM represent the direct stresses at QR. Let OO be the neutral layer and let OM or OQ = y.

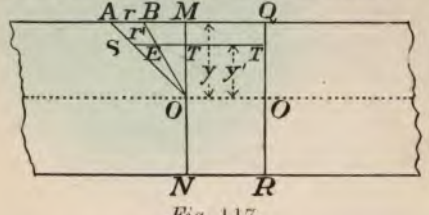


Fig. 117.

Let AB, or the difference of the direct stresses AM, BM, on the outer fibre distant y from the neutral axis = r. Let SE, or the difference of the direct stresses ST, ET, on the fibres distant y' from the neutral axis = r'.

Then the area of the trapezium AE = $\frac{1}{2}(r+r')(y-y') = s$ or the shearing stress on TT′, inasmuch as it represents the difference of the direct stresses acting at QT and MT.

Now from similar triangles—

$$r : r' :: y : y', \text{ or } r' = ry' \div y.$$

Substituting this value of r' in the above value of s—

$$s = \tfrac{1}{2}(r + ry' \div y)(y - y') = \frac{r}{2y}(y^2 - y'^2).$$

This is the equation to a parabola, where the origin of co-ordinates is at a point in the axis, whose distance from the vertex represents the shearing stress on OO′, *i.e.*, $\frac{1}{2}ry$, which is the value of s when $y' = 0$, and also the area of the triangle AOB. As this triangle represents the difference of the two triangles AOM, BOM, or the total direct stresses at the sections MN, QR, gives the total shearing stress on OO′.

The distribution of the shearing stress may be represented graphically as in *Fig.* 118, in which ac represents the shearing stress at the neutral layers, s' represents the shear at TT′ distant y' from OO, and the area of the parabola represents the total shearing stress on the whole section. Now as the area of

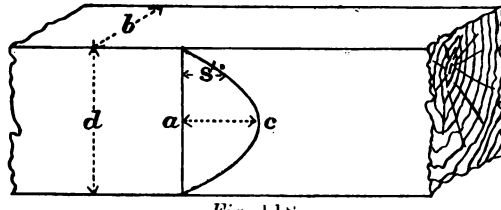


Fig. 118.

a parabola is two-thirds of the circumscribing rectangle, if we write R = the total shearing stress, and S = the maximum intensity at the neutral layer = ac, and if b and d represent, as usual, the breath and depth of the beam, we have

$$R = \tfrac{2}{3}d \times ac = \tfrac{2}{3}d \times Sb \text{ or } S = \frac{3R}{2db}.$$

The above shows the distribution for a particular case. The general equation showing the value of the intensity of the shearing stress in the cross section of a beam of any form is shown by Professor Rankine to be

$$q' = \frac{S}{Ib'} \int_{y'}^{y_1} ybdy,$$

where q' = intensity of shearing stress on any beam,

S = total shearing on the section,

I = Moment of Inertia of the section about its neutral axis,

b' = breadth of section where stress intensity = q',

y' = distance of same plane from the neutral axis,

y_1 = distance of extreme fibre on that side of the neutral axis, from that axis,

b = breadth of section at distances y from NA.

Practically, however, the shearing stress is considered as uniform all over the section.

CHAPTER VII.

Deflection of Beams.

General Remarks.—Principles of Calculation.—Beams of Uniform Strength and Depth, Special Considerations.—Limits of Deflection Admissible.—Appendices.

General Remarks.

The elastic deflection of beams is a subject that, in all permanent engineering works, is second only in importance to the question of strength. The beam or girder must be not only strong enough to sustain the anticipated loads without breaking, but must be sufficiently stiff to bear the loads without being strained beyond certain limits, those limits being determined in the first instance by the consideration that no part of the beam shall be strained beyond the limit of elasticity, and in the second place, that the deflection produced shall not cause injury to any parts of the structure which are dependent on the beam, *e.g.*, the plaster on a ceiling, or the slates on a roof.

In temporary structures this consideration is of comparatively little consequence, because there the requirements are fulfilled if the beam is strong enough (with a margin of safety) to carry the loads, and also because the element of elasticity, which enters into the question, varies very largely in the material of which temporary structures are usually built, viz., timber, so that calculations made upon the basis of stiffness of seasoned stuff would be quite unreliable if applied to freshly cut logs. Hence, for field operations, deflection may be usually neglected, and it may be fairly assumed that all beams used in field engineering, if strong enough, are stiff enough for all practical purposes.

The case, however, is quite different in permanent structures. The laws which govern the elastic deflection of beams are quite different from those which rule the strength, and it very frequently happens that a beam which is quite strong enough to bear, with the usual factor of safety, any ordinary load, will, with the same load, exhibit

leflection that is quite in excess of permissible limits. It is there-
:e necessary to calculate the dimensions of beams both for strength
d stiffness, and to take the dimensions which come out the largest
the scantling to be adopted. This may, at first sight, appear to
'olve a great amount of labour, but as the processes for all ordinary
ses may be resolved into comparatively simple formulæ, as will be
reafter pointed out, it will be seen that the labour involved is not
eat, and not to be compared to the disadvantages of trusting to a
re rule-of-thumb allowance, or to tables given in pocket books,
ich are often only partially applicable to a given case.

In the case of bridges, the rules for deflection often afford a
iable guide as to the speeds allowable for trains crossing; for it
possible to calculate from the observed deflection the maximum
ess intensity on the parts of the structure most exposed to stress,
d if the speed of a passing train produce a deflection that indicates
approach to the working stress for the material, it is clear that
e speed in future must be restricted to something less than that
as observed. For instance, let it be assumed that the deflection
a bridge with a train passing at 40 miles an hour be 1″, and that
is deflection, on calculation, corresponds to a stress intensity of
tons per square inch, then, if the bridge is built of mild steel
nich is only supposed to be subjected to a working stress of 7 tons
r square inch, it is clear that instructions must be issued limiting
e speed to less than 40 miles an hour. An example of this is
ven below.

Another advantage of the deflection principle is that by it one
ay ascertain in some cases the quality of the material. Thus it is
t unusual, in the case of spars required for telegraph poles or some
ch work, to specify that the spars of a given diameter shall, when
d horizontally on supports at a definite distance apart, and loaded
th a definite load, not deflect beyond a certain limit. From this
is possible to calculate the modulus of elasticity of the specimen,
d by comparing it with that of good timber of the same class,
lge of the value of the specimen. Thus the deflection gives
proximately an index of quality.

Principles of Calculation.

When a beam deflects, its neutral line, or line passing through the
utral axis of every section, becomes, if originally straight, concave
pwards, or if originally convex upwards, tends to become hori-
ontal.

If the equation to the curve can be determined by referring co-ordinates, the deflection at any point, and therefore at the p where it is a maximum, can be calculated.

It may be here stated that the deflection is affected by all the siderations which affect the moments both of flexure and resistan and also by the elasticity of the material.

In *Fig.* 119 let OAO represent the neutral line of a bent bea then at any section the radius of curvature may be expressed fr the expression $\frac{1}{\rho} = \frac{M_f}{EI}$, where ρ is the radius of curvature, and M_f E and I the values usually given to these symbols.

The proof of this should be studied, as it explains clearly the principles on which the above fundamental expression is arrived at.

Let BC, DE (*Fig.* 120) be two sections of a bent beam very near together, and let OO be the neutral line of the beam, then, as the sections are close together, AL will be the common radius ρ. The fibre AM, on the neutral line, is the same length that it was originally, and if we draw MF parallel to AB, BF will be equal to AM, and therefore FD will represent the elongation in the fibre BD.

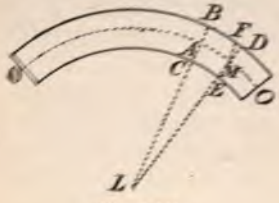


Fig. 119.

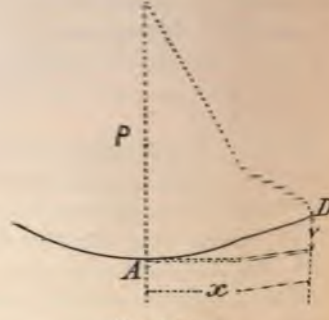


Fig. 120.

But since, by Hooke's law, the strain is proportional to the stress, FD represents the intensity of stress r at the distance y from the neutral axis; for the same reason AM will on the same scale represent the modulus of elasticity E, since from the definition of the term the modulus of elasticity represents that stress intensity which would cause an elongation as great as the original length (see Chapter I.).

Then, from the similar triangles DFM, MAL—

$$FD:FM::AM:AL,$$

or

$$r:y::E:\rho.$$

But we know that

$$M_f = M_r = \frac{rI}{y} \text{ (see Chapter V.).}$$

Hence

$$M_f = \frac{EI}{\rho} \text{ or } \frac{1}{\rho} = \frac{M_f}{EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Thus we have an expression for the radius of the deflection curve in terms with which, from previous investigation, we are already familiar.

To express this in terms of the ordinates x and v (*Fig.* 120) we shall always have, no matter where the origin is taken,

$$\frac{1}{\rho} = \frac{-\frac{d^2v}{dx^2}}{\left(1+\left(\frac{dv}{dx}\right)^2\right)^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

For proof see Appendix I.

This formula is too complicated to be integrated, except by approximation, and that which is usually made is as follows :—

Since the slope of deflection is necessarily small, the angle which the deflected neutral line makes with the horizontal is everywhere small, and $\frac{dv}{dx}$ is very small. $1+\left(\frac{dv}{dx}\right)^2$ is therefore practically $=1$.

Hence, substituting in (2) the value of $\frac{1}{\rho}$ from (1) and making $1+\left(\frac{dv}{dx}\right)^2=1$, we have—

$$\frac{M_f}{EI} = -\frac{d^2v}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

which is the fundamental equation for computing the slope and deflection of a loaded beam.

By one integration of this we find the slope whose tangent is $\frac{dv}{dx}$; by a second integration we find the value of the deflection v at a distance x from the origin, and by substituting any value of x we obtain the value of the deflection in terms of known expressions.

The integration of (3) differs for different forms of beams, on account of the variations which occur in the values of M_f and I. In practice, however, there are two classes of beams which need be considered :—

(1). Beams of uniform cross section throughout, where the value

of I is constant for all sections. To this class belong rectangular beams and rolled joists.

(2). Beams built up so as to have a uniform unit stress on each flange throughout. The *strength* and *depth* is uniform. To this class belong all built up girders with parallel flanges.

There are, of course, other classes of beams, but the above comprise those most usually met with.

An example of the integration of formula (3) in the case most usually met with, viz., beams of uniform cross section supported at both ends and uniformly loaded, is given in Appendix II.

The general formulæ for maximum deflection are as follows :—

$$V = \frac{nWl^3}{EI}, \text{ or } = \frac{n'rl^2}{Ey} \quad \ldots\ldots\ldots\ldots (4), (5),$$

the former being used when it is desired to ascertain the deflection *under a given load*, and the latter when it is desired to ascertain the deflection under a *given intensity of stress.*

The values of the symbols used above are as follows :—

V = the maximum of deflection in inches.

n and n' are factors given in tables below.

l = length of the beam in inches.

W = weight on the beam in lbs. or tons.

E = modulus of elasticity in lbs. or tons (same as W, *i.e.*, if W is in tons, E must be also).

I = Moment of Inertia in inches.

y = distance of extreme fibre from neutral axis in inches.

r = limiting intensity of stress at y, in lbs. per square inch if E is in lbs., and in tons if E is in tons.

TABLE XIX.

Values of n *and* n′ *in the General Formulæ for Deflection.*

	n	n'
Uniform Cross Section.		
1. Cantilever, load at any point	$\frac{1}{3}$	$\frac{1}{3}$
2. ,, loaded uniformly	$\frac{1}{8}$	$\frac{1}{4}$
3. Beam supported, central load	$\frac{1}{48}$	$\frac{1}{12}$
4. ,, ,, uniform ,, ...	$\frac{5}{384}$	$\frac{5}{24}$
5. ,, fixed both ends, central load	$\frac{1}{192}$	$\frac{1}{24}$
6. ,, ,, ,, uniform ,, ...	$\frac{1}{384}$	$\frac{1}{12}$
Uniform Strength and Depth.		
7. Cantilever, load at any point	$\frac{1}{2}$	$\frac{1}{3}$
8. ,, ,, uniform	$\frac{1}{4}$	$\frac{1}{3}$
9. Beam supported, central load	$\frac{1}{24}$	$\frac{1}{3}$
10. ,, ,, uniform load	$\frac{1}{64}$	$\frac{1}{3}$

The value of n' can be easily calculated from the value of n as follows:—For example, take Case 3, *i.e.*, a beam supported at both ends and loaded in the centre, in which $W = \frac{1}{48}$. We know from the principles laid down in Chapter IV. that M_f in this case $= \frac{Wl}{4}$, and

that $$M_f = M_r = \frac{rI}{y}.$$

$$\therefore \quad \frac{Wl}{4} = \frac{rI}{y},$$

hence $$\frac{r}{y} = \frac{Wl}{4I}.$$

Substituting this value of $\frac{Wl}{4I}$ in the equation $V = \frac{1}{48} \frac{Wl^3}{EI}$, we have

$$v = \frac{r}{y} \times \frac{l^2}{12E} = \frac{1}{12} \frac{rl^2}{Ey},$$

hence the value of n', as shown in the column, is $\frac{1}{12}$.

Beams of Uniform Strength and Depth, Special Considerations.

These beams are of such frequent occurrence in bridges that it may be well to devote some further attention to them.

In these girders the design is such that the unit stress, or intensity of stress, is uniform throughout the whole length of the flange. Now as the *strain* is proportional to the *stress*, within elastic limits, the elongation or contraction on the flanges will be uniform through out the length of the flange. Hence the deflection curve produced will be circular, since the circle is the only curve that is due to a uniform cause. (For further proof that the curve is a circle see Appendix III.). When the beam is loaded the unit flange stress may be determined, and the amount of compression and extension in the flanges calculated. From these and from the modulus of elasticity the deflection may be determined by means of a simple equation,* as follows:—

Let *Fig.* 121 represent a girder supported at both ends and loaded. The deflection curve will be the arc of a circle, the centre of which is at O.

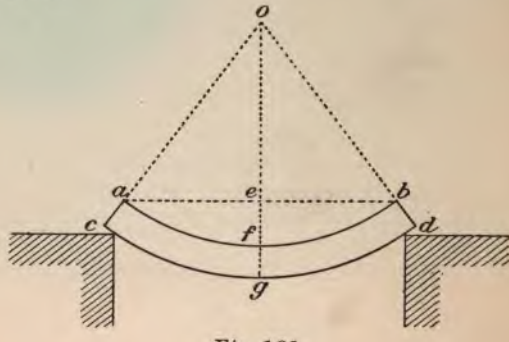


Fig. 121.

Let l = length of top flange *afb*.
l_1 = „ „ bottom flange *cgd*.
V = *ef* the deflection.
d = depth of girder *fg*.
ρ = radius of curvature of top flange.

Then, as the deflection is small compared to the radius, O*e* may be taken as = O*f*, and *aeb* is nearly = l.

* From Anglin's *Design of Structures.*

Then from the properties of the circle (*Euclid*, III., 35)—

$$2\rho \times V = \tfrac{1}{4}l^2 \text{ or } V = \frac{l^2}{8\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6).$$

By similar triangles we get

$$\frac{\rho}{\rho + d} = \frac{l}{l_1} \text{ or } \rho = \frac{dl}{l_1 - l} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7).$$

Substituting this value of ρ in (6)—

$$V = \frac{l(l_1 - l)}{8d} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8),$$

which is a convenient expression for the value of the deflection when the contraction and expansion of the two flanges respectively are known. But as these are generally not easily determined, we have to go a step further and express them in terms of the flange stress and modulus of elasticity. As the length of the top flange is shortened by $\frac{1}{2}(l_1 - l)$ we have

$$r : E :: \tfrac{1}{2}(l_1 - l) : l$$

where r is the flange-stress in tons per square inch, or

$$l_1 - l = \frac{2fl}{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots(9).$$

Substituting this value in equation (3) we obtain

$$V = \tfrac{1}{4}\frac{rl^2}{dE} = \tfrac{1}{8}\frac{rl^2}{Ey} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10).$$

as found previously (see Table XIX.).

If l and d be expressed in feet, Equation (10) becomes

$$V = \frac{3rl^2}{dE} \quad \ldots\ldots\ldots\ldots\ldots\ldots(11).$$

The use of these formulæ may be exemplified by the following:—

EXAMPLE 15.—*A railway bridge of mild steel, 100′ span and 8′ deep, was found, in testing, to deflect under a train passing at 40 miles an hour 2 inches at the centre. Is this speed permissible?*

From Equation (11)

$$r = \frac{2 \times 8 \times 13{,}000}{3 \times 10{,}000} = 6{\cdot}92 \text{ tons per square inch,}$$

which, with mild steel having a tensile resistance of 30 tons, gives a

factor of safety of 4½. The speed of 40 miles an hour will, therefore, be permissible, but should not be exceeded.

EXAMPLE 16.—*What would be the deflection of a bridge* 100′ *span* 8′ *deep, and net sectional flange area at the centre of* 52 *inches, with uniform load of* 2 *tons per foot run?*

Load $= 100 \times 2 = 200$ tons.

Flange stress at centre

$$= \frac{200 \times 100}{8 \times 2},$$

i.e.,

$$\left(\frac{W}{8} \times \frac{l}{d}\right) = 312.$$

Intensity of stress per square inch

$$= \frac{312}{52} = 6 \text{ tons.}$$

Hence we have

$$f = 6,\ l = 100,\ d = 8,\ E = 13{,}000.$$

Substituting these values in Equation (11)—

$$Y = \frac{3 \times 6 \times 1{,}000}{8 \times 13{,}000} = 1{\cdot}68 \text{ inches.}$$

In the above, it will be noticed that only the elongation of the flanges of t girders are considered in estimating the deflection. Whether an engineer justified in neglecting the effect of the web, or of the braces, in resisti deflection, is a matter on which scientific experts differ. On the one ha Professor Rankine has shown that in a girder of the ordinary proportions t deflection due to web strains does not exceed ⅓ of the whole, and l opinion has been adopted by such eminent investigators as Mr. Stoney, M Claxton Fidler, and Professor Warren. On the other hand the most rece American authorities, Professors Johnson, Turneaure, and Mr. Bryan, hc that this view of the case as regards framed structures is "erroneous a grossly misleading." To the ordinary student such conflict of opinion is ve perplexing, but it is at all events satisfactory to find that the results observation, recorded by Professor Warren, on a number of plate web gird bridges of 60′ span, and on a lattice girder of 156′ span, agreed most accurate with, being if anything rather less than, the calculated deflection based o formula very much the same as Formula (10) given above. The tests w carried out by bringing on the girders locomotives, the weights of whi distances between centres, etc., were accurately known. The bridges w carefully measured, and the stress intensities in the flanges calculated.

In the light of the agreement of theory and practice afforded by these tes it is considered that the rules given above may be taken as practically g

enough for the bridges likely to be erected by those for whom this book is intended.

The initial "camber" which is required in order to bring any proposed girder into a horizontal line when loaded may be obtained either by giving it a rise in the centre equal to the calculated deflection, plus 25 per cent., or, in the case of braced structures, by making the initial length of each strut greater than its designed length in the ratio of $\left(1 + \frac{r}{E}\right)$ to 1, and at the same time making every tie shorter than its designed length by $\left(1 - \frac{r}{E}\right)$ to 1. These small additions or dimensions are sometimes specified on the working drawings of the bridge.*

It is well also to note that "the modulus of elasticity in a riveted girder will depend upon the number of joints, and the degree of tightness of the rivets, as well as the material. Experiments made by Sir B. Baker, M.Inst.C.E., on small riveted girders show that the rivets behaved as clamps for ordinary working stresses, the deflections being practically the same as if the girders were rolled in one piece.

"In a plate web girder bridge, however, it will generally happen that the modulus is 20 per cent. less than the material of the girder."†

It must be remembered that the deflection test does not afford a reliable indication of *strength*.

It does not follow that if the structure is weak, owing either to bad workmanship, design or material, it will exhibit any unusual deflection under the test load.

Permissible Limits of Deflection.

In timber beams, such as floors, roofs, etc., the limit usually taken, as laid down in Tredgold's *Carpentry*, is $\frac{1}{40}''$ per foot run of span, or $\frac{1}{480}$ the total span, which is equivalent to $\frac{1}{240}$ of the span in cantilevers. It must be remembered that this is an arbitrary limit; fixed; however, by the opinion of experts.

As a matter of fact, beams calculated on this basis give very satisfactory results, but are generally of a greater scantling than is required for strength, except when fixed.

* Mr. Claxton Fidler, *Bridge Construction*, p. 133.
† Prof. Warren, *Engineering Construction*, p. 156.

With a rectangular beam uniformly loaded, the formula

$$V = \frac{nWl^3}{EI}$$

becomes

$$\frac{l}{480} = \frac{5}{384} \times \frac{Wl^3}{E} \times \frac{bd^3}{12},$$

which, as will be shown in a subsequent chapter, is capable of being simplified to a very convenient form for practical work.

For girders the usual deflection allowed is from $\frac{1}{1500}$ to $\frac{1}{2500}$ of the span. For the proof load on bridges Rankine gives the deflection from $\frac{1}{200}$ to $\frac{1}{600}$ part of the span.

APPENDIX I.

To express the value of the radius of a circle in terms of the ordinates of a point on its circumference referred to an origin at any point.

Let the origin be the point O (*Fig.* 122), and let the ordinates of the centre of the circle be a and b. Let the radius be ρ.

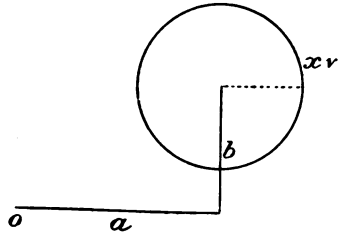


Fig. 122.

Then $$(x-a)^2+(v-b)^2=\rho^2 \quad \ldots\ldots (1).$$

Differentiating twice—

$$2(x-a)+2(v-b)\frac{dv}{dx}=0 \quad \ldots\ldots (2).$$

$$1+(v-b)\frac{d^2v}{dx^2}+\left(\frac{dv}{dx}\right)^2=0.$$

$$\therefore \quad v-b=\frac{1+\left(\frac{dv}{dx}\right)^2}{\frac{d^2v}{dx^2}} \quad \ldots\ldots (3).$$

Substituting this value in (2)—

$$x-a=-\frac{1+\left(\frac{dv}{dx}\right)}{\frac{d^2v}{dx^2}} \times \frac{dv}{dx} \quad \ldots\ldots (4).$$

Hence we have the terms a and b in Equation (1) both eliminated, and may substitute for $x-a$ and $v-b$ in that equation the values above in (3) and (4), *i.e.*—

$$\rho = -\frac{\left\{1+\left(\frac{dv}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2v}{dx^2}}.$$

APPENDIX II.

To prove that the deflection in a beam of uniform cross section, loaded uniformly, is $\frac{5}{384}\frac{Wl^3}{EI}$.

As the origin of the ordinates in previous figures in this chapter has been taken at the lowest point of the curve it will be as well to do the same in this investigation.

Let AOB (*Fig.* 123) be the beam loaded uniformly with a unit weight w per unit of length. Let l be the length, E the modulus of elasticity, and I the Moment of Inertia.

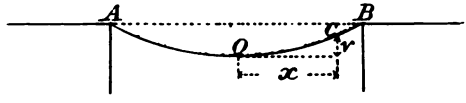


Fig. 123.

Let the ordinates of any point C referred to O be x and v.

Then M_t at C

$$= -\frac{wl}{2}\left(\frac{l}{2}-x\right)+\frac{w}{2}\left(\frac{l}{2}-x\right)^2 = -\frac{wl^2}{8}+\frac{wx^2}{2}.$$

Since $$\frac{d^2v}{dx^2}=\frac{M_t}{EI},\quad EI\frac{d^2v}{dx^2} = -\frac{wl^2}{8}+\frac{wx^2}{2}.$$

Integrating— $$2EI\frac{dv}{dx} = -\frac{wl^2x}{4}+\frac{wx^3}{3}+C.$$

$$\text{When } \frac{dv}{dx}=0,\ x=0 \text{ and } C=0.$$

Integrating again—

$$2EIv = -\frac{wl^2x^2}{8}+\frac{wx^4}{12}+C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a),$$

$$\text{when } x=0,\ C=0.$$

(a) represents the general equation to the deflection curve. To find the maximum deflection, make $x=\frac{l}{2}$.

Then $$2EIv = -\frac{wl^4}{32}+\frac{wl^4}{192} = -\frac{5}{192}wl^4.$$

But $$wl = W.$$

Hence $$wl^4 = Wl^3.$$

Hence $$v = \frac{5}{384}\frac{Wl^3}{EI}.$$

APPENDIX III.

To prove that the radius of curvature in a beam of uniform strength and depth is a circle.

In the general expression $\rho = \frac{EI}{M_f}$, there may be two variables, viz., M_f and I. Before the general equation can be integrated, it is necessary to find an equivalent value for I which shall make the general equation true for every section of the beam.

Let M_{ff} and I_1 be the Moments of Flexure and Inertia at the section of greatest stress, and M_f and I similar moments at any other section.

Then $M_{ff} = \frac{rI_1}{y_1}$ and $M_f = \frac{rI}{y}$. But r is uniform in the case under consideration, hence

$$\frac{M_f}{M_{ff}} = \frac{Iy_1}{I_1 y} \text{ and } M_f = \frac{M_{ff} I y_1}{I_1 y}.$$

Hence the general equation for the radius of curvature

$$= \frac{EI_1 y}{M_{ff} y_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta).$$

But in the case of a beam of uniform strength and depth $y_1 = y_1$ and M_f and I_1 are both constants, hence the radius of curvature is a constant, *i.e.*, the curve is circular.

CHAPTER VIII.

Application of the Principles of Strength and Deflection to Timber and Iron Beams.

Timber.—Useful Formulæ for Ordinary Cases.—Examples.—Proportions of Breadth to Depth.—Flitched Beams.—Cast-Iron Beams.—Examples.—Rolled Wrought-Iron and Steel Beams.—Transverse Strength of Railway Rails.—Table of Scantlings.

Having investigated the rules that govern the strength and deflection of beams under transverse stress, it is now proposed to give a few examples of the application of these rules to ordinary cases.

Timber.

In permanent works, timber beams are always rectangular in section, as it is not usually worth while to cut them to any other form.

As regards *strength*, in similar timber beams of equal lengths, equally loaded, the strength increases directly with the breadth and with the square of the depth. The *stiffness* varies with the breadth and the cube of the depth, and the shearing resistance increases with the sectional area. Hence for equality of strength and stiffness the deeper a beam can be made (provided its sectional area is large enough to resist the shearing forces) the smaller its sectional area will be, and the more economical will be the construction, other things being equal. There are, of course, practical limits to the depth of beams actually used, because (1) in ordinary construction every beam should have breadth sufficient to secure stability, when placed on its edge; (2), it should have breadth enough to enable it to be fastened to other parts of the structure, and to have other parts fastened to it. Another limit (3) is imposed by the dimensions of beams ordinarily obtained in the market.

A familiar instance of beams of the least possible breadth is afforded in the joists of a floor, where, in order to admit of plaster

to the ceiling beneath, the width is limited to as small an amou possible, but yet must be sufficient to admit of the joists l fastened to the wall-plates on the one hand, and on the other l of admitting the ends of two adjacent floor-boards to be nailec the top. Thus the width is generally not less than 2″, and not n than 3″, lateral stability being ensured by strutting.

In the ordinary cases of timber beams used in buildings, the l may be regarded as uniform, and the beam may be regardec supported only.

It is true that in many cases the beams are fixed, but thro imperfect workmanship such fixing may only be partial, and i safer to consider them as supported only.

Further, as all the timbers required to bear transverse loading building, or, indeed, in all the buildings in a locality, are, as a r taken from one class of tree, it is possible to reduce the rules ordinary calculation to one or two very simple formulæ.

One word of caution is necessary. Formulæ are very useful serv to those who understand the principles on which they are based, they become very dangerous in the hands of those who are ignor who often apply them to cases to which they do not refer. L be very clearly understood, therefore, that the following form refer to one particular class, the most usual class, of bea they refer to certain kinds of timber, and that any varia of these conditions must be investigated on first princi and not worked out by these formulæ.

For instance, the timber that is universally used in Englanc the beams, rafters, joists, etc., of dwellings is red fir (*pinus sylve* whose modulus of rupture may be taken at 5,040, and modul elasticity at 1,440,000 lbs. The formula for ascertaining the stre of a beam uniformly loaded and supported is

$$\frac{Wl}{8} = \frac{rbd^2}{6} \quad \text{.....(}$$

and for stiffness

$$V = \frac{nWl^3}{EI} \quad \text{.....(}$$

and as the permissible deflection is $\frac{1}{480}$ of the span, we hav value of V as equal to $\frac{l}{480}$.

Now, still further to simplify matters, as the length of the ord beam is usually stated in feet and not in inches, and as the lo

usually estimated in lbs. per square foot, we may express the formulæ somewhat differently by calling $W = LSw$, where W = weight per square foot, S = distance apart in feet of two adjacent beams, and we may write L as the length in feet, equal to $\frac{1}{12}l$. Substituting in (*a*) these values and giving the value of *r*, with a factor of safety of $4, = \frac{5040}{4} = 1{,}260$, we have

$$\frac{LSw \times 12L}{8} = \frac{1260bd^2}{6},$$

or
$$bd^2 = \frac{L(LSw)}{140} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(c).$$

Similarly for deflection

$$V = \frac{L}{40} \text{ inches},$$

hence
$$\frac{L}{40} = \frac{5}{384} \frac{LSw \times L^3 \times 12^3}{1{,}440{,}000 \times \frac{1}{12} bd^3},$$

from which
$$bd^3 = \frac{L^2(LSw)}{133} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(d).$$

If, instead of using red fir, we had used the timber most usually employed in India for barracks in the plains, viz., sal (*shorea robusta*), which has a modulus of rupture of 6,000 and of elasticity of 1,850,000 (see Table XIV., Chapter III.), the values of the two equations (*c*) and (*d*) would be—

For strength
$$bd^2 = \frac{L(LSw)}{166} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(e),$$

and for stiffness
$$bd^3 = \frac{L^2(LSw)}{170} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(f),$$

the only differences being in the denominators of the fractions.

It is therefore possible to frame formulæ for any class of timber under the conditions named above, provided the moduli of rupture and elasticity are known.

To show the application of the formulæ, a few examples are now given.

EXAMPLE 17.—*The bridging joists of a floor in an ordinary English house, 1 foot apart centre to centre, have a clear span of 12 feet, the weight on the floor being 1¼ cwts. per foot super. Find the scantlings necessary.*

1. For strength, substituting values in formula (*c*)—

$$bd^2 = \frac{144 \times 1 \times 140}{140} = 144.$$

2. For stiffness—

$$bd^3 = \frac{1728 \times 1 \times 140}{133} = 1820.$$

We may either (i.) *assume* a width for b, and find from the values of bd^2 and bd^3 the greatest value of d, or (ii.) we may examine a table of the values of bd^2 and bd^3, and take any scantling that suits. Such tables are given at the end of this chapter.

In the first case, assuming $b = 2''$, we have $d = 8\frac{1}{2}''$ and 9·6 inches, whence we take the greater, say $9\frac{3}{4}''$.

In the second case we find that for strength we may take $2'' \times 8\frac{1}{2}''$, or $2\frac{1}{4}'' \times 8''$, or $3'' \times 7''$. For stiffness we may take $3'' \times 8\frac{1}{2}''$, or $2\frac{1}{2}'' \times 9''$, or $2'' \times 9\frac{3}{4}''$.

The practical advantage of the tables is that without calculation one can pick out a suitable scantling.

In the case in point $2\frac{1}{2}'' \times 9''$ would probably be very suitable in every way. Hurst's pocketbook gives $2\frac{1}{2}'' \times 8\frac{1}{2}''$, but floors with joists of this depth, though quite strong enough, are often deficient in stiffness.

EXAMPLE 18.—*A purlin in a king-post roof where the trusses are 10 feet apart and the length of the principal rafter 16 feet. The weight per square foot on the roof, including wind, snow, etc., may be taken at 45 lbs. Timber, deodar.*

This is the case of an ordinary timber roof for a barrack in the hills in India.

From Table XIV., Chapter III., we see that the Modulus of Elasticity for deodar is the same as for red fir, but that the Modulus of Rupture is considerably greater. Formula (*d*) may be taken as far as stiffness is concerned, while formula (*b*) becomes

$$bd^2 = \frac{L\,(LSw)}{225}.$$

The purlin is most probably "fixed" at the ends, but as such fixing may be modified by joints or framing, it is more usual to consider all roof timbers as supported only.

Here $L = 10'$, *i.e.*, distance apart of trusses.

$S = 8'$, *i.e.*, $\frac{1}{2}$ distance from purlin to pole plate + $\frac{1}{2}$ distance from purlin to ridge.

(1). Then, for strength—

$$bd^2 = \frac{L\,(LSw)}{225}$$

becomes

$$= \frac{100 \times 8 \times 45}{225} = 160.$$

4″ × 6½″ would satisfy this (*vide* Table XXII., p. 155).

(2). For stiffness—

$$bd^3 = \frac{l^3Sw}{133} = \frac{1{,}000 \times 8 \times 45}{133} = 2{,}700.$$

5½″ × 8″ is the scantling required for this.

In this case the dimensions necessary for stiffness are much greater han those for strength.

In purlins the limits of deflection might not necessarily be insisted pon with the same urgency as in the case of floors, or ceiling mbers, especially as, from the nature of the case, they are usually ted. But, on the other hand, careless workmanship may lead to ie cutting and notching of purlins at weak points; hence it is well give a good margin.

Proportions of Breadth to Depth in Timber Beams.

In the above examples it will be noticed that in the case of the ists of a floor the breadth was limited to the practical requirements carpentry, and the depth made as great as possible, whereas in e roof purlin the depth is only about half as much again as the eadth. This may naturally raise the question—What are the best oportions of breadth to depth to be given to a beam so as to cure strength and stiffness?

If the beam were being cut out of a round log so that the material utilized to the best possible advantage, it may be proved that the est proportions for b to d are when $b:d::1:\sqrt{2}$, when *strength* only required, and that $b:d::1:\sqrt{3}$ when *stiffness* is required. Although iese proportions hold good for beams when cut out of logs, it does ot follow that they represent the best proportions consistent with conomy in all cases. For it is evident that both for strength and tiffness the greatest economy will be attained, or least material will be required, when d is a maximum, and we, therefore, must aim at getting the greatest depth in our beams, consistent with practical requirements. For instance, a beam 7″ × 3″, placed with

the longer side vertical, is both stronger and stiffer than one $6'' \times 4''$, although it has less timber in it. Unfortunately, there is a prevalent idea in some quarters that the proportions for beams cut out of round logs are the best in all cases.

In the case of floors, we have seen that the limit of breadth is fixed by the least breadth for fastening floor boards. In other cases the limit is arrived at from considering how much width is required to give the beam a secure hold at the points of support. Generally this security is attained if $d = 2b$.

With purlins in a roof where the beam is placed so that no side is vertical, the proportions of b to d may be as 2 : 3.*

Flitched Beams.

Timber beams are often strengthened by being sawn in two longitudinally, and having a *flitch*, or plate of iron, introduced. Sometimes two flitches are given, at either side, the beam remaining intact, but the former method is the more usual. The various parts are fastened together by bolts (see *Fig.* 124). To investigate the strength of such flitched beams is, of course, a matter of no difficulty.

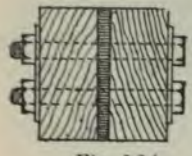


Fig. 124.

Let b be the breadth of the timber beam.

" t " " " " " iron plate.

" r " " modulus of rupture of the timber.

" r' " " " " " " " iron.

Then $M_r = rbd^2 \div 6 + r'td^2 \div 6$

$$= \frac{d^2}{6}(rb + r't).$$

To take an example, we may investigate the comparative strength of a beam of oak $12'' \times 6''$, with and without a flitch of $\frac{1}{4}''$ wrought iron.

The modulus of rupture of oak is 10,000 lbs. per square inch $= r$.

The modulus of rupture of wrought iron is 44,800 lbs. per square inch $= r'$.

* This is generally in accordance with English practice. In India they are generally made square, though there appears to be no adequate reason for so doing. Theoretically the ratio of b to d should be inversely as the resolved portions of the roof load acting along the roof slope, and normal to that slope.

Then with $\frac{1}{4}''$ flitch—

$$M_r = 12 \times 12 \div 6\ (10{,}000 \times 6 + 44{,}800 \times \tfrac{1}{4}) = 1{,}708{,}000 \text{ inch-lbs.},$$

thout flitch—

$$M_r = 144 \div 6 \times 10{,}000 \times 6 = 1{,}440{,}000 \text{ inch-lbs.}$$

The increase in strength is, therefore, 18 per cent. With a $7'' \times 6''$ oak beam, and a $\frac{1}{4}''$ plate, the increase would be about 37 per cent.

Cast-iron Beams.

Cast iron as a material for resisting transverse stress is not to be recommended. The objections to it are enumerated in Chapter III., and need not here be repeated.

Many fatal accidents have occurred through the sudden collapse of cast-iron girders in floors, and other places where they have been used.

One advantage, however, of cast iron is that it may be cast in any required form, and therefore the cross section of the girder may be made to vary with the variations in the M_f at various parts of the beam. There are also no joints in the structure to be a source of weakness. These advantages were considered so great in the early years of railway construction that many bridges were made with cast-iron girders. The failure of many of these (chiefly through hidden flaws in the casting) has caused this class of girder to be wholly prohibited for railway work, more especially as steel of suitable quality can now be so readily obtained.

Although not recommended for use, it is necessary to pay some attention to the principles of the design of cast-iron girders, as they are still often met with.

The difference in the resistance to tension and to compression in cast iron leads to a difference in the size of the tension and compression flanges.

The broad principles of design in metal of all kinds have already been alluded to in a previous chapter, and may be briefly stated as follows:—The material of any cross section is so arranged that it forms two tables or *flanges* at the extreme edges, where the direct stresses are greatest, connected with a vertical layer or *web* capable of resisting the shearing stress. The size of the flanges in wrought iron or steel

beams may be the same, as the resistance of those metals to tension and compression is nearly the same. In cast iron the tension flange is made about three times as large as the compression flange. Theoretically, the tension flange should be about six times as large as the other, but practically the proportion given is generally adopted. The reason for this is that with very small upper flanges there is no room for the load.

In cast-iron beams there should be no sudden variation in the thickness of the metal, because the thinner parts cool sooner than the thicker, and tend to produce internal stresses in the beam before the load comes on, and sometimes to cause hidden cracks. The thickness of the different parts should be such as to allow the fluid metal to fill every portion of the mould before it has time to lose any material part of its heat.

The proportions which have been found to fulfil these conditions approximately are as follows:—For flanges of 2′ in width, it is not wise to have a less thickness than $1\frac{1}{2}$″, and for flanges 18″ wide not less than 1 inch, while for narrower flanges a somewhat less thickness may be used. The thickness of the web should be the same as the flanges, and the angles in all parts of the beam rounded off.

Cast-iron girders seldom exceed 2′ in depth or width, a depth of 3′ being uncommon. The proportion of depth to span should be generally 1 to 12, but never less than 1 to 20. The width of the top flange is usually taken as from $\frac{1}{30}$ to $\frac{1}{40}$ span.

In calculation the principle of equal areas of resistance mentioned in Chapter V. is followed. In this case the resistance of the web is ignored, and it is assumed that the areas of the tension and compression flanges (a_t, a_c), as well as their breadths, are inversely proportional to their working resistances (r_t, r_c) and to their distances from the neutral axis, and that each flange constitutes an area of equal resistance (*Fig.* 125).

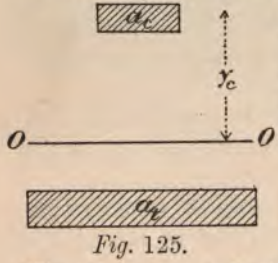


Fig. 125.

The Moment of Resistance may be expressed by the general expression $\frac{rI}{y}$, or less accurately, but more easily, by the area of either flange multiplied by its intensity of resistance, and by the distance between the centres of resistance of the two flanges, *i.e.*—

$$M_r = r_t a_t d = r_c a_c d.$$

Hence for a beam supported at both ends and loaded in the ntre—

If W = breaking weight in tons.

l = length in inches between supports.

d = distance between centres of flanges at central section in inches.

r_t = ultimate resistance in tension in tons per square inch.

r_c = „ „ „ compression „ „ „ „

a_t = area of tension flange in square inches.

a_c = „ „ compression „ „ „ „

Then $$\tfrac{1}{4}Wl = r_t a_t d = r_c a_c d.$$

If r_t be taken at 6·83 tons, the lowest value given by Professor nwin (see Chapter III.), we have W tons $= 6{\cdot}83 \times 4ad \div l$, or, putg l inches into L feet, $W = \dfrac{2{\cdot}3ad}{L}$.*

In this formula it is well to take a low value for r_t, because the rmula has been framed on the assumption that the whole flange is erting its full working resistance, whereas that resistance is exerted the fibres at the extreme edge only. And as the metal in the eb is neglected, it is usual to take the depth d as the *total* depth at e centre, and not the depth between the centres of the flanges.

The steps to be taken in the design of a cast-iron girder for a given an would then be as follows :—

(1). From the given span determine the depth of the girder at the ntre from the proportion $d = \frac{1}{12}$ span.

(2). Equate the M_f at the centre with $r_t a_t d$. In this equation is the only unknown, and r_t should be taken as the *safe* tensile sistance per square inch.

(3). Having found a_t, find the *width* of the tension flange by llowing the rules mentioned above for the thickness of the flanges, ., find from the ascertained area and the thickness the *width* of the nsion flange.

(4). Or, having a_t the area of the tension flange, make a_c the area the compression flange = one-third the area of a_t. Then making e width of the compression flange = from $\frac{1}{30}$ to $\frac{1}{40}$ of the span, btain the thickness of that flange. This thickness may be the me for the whole cross section, though in practice it sometimes aries.

* General Wray's *Instruction in Construction*, p. 136. For safe loads take a ctor of safety of $\frac{1}{4}$.

(5). Keeping the areas of the top flange and of the bottom flange constant, the depth of the girder at points other than the centre may be ascertained by equating the M_f at those places with *rad*, where r and a are constant, but d varies.

(6). The above will give a girder that will approximately fulfil required conditions. The weight of the girder itself should be included in the calculations, properly speaking. To find the extra weight which must be added to the supported weight W_s, so as to include the weight of the girder, we must first find the ratio between W_s, and W_g = the weight of the girder itself, as approximately obtained above. Let $W_s : W_g :: x : 1$. Then W_s must be increased by $\frac{1}{x}\frac{(x+1)}{(x-1)}$.*

EXAMPLE 19.—*Cast-iron cantilever.*

These are frequently used in balconies, the area of the upper flange being constant, and the depth being varied from the point of fixing to the outer end. In this case let it be assumed that the length of the projecting part is 6 feet, and that the load on the balcony is 80 lbs. per square foot, the cantilevers being 5 feet apart. It is required to design a suitable form.

Let l = length, 72 inches.

a_t = area in square inches of tension flange.

r_t = safe intensity of tensile resistance, *i.e.*, 1·5 tons per square inch.

d = depth at fixing in inches.

The total weight (dead load) on the cantilever is

$$5 \times 6 \times 80 = 2400 \text{ lbs.}$$

The M_{ff} will be at the point of fixing, and will be

$$\tfrac{1}{2}Wl = \tfrac{1}{2}\ 2400 \times 72''.$$

This must be equal to $r_t a_t d$, where r_t is the only known quantity = 1·5 tons = 3360 lbs. We may find a value of a_t, however, by considering the breadth as 3", and the thickness as 1". If we find these dimensions not suitable they can be modified, but they are at all events convenient for allowing room for the load. Then $r_t a_t d = 3360 \times 3 \times d$, and is equal to $\frac{1}{2}Wl = \frac{1}{2}2400 \times 72$, whence $d = 8{\cdot}25$ inches. Make $8\frac{1}{2}$ inches. The depths throughout the

* The ultimate value of an expanding series of small increments to W_s due to increments in W_g.

remainder of the beam will vary as the M_6 *i.e.*, as the ordinates of a parabola (see Case 3, Chapter IV.). The weight of the girder as approximately designed can then be arrived at. For, the area of the compression flange being one-third that of the tension flange, in this case it will be only 1″ wide, *i.e.*, the width of the flange-thickness.

The approximate section will then be as in *Fig.* 126, and the weight W_g of the girder will be—

The weight of top flange $72'' \times 3'' \times 1'' \times 0{\cdot}27$ lbs. (weight per cubic inch of cast iron) = 60 lbs. The weight of the web (being in elevation a parabola, this will have $\frac{1}{3}$ the area of the rectangle formed by the length and greatest depth) $= \frac{1}{3} \times 72'' \times 8\frac{1}{2}'' \times 1'' \times {\cdot}27 = 57$ lbs. Hence total weight $= 60 + 57 = 117$ lbs. $= W_g$. Thus we have $W_s = 2400$, and $W_g = 117$, hence the ratio $x = 14$. Therefore the increment to be added to W_s, from the expression $\frac{1\,(x \times 1)}{x\,(x-1)} = \frac{1 \times 15}{14 \times 13} \times W_s\ 2400 = 200$. Hence W becomes 2600 lbs.

Taking this new value for W in the equation $\frac{1}{2}Wl$, we have

$$\tfrac{1}{2}2600 \times 72 = rad = 1{\cdot}5 \times a \times 8{\cdot}5.$$

Whence $a = 26$, say $3\frac{1}{2}$ inches wide.

The section of the beam at the wall is as shown in *Fig.* 127, and the side elevation in *Fig.* 128.

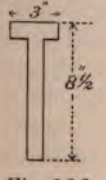


Fig. 126.

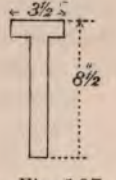


Fig. 127.

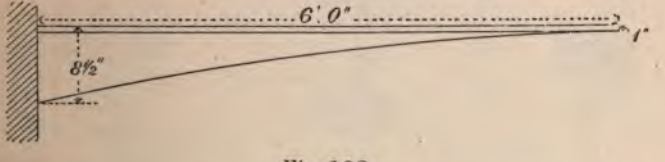


Fig. 128.

To ascertain the deflection, we must find the value of y, the distance of the neutral axis from the extreme fibre. This may be found from the rules given in Chapter IV., and in this case $y = 3{\cdot}4$ inches.

The deflection $V = \frac{n'rl^2}{Ey}$, where $n' = 1$, $r = 1{\cdot}5$ tons per square in
$= 72$ inches, and $E = 8300$ tons.

Hence $V = {\cdot}3$ inches, which is within the permissible limits
deflection.

The calculation of deflection with cast iron is not very satisfa
tory, however, on account of the variation that exists both in t
tensile strength and in the elasticity of cast iron.

It must be remembered that if the flange of the cantilever has
be drilled for bolt holes their diameter must be deducted in calc
lating the width of the flange. Usually the bolt holes are arrange
in lugs cast on at certain intervals, extra to the width of the flang

The dimensions in this case will give a far greater shearing resi
tance than the beam will be called upon to bear. The safe shearin
resistance of cast iron is about 2·4 tons per square inch. The we
in this case has 7·5 square inches in cross section, hence its resistanc
is $7{\cdot}5 \times 2{\cdot}4 = 18$ tons, and the actual shearing force is a little mo
than 1 ton. The solid web might, therefore, be replaced by op
scroll work.

Wrought Iron.

The principles of calculating rolled wrought-iron or steel be
have already been considered in Chapter V., and need not here
repeated. Angle-iron and T-iron beams, when loaded transvers
should be calculated on the principles enumerated in that chapte

Rolled iron or steel beams are generally of uniform sec
throughout. Sometimes more strength is given by riveting e
plates to the flanges, as in *Fig.* 129, or by riveting two togethe
in *Fig.* 130, or by forming them into a box girder, as in *Fig.*
As a rule, however, such contrivances are not economical of m

Fig. 129.

Fig. 130.

Fig. 1

The limit of depth in such girders is about 20", hen

not, as a rule, suitable for spans of much more than 30′. Above that span it is more economical to build up the girder with plates and angle irons. The consideration of design of such structures is necessarily deferred till the subject of joints and fastenings has been dealt with, and is dealt with in Part II.

An advantage of rolled joists is that the ends can be *fixed* with advantage. The section is uniform, and the flanges arranged to take either tension or compression.

In selecting a section from a manufacturer's catalogue, it would be advisable to choose (1) for stiffness a deep section with broad flanges, the breadth of the flange being from $\frac{1}{30}$ to $\frac{1}{40}$ of the span, and (2) a section with a thin web.

The following table, taken from the handbook of Messrs. Dorman, Long and Co., of Middlesborough, shows the weights, dimensions, and Moments of Inertia of various sections.

TABLE XX.

Statement of Weights, Values of I, *etc., in Rolled Steel Beams.*

No. in Catalogue.	Weight per foot run in lbs.	Breadth of Flange, inches.	Depth of Beam, inches.	Thickness of Web, inches.	Mean Thickness of Flange.	Value of I (Vertical)	Distributed Load in Tons that 1 foot will carry when r=8 tons and Beam is Supported.
G 1	89	7·5	20	·6	1·0	1646	878
G 2	75	7·	18	·55	·94	1150	681
G 3	62	6·	18	·56	·85	722	481
G 3a	50	5·	16	·51	·73	556	371
G 4	59	6·	15	·54	·85	617	438
G 5	42	5·	15	·42	·625	414	294
G 6	57	6·	14	·51	·85	520	396
G 7	54	6·	12	·51	·87	370	328

Transverse Strength of Railway Rails.

It is sometimes necessary to use ordinary rails as beams under transverse load.

It is possible to find their M_r by taking an accurate cross section and dividing it into equivalent rectangles, and thus ascertaining the value of I.

An approximate and very fairly accurate idea of the resistance, however, may be inferred from the considerations of the ordinary proportions given to railway rails.

If S = the weight of the rail, as usually stated, in lbs. per yard run,

A = the area of the cross section in square inches.

Then A × ·288 = the weight of one inch in length of the rail, since steel weighs ·288 lb. per cubic inch.

But the weight of one inch in length $= \frac{S}{36}$.

Hence $$A = \frac{S}{36 \times \cdot 288} = \frac{S}{10 \cdot 4}.$$

Now the proportions of the areas of the cross section in average rails are :—Top flange, 3. Bottom flange, 2. Web, 1·6. The flanges combined : web : : 3·13 : 1.

The area of the bottom flange, therefore, is $\frac{2}{6 \cdot 6} = \cdot 3$, or $\frac{1}{3}$ of the total area $= \frac{A}{3}$ or $\frac{S}{30}$ approximately. This may then be substituted for a in the usual formula for the resistance of a rolled beam *rad*.

Further, as the usual depth in English rails is 5·5″, and as the *safe* Modulus of Rupture for steel is 6 tons* per square inch, we may substitute for d and r 5·5 and 6 respectively. Thus *rad* becomes

$$6 \times \frac{S}{30} \times 5 \cdot 5 = \frac{5 \cdot 5}{5} S,$$

which, for approximate calculations, may be taken as = S.

Approximately, therefore, the safe *moment of resistance in inch-tons for an ordinary English rail = the weight in lbs. per yard run.*

This may be expressed in terms of the safe weight in tons on the centre. In such a case the $M_{ft} = \frac{Wl}{4}$. Hence $W = \frac{4S}{l}$ where l is expressed in inches. For instance, a 90 lb. rail 15′ span could bear $\frac{4 \times 90}{15 \times 12} = 2$ tons.

Where the depth is not 5·5″ *rad* becomes

$$6 \times \frac{S}{30} \times d = \frac{Sd}{5}.$$

* This allows a factor of safety of about 5, since mild steel will almost always give resistance of 30 tons per square inch.

EXAMPLE 20.—*A casemate* 10′ *span is covered with* 3′ *of concrete* (*at* 130 *lbs. per cubic foot*), *and* 90-*lb. rails are to be used for supporting them. How far apart should they be?*

Let x = distance apart.

Then
$$W = 10 \times x \times 130 \times 3 \div 2{,}240 = 1{\cdot}74x \text{ tons},$$
$$M_{ft} = \frac{Wl}{8} = \frac{1{\cdot}74x \times 10 \times 12}{8} = 26{\cdot}2x,$$
$$rad = 90 = M_{ft} = 26{\cdot}2x;$$
$$\therefore \quad x = 3{\cdot}45,$$

say 3′ apart.

As the ends of the rails would be probably fixed by being built into the concrete, they might really be placed further apart with safety.

A table showing the values of bd^2 and bd^3 for different dimensions of timber ordinarily used is here given. From it one can select a suitable scantling for a given value.

TABLE XXI.

Values of bd^3, b *varying from* 1 *to* 6, d *varying from* 1 *to* 12.

DEPTH.	BREADTH.																			
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
1	1·0	1·2	1·5	1·7	2·0	2·2	2·5	2·7	3·0	3·2	3·5	3·7	4·0	4·2	4·5	4·7	5·0	5 2	5·5	5·7	6·0
1¼	2·0	2·4	2·9	3·4	3·9	4·4	4·9	5·4	5·9	6·3	6·8	7·3	7·8	8·3	8·8	9·3	9·8	10·3	10·7	11·2	11·7
1½	3·4	4·2	5·1	5·9	6·7	7·6	8·4	9·3	10·1	11·0	11·8	12·7	13·5	14·3	15·2	16·0	16·9	17·7	18·6	19·4	20·2
1¾	5·4	6·7	8·0	9·4	10·7	12·1	13·4	14·7	16·1	17·4	18·8	20·1	21·4	22·8	24·1	25·5	26·8	28·1	29·5	30·8	32·2
2	8·0	10·0	12·0	14·0	16·0	18·0	20·0	22·0	24·0	26·0	28·0	30·0	32·0	34·0	36·0	38·0	40·0	42·0	44·0	46·0	48·0
2¼	11·4	14·2	17·1	19·9	22·8	25·6	28·5	31·3	34·2	37·0	39·9	42·7	45·6	49·4	51·3	54·1	57·0	59·8	62·6	65·5	68·3
2½	15·6	19·5	23·4	27·3	31·2	35·2	39·1	43·0	46·9	50·8	54·7	58·6	62·5	66·4	70·3	74·2	78·1	82·0	85·9	89·8	93·7
2¾	20·8	26·0	31·2	36·4	41·6	46·8	52·0	57·2	62·4	67·6	72·8	78·0	83·2	88·4	93·6	98·8	104	109	114	120	125
3	27·0	33·7	40·5	47·2	54·0	60·7	67·5	74·2	81·0	87·7	94·5	101	108	115	121	128	135	142	148	155	162
3¼	34·3	42·9	51·5	60·1	68·7	77·2	85·8	94·4	103	112	120	129	137	146	154	163	172	180	189	197	206
3½	42·9	53·6	64·3	75·0	85·8	96·5	107	118	129	139	150	161	171	182	193	204	214	225	236	247	257
3¾	52·7	65·9	79·1	92·3	105	119	132	145	158	171	185	198	211	224	237	250	264	277	290	303	316
4	64·0	80·0	96·0	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384
4¼	76·8	96·0	115	134	154	173	192	211	230	249	269	288	307	326	345	365	384	403	422	441	461
4½	91·1	114	137	159	182	205	228	251	273	296	319	342	364	387	410	433	456	478	501	524	547

TABLE XXI.—*(Continued).*

Values of bd^3, b *varying from* 1 *to* 6, d *varying from* 1 *to* 12.

DEPTH.	BREADTH.																				
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
6	216	270	324	378	432	486	540	594	648	702	756	810	864	918	972	1026	1080	1134	1188	1242	1296
6¼	244	305	366	427	488	549	610	671	732	793	855	916	977	1038	1099	1160	1221	1282	1343	1404	1465
6½	275	343	412	481	549	618	687	755	824	893	961	1030	1098	1167	1236	1304	1373	1442	1510	1579	1648
6¾	308	384	461	538	615	692	769	846	923	1000	1076	1153	1230	1307	1384	1461	1538	1615	1692	1768	1845
7	343	429	514	600	686	772	857	943	1029	1115	1200	1286	1372	1458	1543	1629	1715	1801	1886	1972	2058
7¼	381	476	572	667	762	857	953	1048	1143	1238	1334	1429	1524	1620	1715	1810	1905	2001	2096	2191	2286
7½	422	527	633	738	844	949	1055	1160	1266	1371	1477	1582	1687	1793	1898	2004	2109	2215	2320	2426	2531
7¾	465	582	698	815	931	1047	1164	1280	1396	1513	1629	1746	1862	1978	2095	2211	2327	2444	2560	2677	2793
8	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048	2176	2304	2432	2560	2688	2816	2944	3072
8¼	562	702	842	983	1123	1263	1404	1544	1685	1825	1965	2106	2246	2386	2527	2667	2808	2948	3088	3229	3369
8½	614	768	921	1075	1228	1382	1535	1689	1842	1996	2149	2303	2456	2610	2763	2917	3071	3224	3378	3531	3685
8¾	670	837	1005	1172	1340	1507	1675	1842	2010	2177	2345	2512	2680	2847	3015	3182	3350	3517	3685	3852	4020
9	729	911	1093	1276	1458	1640	1822	2005	2187	2369	2551	2734	2916	3098	3280	3463	3645	3827	4009	4192	4374
9¼	791	989	1187	1385	1583	1781	1979	2176	2374	2572	2770	2968	3166	3364	3562	3759	3957	4155	4353	4551	4749
9½	857	1072	1286	1500	1715	1929	2143	2358	2572	2786	3001	3215	3429	3644	3858	4073	4289	4501	4716	4930	5144
9¾	927	1159	1390	1622	1854	2085	2317	2549	2781	3012	3244	3476	3707	3939	4171	4403	4634	4866	5098	5329	5561
10	1000	1250	1500	1750	2000	2250	2500	2750	3000	3250	3500	3750	4000	4250	4500	4750	5000	5250	5500	5750	6000
11	1331	1664	1997	2329	2662	2995	3328	3660	3993	4326	4659	4991	5324	5657	5990	6322	6655	6988	7321	7653	7986
12	1728	2160	2592	3024	3456	3888	4320	4752	5184	5616	6048	6480	6912	7344	7776	8208	8640	9072	9504	9936	10368

TABLE XXI.

Values of bd^3, b *varying from* 1 *to* 6, d *varying from* 1 *to* 12.

Depth.	Breadth.																					
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
1	1·0	1·2	1·5	1·7	2·0	2·2	2·5	2·7	3·0	3·2	3·5	3·7	4·0	4·2	4·5	4·7	5·0	5 2	5·5	5·7	6·0
1¼	2·0	2·4	2·9	3·4	3·9	4·4	4·9	5·4	5·9	6·3	6·8	7·3	7·8	8·3	8·8	9·3	9·8	10·3	10·7	11·2	11·7
1½	3·4	4·2	5·1	5·9	6·7	7·6	8·4	9·3	10·1	11·0	11·8	12·7	13·5	14·3	15·2	16·0	16·9	17·7	18·6	19·4	20·2
1¾	4·2	4·2	6·7	8·0	9·4	10·7	12·1	13·4	14·7	16·1	17·4	18·8	20·1	22·8	24·1	25·5	26·8	28·1	29·5	30·8	32·2
2	5·3	6·7	8·0	12·0	14·0	16·0	18·0	20·0	22·0	24·0	26·0	28·0	30·0	32·0	34·0	36·0	38·0	42·0	44·0	46·0	48·0
2¼	6·7	8·4	10·1	11·8	16·0	18·0	20·2	22·5	24·7	27·0	29·2	31·5	33·7	36·0	38·2	40·5	42·7	47·2	49·5	51·7	54·0
2½	8·4	10·5	12·7	14·8	18·0	20·2	22·5	25·3	27·5	29·8	32·0	34·3	36·5	39·8	42·0	44·3	46·5	51·0	53·3	55·5	58·3
2¾	10·5	13·2	15·8	18·4	22·8	25·6	28·5	31·3	34·2	37·0	39·9	42·7	45·6	49·4	52·3	55·1	58·0	63·7	66·5	69·4	72·2
3	12·7	15·9	19·1	22·3	25·6	31·0	34·2	38·0	41·3	45·0	48·2	51·5	54·7	59·8	63·0	66·7	69·9	76·8	80·1	83·8	87·0
3¼	15·2	19·0	22·8	26·6	30·4	34·2	38·0	41·8	45·6	49·4	53·2	57·0	60·8	66·4	70·2	74·0	77·8	85·4	89·2	93·0	96·8
3½	18·0	22·5	27·0	31·5	36·0	40·5	45·0	49·5	54·0	58·5	63·0	67·5	72·0	78·7	83·2	87·7	92·2	101	106	110	115
3¾	21·1	26·4	31·6	36·9	42·2	47·4	52·7	58·0	63·3	68·5	73·8	79·1	84·4	92·2	97·5	103	108	119	124	129	135
4	24·4	30·4	36·5	42·6	48·7	54·7	60·8	66·9	73·0	79·0	85·1	91·2	97·3	106	112	119	125	137	143	149	155
4¼	27·8	34·8	41·7	48·7	55·6	62·6	69·5	76·5	83·4	90·4	97·3	104	111	122	128	136	142	156	163	170	177
4½	31·5	39·4	47·3	55·1	63·0	70·9	78·8	86·6	94·5	102	110	118	126	138	146	154	162	177	185	193	201
4¾	35·5	44·4	53·3	62·2	71·1	80·0	88·8	97·7	106	115	124	133	142	156	164	173	182	200	209	218	227
5	39·7	49·6	59·5	69·4	79·3	89·2	99·1	109	119	129	139	149	158	174	184	194	204	224	234	244	254
5¼	44·2	55·2	66·2	77·2	88·3	99·3	110	121	132	143	154	165	176	193	204	215	226	248	259	270	281
5½	49·0	61·2	73·4	85·6	97·9	110	122	134	147	159	171	183	196	214	226	238	251	275	287	300	312
5¾	54·0	67·5	81·0	94·5	108	121	135	148	162	175	189	202	216	237	250	263	277	304	317	331	344
6	59·4	74·2	89·1	104	119	134	148	163	178	193	208	222	237	261	275	290	304	334	349	364	378
6¼	65·0	81·2	97·5	114	130	146	162	179	195	211	227	244	260	285	301	317	333	366	382	398	414
6½	70·9	88·6	106	124	142	159	177	195	213	230	248	266	284	312	329	347	364	400	418	435	453
6¾	77·0	96·2	115	135	154	173	192	211	231	250	269	288	308	338	357	376	396	434	453	473	492
7	83·3	104	125	146	167	187	208	229	250	271	291	312	333	366	387	408	429	470	491	512	533
7¼	89·9	112	135	157	180	202	225	247	270	292	315	337	360	396	418	441	463	508	530	553	575
7½	96·7	121	145	169	193	217	241	265	290	314	338	362	386	425	449	473	497	545	569	593	617
7¾	104	129	155	181	207	233	259	284	310	336	362	388	414	455	481	507	533	584	610	636	662
8	111	138	166	194	221	249	277	304	332	359	387	415	443	487	514	542	570	625	653	680	708
8¼	118	148	177	207	236	266	295	325	354	384	413	443	472	519	548	578	607	666	695	725	754
8½	126	158	189	221	252	284	315	347	378	410	441	473	504	554	585	616	648	711	742	774	805
8¾	135	168	202	236	269	303	337	370	404	438	471	505	539	592	625	659	693	760	794	827	861
9	143	179	215	250	286	322	358	394	429	465	501	537	572	630	665	701	737	808	844	880	916
9¼	152	190	228	266	304	342	380	418	456	494	532	570	608	668	706	744	782	858	896	934	972
9½	161	202	242	282	323	363	403	443	484	524	564	605	645	709	749	789	830	910	950	991	1031
9¾	171	213	256	299	341	384	427	469	512	555	597	640	683	751	793	836	879	963	1006	1049	1092
10	180	225	270	315	360	405	450	495	540	585	630	675	720	792	837	882	927	1017	1062	1107	1152
10¼	190	238	285	333	380	428	475	523	570	618	665	713	760	836	883	931	978	1073	1121	1168	1216
10½	200	250	300	350	400	450	500	551	601	651	701	751	801	881	931	981	1031	1131	1181	1231	1281
10¾	211	263	316	369	422	474	527	580	633	686	738	791	844	928	980	1033	1086	1191	1244	1297	1350
11	221	277	332	387	443	498	553	609	664	719	775	830	885	974	1029	1084	1140	1250	1305	1361	1416
11¼	232	290	348	406	464	522	580	638	696	754	812	870	928	1021	1079	1137	1195	1311	1369	1427	1485
11½	243	304	365	425	486	547	608	668	729	790	851	911	972	1069	1130	1191	1251	1373	1434	1495	1556
11¾	254	317	381	444	508	571	635	698	762	825	889	952	1016	1117	1181	1244	1308	1435	1498	1562	1625
12	266	332	398	464	531	597	664	730	797	863	929	996	1062	1168	1234	1301	1367	1500	1566	1633	1699

Values of bd^3, b *varying from* 1 *to* 6, d *varying from* 1 *to* 12.

Depth.	Breadth.																				
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
6	216	270	324	378	432	486	540	594	648	702	756	810	864	918	972	1026	1080	1134	1188	1242	1296
6¼	244	305	366	427	488	549	610	671	732	793	855	916	977	1038	1099	1160	1221	1282	1343	1404	1465
6½	275	343	412	481	549	618	687	755	824	893	961	1030	1098	1167	1236	1304	1373	1442	1510	1579	1648
6¾	308	384	461	538	615	692	769	846	923	1000	1076	1153	1230	1307	1384	1461	1538	1615	1692	1768	1845
7	343	429	514	600	686	772	857	943	1029	1115	1200	1286	1372	1458	1543	1629	1715	1801	1886	1972	2058
7¼	381	476	572	667	762	857	953	1048	1143	1238	1334	1429	1524	1620	1715	1810	1905	2001	2096	2191	2286
7½	422	527	633	738	844	949	1055	1160	1266	1371	1477	1582	1687	1793	1898	2004	2109	2215	2320	2426	2531
7¾	465	582	698	815	931	1047	1164	1280	1396	1513	1629	1746	1862	1978	2095	2211	2327	2444	2560	2677	2793
8	512	640	768	896	1024	1152	1280	1408	1536	1664	1792	1920	2048	2176	2304	2432	2560	2688	2816	2944	3072
8¼	562	702	842	983	1123	1263	1404	1544	1685	1825	1965	2106	2246	2386	2527	2667	2808	2948	3088	3229	3369
8½	614	768	921	1075	1228	1382	1535	1689	1842	1996	2149	2303	2456	2610	2763	2917	3071	3224	3378	3531	3685
8¾	670	837	1005	1172	1340	1507	1675	1842	2010	2177	2345	2512	2680	2847	3015	3182	3350	3517	3685	3852	4020
9	729	911	1093	1276	1458	1640	1822	2005	2187	2369	2551	2734	2916	3098	3280	3463	3645	3827	4009	4192	4374
9¼	791	989	1187	1385	1583	1781	1979	2176	2374	2572	2770	2968	3166	3364	3562	3759	3957	4155	4353	4551	4749
9½	857	1072	1286	1500	1715	1929	2143	2358	2572	2786	3001	3215	3429	3644	3858	4073	4289	4501	4716	4930	5144
9¾	927	1159	1390	1622	1854	2085	2317	2549	2781	3012	3244	3476	3707	3939	4171	4403	4634	4866	5098	5329	5561
10	1000	1250	1500	1750	2000	2250	2500	2750	3000	3250	3500	3750	4000	4250	4500	4750	5000	5250	5500	5750	6000
11	1331	1664	1997	2329	2662	2995	3328	3660	3993	4326	4659	4991	5324	5657	5990	6322	6655	6988	7321	7653	7986
12	1728	2160	2592	3024	3456	3888	4320	4752	5184	5616	6048	6480	6912	7344	7776	8208	8640	9072	9504	9936	10368

TABLE XXII.

Values of bd^2, b *varying from* 1 *to* 6, d *varying from* 1 *to* 12.

DEPTH.	BREADTH.																			
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
1	1·0	1·2	1·5	1·7	2·0	2·2	2·5	2·7	3·0	3·2	3·5	3·7	4·0	4·2	4·5	4·7	5·0	5·2	5 5	5·7	6·0
1¼	1·6	2·0	2·3	2·7	3·1	3·5	3·9	4·3	4·7	5·1	5·5	5·9	6·2	6·6	7·0	7·4	7·8	8·2	8·6	9·0	9·4
1½	2·2	2·8	3·4	3·9	4·5	5·1	5·6	6·2	6·7	7·3	7·9	8·4	9·0	9·6	10·1	10·7	11·2	11·8	12·4	12·9	13·5
1¾	3·1	3·8	4·6	5·4	6·1	6·9	7·7	8·4	9·2	10·0	10·7	11·5	12·3	13·0	13·8	14·6	15·3	16·1	16·8	17·6	18·4
2	4·0	5·0	6·0	7·0	8·0	9·0	10·0	11·0	12·0	13·0	14·0	15·0	16·0	17·0	18·0	19·0	20·0	21·0	22·0	23·0	24·0
2¼	5·1	6·3	7·6	8·9	10·1	11·4	12·7	13·9	15·2	16·2	17·7	19·0	20·2	21·5	22·8	24·0	25·3	26·6	27·8	29·1	30·4
2½	6·2	7·8	9·4	10·9	12·5	14·1	15·6	17·2	18·7	20·3	21·9	23·4	25·0	26·6	28·1	29·7	31·2	32·8	34·4	35·9	37·5
2¾	7·6	9·5	11·3	13·2	15·1	17·0	18·9	20·8	22·7	24·6	26·5	28·4	30·2	32·1	34·0	35·9	37·8	39·7	41·6	43·5	45·4
3	9·0	11·2	13·5	15·7	18·0	20·2	22·5	24·7	27·0	29·2	31·5	33·7	36·0	38·2	40·5	42·7	45·0	47·2	49·5	51·7	54·0
3¼	10·6	13·2	15·8	18·5	21·1	23·8	26·4	29·0	31·7	34·3	37·0	39·6	42·2	44·9	47·5	50·2	52·8	55·5	58·1	60·7	63·4
3½	12·2	15·3	18·4	21·4	24·5	27·6	30·6	33·7	36·7	39·8	42·9	45·9	49·0	52·1	55·1	58·2	61·2	64·3	67·4	70·4	73·5
3¾	14·1	17·6	21·1	24·6	28·1	31·6	35·2	38·7	42·2	45·7	49·2	52·7	56·2	59·8	63·3	66·8	70·3	73·8	77·3	80·9	84·4
4	16·0	20·0	24·0	28·0	32·0	36·0	40·0	44·0	48·0	52·0	56·0	60·0	64·0	68·0	72·0	76·0	80·0	84·0	88·0	92·0	96·0
4¼	18·1	22·6	27·1	31·6	36·1	40·6	45·2	49·7	54·2	58·7	63·2	67·0	72·2	76·8	81·3	85·8	90·3	94·8	99·3	104	108
4½	20·2	25·3	30·4	35·4	40·5	45·6	50·6	55·7	60·7	65·8	70·9	75·9	81·0	86·1	91·1	96·2	101	106	111	116	121
4¾	22·6	28·2	33·8	39·5	45·1	50·8	56·4	62·0	67·7	73·3	79·0	84·6	90·2	95·9	102	107	113	118	124	130	135
5	25·0	31·2	37·5	43·7	50·0	56·2	62·5	68·7	75·0	81·2	87·5	93·7	100	106	112	119	125	131	137	144	150
5¼	27·6	34·5	41·3	48·2	55·1	62·0	68·9	75·8	82·7	89·6	96·5	103	110	117	124	131	138	145	152	158	165
5½	30·2	37·8	45·4	52·9	60·5	68·1	75·6	83·2	90·7	98·3	106	113	121	129	136	144	151	159	166	174	181

Depth.	Breadth.																				
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
6	36·0	45·0	54·0	63·0	72·0	81·0	90·0	99·0	108	117	126	135	144	153	162	171	180	189	198	207	216
6¼	39·1	48·8	58·6	68·4	78·1	87·9	97·7	107	117	127	137	146	156	166	176	186	195	205	215	225	234
6½	42·2	52·8	63·4	73·9	84·5	95·1	106	116	127	137	148	158	169	180	190	201	211	222	232	243	253
6¾	45·6	57·0	68·3	79·7	91·1	103	114	125	137	148	159	171	182	194	205	216	228	239	251	262	273
7	49·0	61·2	73·5	85·7	98·0	110	122	135	147	159	171	184	196	208	220	233	245	257	269	282	294
7¼	52·6	65·7	78·9	92·0	105	118	131	145	158	171	184	197	210	223	237	250	263	276	289	302	315
7½	56·2	70·3	84·4	98·4	112	127	141	155	169	183	197	211	225	239	253	267	281	295	309	323	337
7¾	60·1	75·1	90·1	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
8	64·0	80·0	96·0	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384
8¼	68·1	85·1	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408
8½	72·2	90·3	108	126	144	163	181	199	217	235	253	271	289	307	325	343	361	379	397	415	433
8¾	76·6	95·7	115	134	153	172	191	211	230	249	268	287	306	325	345	364	383	402	421	440	459
9	81·0	101	121	142	162	182	202	223	243	263	283	304	324	344	364	385	405	425	445	466	486
9¼	85·6	107	128	150	171	193	214	235	257	278	299	321	342	364	385	406	428	449	470	492	513
9½	90·2	113	135	158	180	203	226	248	271	293	316	338	361	384	406	429	451	474	496	519	541
9¾	95·1	119	143	166	190	214	238	261	285	309	333	356	380	404	428	452	475	499	523	547	570
10	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600
11	121	151	181	212	242	272	302	333	363	393	423	454	484	514	544	575	605	635	665	696	726
12	144	180	216	252	288	324	360	396	432	468	504	540	576	612	648	684	720	756	792	828	864

CHAPTER IX.

Tension and Compression Bars.

Tension Bars or Ties.—Compression Bars.—Flexure of Long Columns.—Radius of Gyration.—Failure by Bending and Crushing Combined.—Professor Johnson and Mr. Claxton Fidler's Investigations.—Timber Columns.—Effect of End Conditions.—Summary of Investigation.—Combined Bending and Direct Stresses.—General Deductions.—Examples.—Appendices.

In the beams hitherto considered, the stresses have been of a compound character, *i.e.*, the transverse stress produces compression in the fibres at one side and tension in those at the other. We have now to consider the effect of *direct stresses*, either tension or compression, where the direction of the force is no longer, as in the cases previously considered, at right angles to the axis of the beam, but parallel to that axis. In a truss, for instance, the various parts are arranged so that certain members bear one form of stress, and others a different form, hence it is necessary to examine the best form of structure calculated to withstand these different kinds of direct stresses, as a preliminary to the design of trusses, columns, and braced girders. In such design it is evident that there must be three operations, viz.: (1), to ascertain the amount, nature and direction of the stresses which each member has to bear; (2), to design the member so as to bear the stress in question safely; and (3) so to design the joints or fastenings of the various parts that there shall be no weakness at that part.

In this chapter we shall confine our attention to the second of these operations, leaving the others for future consideration.

Tension Bars, or Ties.

If a bar is subject to a force tending to pull it apart, and if the material of the bar be homogeneous throughout, the resistance to the pull arising from the mutual cohesion of one particle of the material with that next to it will be uniform for each unit of the area of the

cross section, provided the application of the force coincide with the axis of the bar. Under these conditions, therefore, there is a series of equal forces r, r, r, r, resisting the total force P (*Fig.* 132), and if A be the area of the cross section of the bar, and r_t the resistance per unit of area of material—

$$P = A \times r_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

There are, however, in this reasoning certain assumptions which, in practice, cannot always be relied upon. Few materials are homogeneous, and it is not always possible so to arrange the fastening of ties as to bring the pull exactly coinciding with the axis of the bar. The result is unequal distribution of the resisting force, as indicated in *Fig.* 133, and less ultimate strength for the bar as a whole than would be obtained from the application of the rules given above.

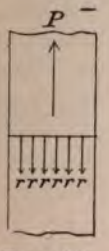


Fig. 132.

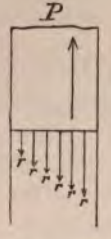


Fig. 133.

In timber especially is it necessary to bear this in mind. The experiments on the ultimate strength of various timbers given to us in textbooks have been in most cases made on small specimens of good homogeneous material, such as would rarely be obtainable in large scantlings. Hence the necessity for an ample margin of safety.

It is evident, also, that where the area of the cross section at any part of the bar is reduced by holes, such as those required for bolts or rivets, the effective section is proportionately reduced. In *Fig.* 134 the cross section at D is clearly the weakest in the bar, and it is there, if anywhere, that rupture will take place. If several holes be made, as in *Fig.* 135, the question will be whether rupture will take place at right angles to the pull or along the zig-zag line of the hole, *i.e.*, along *abcdef*. Experiment has shown that fracture will take place along that part where the area is a minimum.

If the stress be applied unsymmetrically, the intensity of resistance will no longer be uniform, but will be greater near the point of application than further away, hence, if the force is applied through a bolt or other fastening, which passes through the bar eccentrically, as in *Fig.* 136, the bar will reach its ultimate limit of resistance sooner than would be the case if the pressure were applied centrally. It should be the aim, therefore, in the design of tension members so to arrange them that the necessary parts removed for connections are grouped symmetrically to the mean fibre. If this cannot be done it is necessary to increase the section slightly.

Where the end connection is a screw, it should be borne in mind that the effective section is the area at the bottom of the threads.

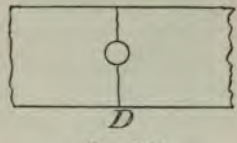


Fig. 134.

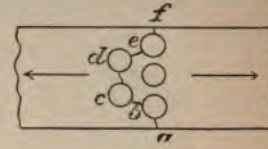


Fig. 135.

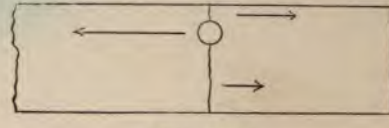


Fig. 136.

Generally speaking, in tension members the points requiring practical attention are the arrangements for the fastenings, a subject to be dealt with in detail in a later chapter. It is at the fastening that such members usually fail, the main part of the bar being usually stronger than the joint.

Compression Bars.

When we come to investigate the resistance of a bar to crushing, we find ourselves face to face with a much more complex problem than the case of a tension bar.

In order clearly to understand this problem, it is necessary to have a clear conception of what constitutes failure.

The compression members of a structure may have almost any dimensions between a slender column and a flat plate, and the con

ditions of failure in one case are different from failure in the other. In the slender column, failure may be said to take place when it bends or deflects from the direction of the force, corresponding to the original axis of the column. In the case of a flat plate, failure cannot be said to take place until, in the case of such materials as cast iron or timber or stone, the material is split or broken, or, in the case of plastic materials as wrought iron or steel, the material bulges by the displacement, but not separation, of the component particles.

Inasmuch as it is impossible to squeeze two particles into one, there can be no such thing as failure by direct compressive force, and the only way whereby we can conceive of failure is the lateral motion of particles in some way or other. We know that even with the most friable material, such as sand, where there is little mutual cohesion, almost any amount of compression can be brought to bear upon it, if only it is prevented from escaping laterally. So if we consider a thin plate of metal, there is hardly any limit to the load it will carry, for the interior of the plate is prevented from spreading laterally by the surrounding portions.

If we pass, however, from short plates to long columns, we find that the column finds relief from the compressive stress by lateral bending, which for practical purposes means failure.

What we require to ascertain in the case of any strut is the force that will cause that strut to fail, either (1) by the disintegration of the particles, or (2) by bending, or (3) by both causes combined.

In very short columns or cubes, the practical measure of their strength is the force that will just produce failure by bulging or splitting, or other separation of the particles.* The intensity per square unit thus found from experiment is shown in tables as *the compressive strength* of the material, and is usually indicated by the symbol r_c.

Numerous tests on the rupture of long columns have shown that the load causing rupture is approximately inversely proportional to the square of the length of the column.

It was formerly customary to divide all columns into two classes, viz., "long columns" and "short columns," the difference being determined by a consideration of the ratio of the length to the least diameter, and to calculate the "short" columns from the formula

$$P = r_c \times A,$$

where P = the force crushing the beam, A the area of the cross

* See *Fig.* 39, Chapter III.

section, and r_c the compressive strength per square unit. columns" were calculated by another formula, based on d investigation, and giving very different results. The definiti "short" as compared with a "long" column was as follows:-

For timber, a "short" column was one where the length v greater than 20 times the least dimension of the cross secti cast iron the length was not greater than 5 times and for w iron or steel the length was not greater than 10 times, th dimension.

It is evident that this method of reasoning is contrary to co sense, for there can be no abrupt change from "short" to " columns, nor from one method of failure to the other. In *Plate* II., the absurdity of this reasoning is apparent from the g representation of the ultimate resistance of a column of timbe various ratios. If the strength of a timber post, say 10' 6" × 6" square, be represented by AC, its strength cannot, 10' 6" long, suddenly drop to BC. Yet the post in the one c according to the above definition, a "short" column, in the case a "long" column.

A gradual diminution of the resistance per square inch c consideration of the increase of the ratio of length to least d sion has been empirically arrived at for timber struts, and is q in the Indian M.W.D. Handbook. Though not theoret accurate, the resulting formulæ are approximately correct, an useful for practical purposes. The formulæ are as follows :—

When l is less than $8h$—

$$P = r_c \times A \quad \ldots\ldots(1)$$

" " between $8h$ and $12h$—

$$P = \tfrac{5}{6}\, r_c \times A \quad \ldots\ldots(2)$$

" " " $12h$ " $24h$—

$$P = \tfrac{1}{2}\, r_c \times A \quad \ldots\ldots(3)$$

" " " $24h$ " $36h$—

$$P = \tfrac{1}{4}\, r_c \times A \quad \ldots\ldots(4)$$

" " " $36h$ " $48h$—

$$P = \tfrac{1}{6}\, r_c \times A \quad \ldots\ldots(5)$$

Fig. 3, *Plate* II., shows the graphic representation of formulæ.

The accompanying table (XXIII.), worked out by Captain Sa late R.E., gives the ultimate resistance of long columns of Ri deduced from experiments by Mr. Kirkaldy.

TABLE XXIII.

Ultimate Resistance of Long Columns of Riga Fir.

$= \frac{\text{length}}{\text{least dimension}}$; p = intensity of pressure in cwts. per square inch that will break column $= \frac{P}{S}$ in Gordon's formulæ, or total ultimate pressure ÷ sectional area.

$\frac{l}{d}$, Ends Rounded.	p, Or ultimate pressure per square inch.	$\frac{l}{d}$, Ends Rounded.	p, Or ultimate pressure per square inch.	$\frac{l}{d}$, Ends Rounded.	p, Or ultimate pressure per square inch.
	cwts.		cwts.		cwts.
5	20·0	24	9·3	46	3·8
6	19·3	25	8·9	48	3·6
7	18·6	26	8·5	50	3·3
8	17·9	27	8·1	52	3·1
9	17·2	28	7·8	54	2·9
10	16·7	29	7·5	56	2·7
11	16·1	30	7·1	58	2·6
12	15·5	31	6·8	60	2·4
13	14·9	32	6·6	62	2·3
14	14·4	33	6·3	64	2·2
15	13·8	34	6·0	66	2·1
16	13·2	35	5·8	68	2·0
17	12·7	36	5·6	70	1·9
18	12·1	37	5·3	72	1·8
19	11·6	38	5·1	74	1·7
20	11·1	39	4·9	76	1·6
21	10·6	40	4·8	78	1·5
22	10·2	42	4·4	80	1·4
23	9·7	44	4·1	82	1·3

This Table was drawn up by Capt. H. R. Sankey, R.E., from the formula $p = \frac{10{,}000}{500 + \frac{l^2}{d^2}}$ (similar to Gordon's), deduced from experiments by Mr. Kirkaldy (see *R.E. Corps Papers* (New Series), Vols. XXII. and XXIII.).

For columns with flat ends the value corresponding to *half* the ratio of length to least dimension should be taken.

Flexure of Long Columns.

The following remarks on the flexure of long struts are taken from Mr. Claxton Fidler's *Treatise on Bridge Construction* :—

"The bending of a long elastic strut may be illustrated in a very familiar way by the behaviour of an elastic walking cane under the vertical pressure of the hand. If we take a *perfectly straight* cane with a round head, and apply a gradually increasing pressure upon its top, using the sense of touch as a measure of the force applied, we find, that up to a certain point, the cane offers a solid and *sensibly unyielding* resistance to the pressure; but when it does give way it gives way all at once, and the cane may be bent double, or perhaps broken, without any further *increase* of the pressure, but only by the continued application of the same pressure following the depression of the head.

"But if the cane is not *perfectly straight*, or if we apply the pressure eccentrically upon a tee-headed stick, its behaviour will be quite different; the resistance will never have that solid unyielding character before noticed, but the deflection will begin at once, and from the first application of pressure the resistance will be felt to be an *elastic* resistance, increasing continually with the increasing deflection of the cane."

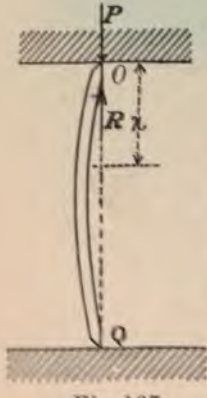


Fig. 137.

Let us suppose that we have a bar which is perfectly straight, of material that is perfectly homogeneous throughout, and where the axis coincides with the line of pressure—such a bar as is termed by writers on this subject as "an ideal column"—and let us suppose that it is subject to a compressive force, as indicated in *Fig.* 137 by P, then there will be excited in the bar a certain resilient force R, which corresponds to the spring of a bow, or the tension in the string. The behaviour of the bar will be as follows:—

(1). As long as P is less than a certain quantity R, there will be no deflection. If lateral pressure is applied to the bar it will deflect, but if that lateral force is withdrawn, the bar will straighten itself.

(2). If P is increased until it is equal to R, the load will not produce deflection, but any, even the smallest, force applied to the middle of the strut will be sufficient to cause it to bend, and if such

›rce be now withdrawn, the column will not now be able to raighten itself.

(3). If the force P be still further increased the column will be in state of unstable equilibrium; it will bend in *some* direction with-›ut the application of any lateral force, and will continue to bend, ›r be broken.

What we evidently want to find is the value of the resilient force R. This value has been ascertained to be expressed by the following:—

$$R = EI \frac{\pi^2}{l^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7),$$

which is known as Euler's formula, from the discoverer who presented it to the Berlin Academy in 1759. It is sometimes written

$$p = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8),$$

where E = modulus of elasticity of the material.

l = length of column.

p = pressure per square unit, *i.e.*, $\frac{R}{A}$.

k = least radius of gyration of the cross section, *i.e.*, $\sqrt{\frac{I}{A}}$,

I being the least Moment of Inertia, and A the area, of the cross section.

This last term is defined as follows:—

Suppose a body free to revolve round an axis which passes through it in any direction. Then suppose a certain given amount of force to be applied to the body at a certain given distance from the axis, so as to impart to the body an angular velocity, there will be in the body a certain point, such that if its entire weight were concentrated there, the same force as before, applied at the same distance from the axis, would impart to the body the same angular velocity. This distance from the given axis is the *radius of gyration.** In the case of areas the surface is supposed to revolve round the neutral axis of the area. In a circle the radius of gyration remains the same, no matter in what direction the neutral axis may be

* For proof that this $= \sqrt{\frac{I}{A}}$, see Appendix I. to this chapter.

drawn. In other figures the length is different for different positions of axis drawn through the centre of gravity. In the case of pillars the *least* radius of gyration of the cross section must be used.

Table XXIV. shows the values of k for various shapes of cross section.

Table XXIV.

Showing Value of k (the Radius of Gyration) for Various Shapes of Cross Section (Trautwine).

Shape of Cross Section.	Least Radius of Gyration.	Remarks.
Solid square.	$\frac{Side}{\sqrt{12}}$	$\sqrt{12} = 3{\cdot}46$
Hollow square (thickness uniform).	$\sqrt{\frac{D^2+d^2}{12}}$	
Solid rectangle.	$\frac{Least\ side}{\sqrt{12}}$	
Hollow rectangle (thickness uniform).	$\sqrt{\frac{BD_3 - bd_3}{12\,(BD - bd)}}$	
Solid circle.	$\frac{Diameter}{4}$	
Hollow circle (thickness uniform).	$\sqrt{\frac{D^2+d^2}{16}}$	
Phœnix column.	$D \times 0{\cdot}3636$	
I beam.	$\frac{B}{4{\cdot}58}$	
Channel iron.	$\frac{F}{3{\cdot}54}$	
Angle with equal legs.	$\frac{F}{5}$	
Angle with unequal legs.	$\frac{F\,f}{2{\cdot}6\,(F+f)}$	
T with $F=f$.	$\frac{F}{4{\cdot}74}$	
Cross with $F=f$.		

For other forms of cross section it is necessary to ascertain the least value of I (see Table XVII., Chapter V., for ordinary forms) and divide by the area. The square root of the result gives the value of the radius of gyration.

The proof of Euler's formula is given in Appendix II. to this chapter. Other proofs have been given by various scientists.

From formula (8) it is possible to represent graphically the breaking weight of ideal columns concentrically loaded of different proportions, by a curve drawn as in *Fig.* 138, where the horizontal line represents values of the ratio $l:k$, and the vertical line shows lbs. per square inch. Ordinates drawn from the horizontal line to the curve indicate the value of p for that ratio of l to k, measured to the curve. From such diagrams as those on *Plate* II. it will be seen that it requires very little force per square inch to bend the column when the ratio of l to k is very great, but this increases very rapidly when that ratio is reduced, and that it requires almost infinite force to bend a column when that ratio approaches the state of affairs in a very short strut. We know, however, that if the compressive stress exceeds a certain amount, the material will fail by bulging or splitting; hence, if we mark upon the diagram the line AD, indicating the ultimate resistance per square unit to crushing, this line will indicate this alternative limit to the strength of the strut, and the line ABC will represent the failure of the strut.

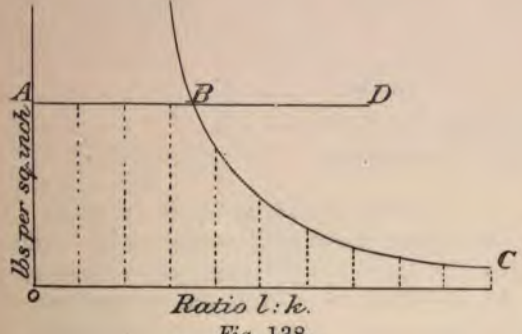


Fig. 138.

Such graphic representations are very convenient for comparison of results obtained by various investigations, and this method of showing the strength of columns is shown on *Plate* II.

Failure by Bending and Crushing Combined.—Professor Johnson's, and Mr. Claxton Fidler's Investigations.

In the investigation above we began with the assumption that the column was perfectly straight, homogeneous and centrally loaded. It is needless to say that such a combination of circumstances never really occurs in practice. It has also been assumed that the column is free to move round its end bearings, as would be the case if it were fastened at both ends by perfectly frictionless pin connections. This, too, is an assumption not met with in practice. We shall consider the very important practical matter of the fixing of the ends presently, but for the present may put that question on one side, and direct our attention to the consideration of the effect of the load being eccentric, or the material being heterogeneous.

In such a case the column will be subject to compressive stress from three causes,* after bending has begun :—

(1). From the uniformly distributed stress, $r_c = \frac{P}{A}$.

(2). From the bending moment caused by the eccentric position of the load. If v be the eccentric displacement (*Fig.* 139) then

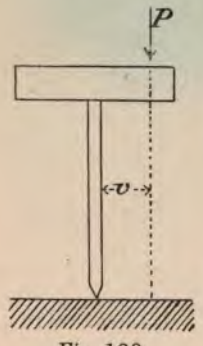


Fig. 139.

$$Pv = \frac{r_c' I}{y},$$

where r_c' is the (compressive) stress intensity due to the eccentric position of the load. Hence

$$r_c' = \frac{Pvy}{I} = \frac{Pvy}{Ak^2},$$

and as $$\frac{P}{A} = r_c,$$

we have $$r_c' = \frac{r_c vy}{k^2},$$

where k is the radius of gyration.

(3). The two above causes will operate until bending actually begins. After this point a third cause of compression comes into play, viz., the increment of moment due to the deflection. Here the case differs from that of a beam transversely loaded. There the load sinks with the deflected beam, and no increase of moment is

* In this investigation, borrowed from Professor Johnson, the notation has been altered to conform with the remainder of this chapter.

caused by the deflection; but in the case of a column, the position of the load remaining the same, every increment of deflection causes increase of the M_f, and therefore of the M_r. The value of the increase in the intensity r_c'' can be found if we know the value of the deflection, for if v' be that value, then $P(v+v') = r_c''I \div y$. But we do know the value of the deflection, inasmuch as it is that due to a uniform load throughout the whole column, and is therefore expressed by the formula $v = \frac{5}{48}\frac{rl^2}{Ey}$ (Table XIX., Chapter VII.).

Hence it is possible to assign to r_c'' a value in terms of known quantities. When the bending is such that the elastic limit f of the material is reached, we have

$$r_c'' = \frac{f - r_c}{10E}\left(\frac{l}{k}\right)^2 r_c.$$

For proof of this expression see Appendix III.

Hence the total compressive stress at the elastic limit, that is, at the point of failure, is

$$r + r' + r'' = f = r_c\left\{1 + \frac{vy}{k^2} + \frac{(f - r_c)}{10\,E}\left(\frac{l}{k}\right)^2\right\} \quad \ldots\ldots(9).$$

Whence

$$r_c = \frac{f}{1 + \frac{vy}{k^2} + \frac{f - r_c}{10E}\left(\frac{l}{k}\right)^2} \quad \ldots\ldots\ldots\ldots(10).$$

This formula is that given by Professor Johnson, of St. Louis, U.S.A. He says, "This is a strictly rational formula, with no purely empirical constants, for a column free to revolve at the ends, and will give good results. It must be solved by trial, since r_c is found on both sides of the equation. If $y = \frac{4}{5}k$, which is about the ordinary ratio, and if $f = 34000$ for wrought iron, and 42000 for steel, with $E = 27{,}000{,}000$ for iron and 28,500,000 for steel this formula would become

$$r_c = \frac{34000}{1 + \frac{4}{3}\frac{v}{k} + \frac{34000 - r_c}{270{,}000{,}000}\left(\frac{l}{k}\right)^2} \quad \ldots\ldots\ldots\ldots(11)$$

for wrought iron, and

$$r_c = \frac{42000}{1 + \frac{4}{3}\frac{v}{k} + \frac{42000 - r_c}{285{,}000{,}000}\left(\frac{l}{k}\right)^2} \quad \ldots\ldots\ldots\ldots(12)$$

for steel.

"These would apply only to columns pivoted on knife-edges. They would not apply to rounded columns, because the point of application of the load shifts as the column bends. They would not apply to a hinged, or pin-connected column, because the resistance to motion here is very considerable, which greatly increases the strength of the column by preventing lateral deflection.

"The second term in the denominator $\frac{vy}{k^2}$ must include all effects of initial bends or kinks in the column, and differences in the moduli of elasticity of the elementary forms of which it is composed, as well as eccentric position of load. These are usually unknown functions, and hence this term cannot commonly be evaluated. If this term be omitted,* and a constant empirical coefficient used for $\frac{f-r_c}{10\,E}$, in the next term we have

$$p=\frac{r_c}{1+a\left(\frac{l}{k}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13),$$

which is commonly known as Gordon's or Rankine's formula."

Gordon's formula is that which has been principally used in this country for long columns. It is usually expressed slightly differently from what is given above, viz. :—

$$P=\frac{r_c \times A}{1+a\left(\frac{l}{d}\right)^2},$$

where P = total breaking pressure on the column in lbs. or tons.

r_c = intensity of resistance (in lbs. or tons) to crushing of a cube of the material.

d = least transverse dimension of the column.

l = length of the bar, in same unit as d.

a = empirical constant.

The value of a varies both with the material and with the shape of the column. Table XXV. gives some of these values.

* This supposition is made only to show what changes would be necessary to obtain Rankine's formula.

TABLE XXV.

Table of the Values of Coefficient a *in Gordon's Formula for Struts.*

Nature of Material.	Form of Strut.	Value of r_c. Ultimate Resistance of a Small Cube to Crushing Tons per square inch.	Values of a.			Authority.
			Ends flat or fixed.	Ends round or hinged.	One end round, the other fixed	
Wrought Iron	L, T, channel, I, and square hollow.	19	$\frac{1}{900}$	$\frac{1}{400}$	$\frac{1}{600}$	Prof. Unwin.
,,	Hollow round...	$17\frac{1}{2}$	$\frac{1}{8000}$	$\frac{1}{3600}$	$\frac{1}{5000}$	,,
,,	Solid round ...	16	$\frac{1}{7200}$	$\frac{1}{3200}$	$\frac{1}{4800}$	,,
,,	Solid rectangular.	16	$\frac{1}{9000}$	$\frac{1}{4000}$	$\frac{1}{6000}$	,,
Mild Steel ...	Solid round ...	30	$\frac{1}{13500}$	$\frac{1}{6000}$	$\frac{1}{9000}$	Sir B. Baker.
,,	Solid rectangular.	30	$\frac{1}{6750}$	$\frac{1}{3000}$	$\frac{1}{4500}$	,,
Dry Fir	Solid round ...	$2\frac{1}{4}$	$\frac{1}{1000}$	$\frac{1}{444}$	$\frac{1}{666}$	
,,	Solid rectangular.	30	$\frac{1}{3000}$	$\frac{1}{667}$	$\frac{1}{1000}$	
Cast Iron.....	Solid round ...	$35\frac{3}{4}$	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{1}{1000}$	
,,	Hollow round...	$35\frac{3}{4}$	$\frac{1}{3000}$	$\frac{1}{3000}$	$\frac{1}{4000}$	

Mr. Claxton Fidler has investigated the matter also very exhaustively, with a view to ascertaining some expression which would give the breaking stress under the worst conditions of unequal, *i.e.*, most eccentric, loading, and variety of elasticity. His formula is as follows :—

$$P = \frac{\rho + f - \sqrt{(\rho+f)^2 - 4f\rho\,(1-\phi)}}{2\,(1-\phi)} \quad \ldots\ldots\ldots(14),$$

where P is the load that would produce any stress f, ρ is the value of the resilient force as found from Euler's formula, *i.e.*, $\frac{\pi^2 E k^2}{l^2}$, and ϕ is a symbol used to include the expression $\frac{\pi}{2}\,\frac{y}{k}\,\frac{e_2 - e_1}{e_2 + e_1}$, where e_1 and e_2 represent the variations of elasticity in the edges of the cross

section. It has been found that with the extreme variations of the Modulus of Elasticity the value of ϕ varies from 0·3 to 0·4.

Curves traced from this formula for cast iron, wrought iron and steel, both with rounded and with fixed ends, are given in *Molesworth's Pocket Book*, pp. 121 and 122 (Edition 1893).

As this subject is of importance, it is necessary to consider it further.

Fig. 1, *Plate* II., shows by the curve AE the breaking load of a column of mild steel with pin ends, as obtained from Fidler's formula. The breaking load of an ideal column, as ascertained from Euler's formula, is also shown by the curve BTC. It has been found by experiment that in almost any given case the stress-intensity that would cause failure in a column, or a series of columns, will not be represented by the ordinates drawn to either of these curves, *but will fall within the area ATCE.*

For a theoretically perfect column and concentric load, failure would occur along the line AT, and then along the curve TC, as the ratio of $l:k$ increased. In other words, the column would fail by direct crushing up to a certain ratio of $l:k$, and beyond that point it would fail by bending. But if there were any eccentricity of load, or any want of homogeneity in the column, the failure would occur by combined bending and crushing, and its strength would be represented by some ordinate below ATC, but above AE, within the area ATCE.

This consideration has suggested the idea to Professor Johnson that it would be possible to obtain the equation to a curve having its locus centrally situated in ATCE, which, while satisfying the theoretical requirements for very long and very short columns (about which there is no difficulty), would give generally true results for the intermediate lengths, where the value of $l:k$ varies between 50 and 200. He proposes to consider this curve tangential to Euler's, where $l:k=150$, and with its apex at the elastic limit of the metal (see *Fig.* 1, *Plate* II.), and the equation to it is

$$p=f-b\left(\frac{l}{k}\right)^2.$$

In this case b is a constant varying with the materials and the method of fixing, f is the elastic limit for the material, p is the stress in lbs. per square inch.

For ordinary lengths, *with a factor of safety of* 4 :—

Wrought-iron pin ends ... ... $p = 8{,}500 - 0{\cdot}17\left(\frac{l}{k}\right)^2$ .....(15).

„ „ fixed or flat ends $p = 8{,}500 - 0{\cdot}11\left(\frac{l}{k}\right)^2$ .....(16).

Mild steel pin ends... ... ... $p = 10{,}500 - 0{\cdot}24\left(\frac{l}{k}\right)^2$ .....(17).

„ „ fixed or flat ends ... $p = 10{,}500 - 0{\cdot}16\left(\frac{l}{k}\right)^2$ ..... (18).

These formulæ are very easily applied. The resulting values for mild steel pin ends are shown on *Fig.* 1, *Plate* II., and may be compared with Mr. Claxton Fidler's results.

Other Formulæ for Long Columns.

Other general formulæ which have been proposed are :—

(1). Mr. T. H. Johnson's straight line formulæ. This principle was proposed in 1886 as an approximation instead of the curve mentioned in the preceding paragraph.

The formulæ are very simple, and easily applied, but they give too great values of the ultimate strength for the shorter lengths. The results are shown on *Fig.* 2., *Plate* II. The formulæ are as follows :—

For wrought-iron hinged ends ... ... $p = 42{,}000 - 157\frac{l}{k}$.

„ „ flat ends ... ... $p = 42{,}000 - 128\frac{l}{k}$.

For steel hinged ends ... ... $p = 52{,}500 - 220\frac{l}{k}$.

„ „ flat ends ... ... $p = 52{,}500 - 179\frac{l}{k}$.

(2). Professor Merriman in 1889 proposed a formula which is used by some engineers—

$$\frac{P}{A} = \frac{r_c}{1 + \frac{r_c}{m\pi^2 E}\left(\frac{l}{K}\right)^2}.$$

Here the notation is as previously used in this chapter, $m = 1$ for rounded ends, $2\frac{1}{4}$ for one end round and one fixed, and 4 for both ends fixed.

This formula is derived from a process of reasoning much the same as that of Professor Johnson in Formulæ (15) to (19) above, *i.e.*, Rankine's and Euler's curves are both taken as correct, and an intermediate curve tangent to both is represented by the formula.

This formula appears to be based on unsound premises (as Rankine's is only approximately true), and it is, moreover, cumbrous to use. There appears, therefore, to be no advantage in it.

Fig. 2, *Plate* II., shows graphically the results obtained by different formulæ.

Timber Columns.

On *Fig.* 3, *Plate* II., are shown graphically the values of the weight per square inch on columns of red fir (*pinus sylvestris*) as obtained from (1) Gordon's formula; (2), the approximate rules from the M.W.D. Handbook; (3), the old rule of short columns up to $\frac{l}{h}=20$; (4), Euler's ideal column; (5), Capt. Sankey's investigations (Table XXIII.). A comparison of these values show that evidently (2) has been based on (1), that both (1) and (2) are apparently on the safe side as compared with (4) and (5), and that (3) is quite inadmissible. On the same diagram is shown results for "long-leaved yellow pine" and "white oak," as worked out by the following formulæ by Professor Johnson, based on the principle above described, whereby he obtains a mean value between the ideal column of Euler and the column of greatest eccentricity, viz. :—

Long-leaf yellow pine $\frac{l}{d}=<60$ ... ... $p=4{,}000-0{\cdot}8\left(\frac{l}{d}\right)^2$.

White oak, flat ends $\frac{l}{d}=<60$ ... ... $p=3{,}500-0{\cdot}8\left(\frac{l}{d}\right)^2$.

Unfortunately, the American habit of giving local and not scientific names to the timber has deprived these formulæ of much of their comparative value. Taking them, however, as fairly representative of soft wood, we see that they represent a value for many ordinary ratios far in excess of those obtained by the M.W.D. formulæ.

At all events, the latter are on the safe side, and may be taken for all practical purposes as fairly satisfactory.

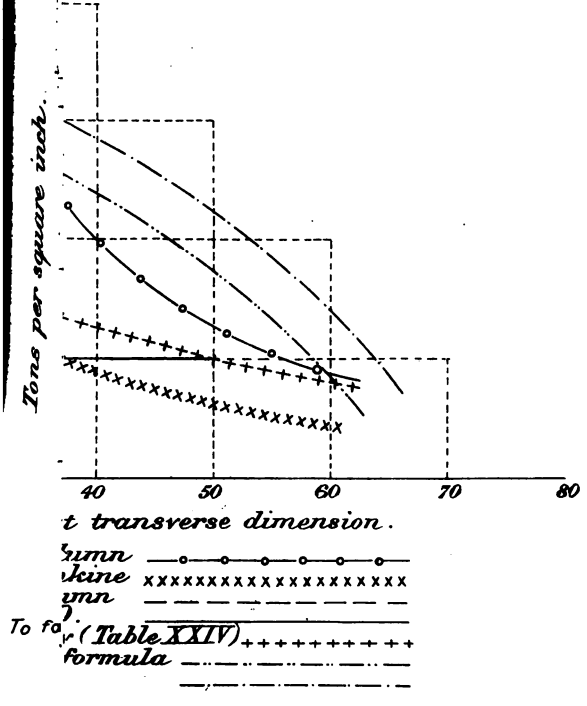
Effect of End Conditions.

As in the case of a beam loaded transversely, so also in a column the manner in which the ends are secured has a great influence o

PLATE II.

timate strength
, calculated by

g Resistance of a small cube



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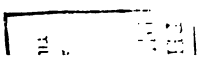


the resistance which it is capable of affording. If the ends are hinged or rounded, as in *Fig.* 140, it will bend as indicated, but if the ends are fixed or flat, the bending will be as shown in *Fig.* 141. One end may be fixed and the other rounded, as in *Fig.* 142. It has been stated above that a bar is very rarely perfectly hinged, but it is equally true that a bar is often assumed to be perfectly fixed, when in reality its fixing is very imperfect. It may be fixed as far as its attachment to the member of the truss at its end is concerned, but if that member be free to move, the result will not be so stiff as if the fixing were perfect. If the end of the bar is flat, as in the case of a timber post or cast-iron column, the ends may be considered as fixed, though probably there may be some imperfection in the fixing.

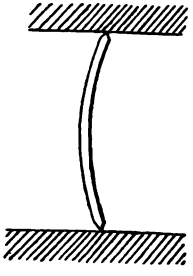


Fig. 140.

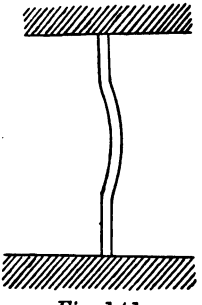


Fig. 141.

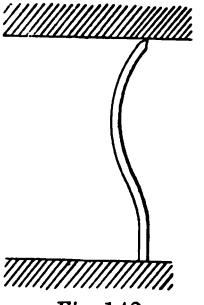


Fig. 142.

A bar that is fixed is stronger than one that is rounded in the ratio of 4 to 1. A bar that is fixed at one end and hinged at the other is stronger than one that is hinged at both ends in the ratio of 2* to 1. As a rule, columns are considered either as fixed wholly, or hinged at both ends. The other case is a mean between the two.

In the construction of trusses of iron or steel, therefore, it is important to bear in mind that the ends of struts, if feasible, should be fastened by *at least two bolts and rivets.* If fastened with one rivet only, they must be considered as hinged only, for although friction prevents them from moving freely round the one rivet as a pivot, yet the amount of that friction is so indefinite that it cannot be allowed to enter into calculations, and the weaker condition must be taken.

It is also important that the load should be brought as concentri-

* Professor Merriman considers 2¼.

cally as possible on the strut by means of the end connections. For though it is generally assumed that the strut will bend, if anywhere, in the direction of the least dimension, yet, as a matter of fact, if the load be brought to bear in a very eccentric direction, the bending may take place in the direction away from the load, even though that may not be in the direction of the least dimension.

In the case of pillars of timber and of cast iron it is important to see that the ends are truly square. In the case of a timber post carrying a bressumer, as in *Fig.* 143, with a mortice and tenon joint, it is important that the end of the tenon does not reach to the bottom of the mortice, but that the weight be borne by the shoulder of the tenon, and thus tend to bring the load concentrically on the strut. With cast-iron pillars it is sometimes customary to specify that the ends shall be planed. If not, and the pillar is based upon a stone slab or cube, it is a salutary precaution to lay a layer of some material as Portland cement mortar, or lead, so that all inequalities may be filled up and the weight distributed.

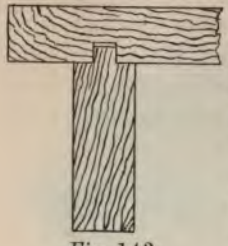


Fig. 143.

It must be remembered that the *least dimension* of a section is the least tranverse dimension of the rectangle, triangle, or other figure circumscribing the section. These are generally indicated on Table XXVI. The correct investigation, however, would be to find the value of the least radius of gyration of the cross section, as in Table XXIV.

Summary of the Foregoing Investigation.

To sum up the foregoing points:—

(1). Struts, columns, etc., may fail either by crushing, bending, or by crushing and bending combined. If the length is relatively short they will fail by crushing, if the length is relatively long they will fail by bending; but it is not practicable to determine any point where the one method of failure will cease and the other begin, for, as a matter of fact, both of investigation and of practice, many will fail by a combination of both causes.

(2). It is possible to investigate the failure of ideal columns, where the material is perfectly homogeneous, and the load absolutely concentric, and to represent such failure by means of curves both for struts fixed and hinged.

(3). It is also possible to investigate the failure of struts under the most unfavourable conditions of eccentricity of load and variation

of elasticity, and to represent such failure on the same graphic diagram as the curve mentioned in (2).

(4). Although no one formula will tell us exactly the behaviour of a strut under any given conditions, we may ascertain from an inspection of the graphic diagram that its ultimate strength will fall within a well-defined area, viz., that contained between the curves showing the behaviour of the ideal column and that corresponding to the column of greatest eccentricity, etc., and approximate formulæ, based on average results, may be applied to all ordinary cases.

(5). The end condition of the strut has a most marked effect on its strength, as upon this condition depends the tendency of the strut to begin to bend.

(6). For timber, the rules given above from the M.W.D. handbook are practical, correspond sufficiently closely with theory, and may be adopted in all ordinary cases of construction.

(7). For wrought iron and steel the following table (XXVI.) will be found to cover almost all the cases of construction in bridges, roofs, etc. This has been based on Professor Fidler's formula (14), and is probably as accurate as is practically possible :—

TABLE XXVI.

Showing Strength of Columns of Cast Iron, Wrought Iron, and Steel.

Here l = length of strut between ends in inches.
k = least radius of gyration, *i.e.*, the square root of Moment of Inertia ÷ area of cross section.
r_c = *safe* stress in tons per square inch.

$\frac{l}{k}$	r_c					
	Cast Iron.		Wrought Iron.		Mild Steel.	
	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.
10	8·68	8·85	4·00	4·00	5·33	5·34
15	8·41	8·76	3·98	4·00	5·26	5·31
20	8·07	8·65	3·92	3·99	5·20	5·29
25	7·58	8·46	3·88	3·98	5·13	5·24
30	6·98	8·21	3·80	3·95	5·02	5·20
35	6·32	7·91	3·72	3·92	4·90	5 15
40	5·68	7·56	3·64	3·89	4·76	5·09
45	5·02	7·19	3·54	3·86	4·58	5·03
50	4·43	6·82	3·44	3·82	4·40	4·98

TABLE XXVI.—(*Continued*).

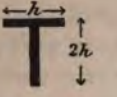
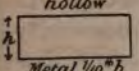
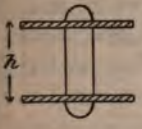
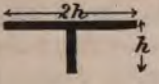
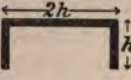
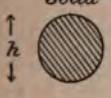
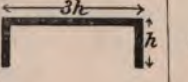
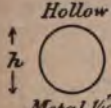
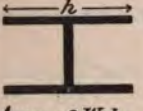
Showing Strength of Columns of Cast Iron, Wrought Iron, and Steel.

$\frac{l}{k}$	r_c					
	Cast Iron.		Wrought Iron.		Mild Steel.	
	Ends rounded.	Ends fixed.	Ends rounded	Ends fixed.	Ends rounded.	Ends fixed.
55	3·84	6·46	3·31	3·78	4·22	4·92
60	3·35	6·10	3·17	3·73	4·02	4·83
65	2·92	5·75	3·04	3·68	3·80	4·75
70	2·57	5·39	2·90	3·63	3·59	4·67
75	2·23	5·02	2·76	3·55	3·37	4·56
80	1·96	4·68	2·60	3·48	3·15	4·45
85	1·74	4·33	2·46	3·40	2·96	4·35
90	1·56	4·00	2·33	3·32	2·77	4·25
95	1·42	3·66	2·18	3·22	2·56	4·13
100	1·29	3·35	2·03	3·17	2·40	4·00
105	1·17	3·07	1·92	3·08	2·24	3·88
110	1·07	2·80	1·79	3·00	2·08	3·74
115	0·99	2·57	1·67	2·91	1·95	3·61
120	0·93	2.37	1·57	2·82	1·83	3·46
125	0·86	2·19	1·47	2·74	1·71	3·32
130	0·80	2·03	1·39	2·66	1·61	3·21
135	0·75	1·90	1·32	2·58	1·50	3·09
140	0·70	1·78	1·24	2·48	1·42	2·96
145	0·66	1·66	1·17	2·40	1·36	2·85
150	0·61	1·56	1·10	2·32	1·28	2·72
160	0·56	1·40	0·98	2·14	1·13	2·51
170	0·49	1·25	0·88	2·00	1·01	2·32
180	0·43	1·14	0·80	1·84	0·91	2·13
190	0·39	1·03	0·72	1·70	0·83	1·97
200	0·36	0·93	0·66	1·57	0·75	1·83
210	0·32	0·84	0·58	1·46	0·68	1·68
220	0·30	0·77	0·55	1·35	0·62	1·55
230	0·28	0·70	0·50	1·26	0·58	1·44
240	0·25	0·64	0 46	1·18	0·53	1·34
250	0·23	0·59	0·42	1·11	0·49	1·25
260	0·22	0·56	0·40	1·04	0·46	1·16
270	0·20	0·52	0·37	0·97	0·42	1·08
280	0·19	0·49	0·35	0·91	0·39	1·01
290	0·18	0·46	0·32	0·86	0·37	0·96
300	0·17	0·43	0·30	0·80	0·35	0·92

For practical purposes the value of $\frac{l}{k}$ can be found with sufficient accuracy from

$$\frac{l}{k} = n\frac{l}{h},$$

where n and h have the following values for various sections in general use :—

Section.	n	Section.	n	Section.	n
	3·5		6·0		4·9
	2·5		4·9		3·9
	3·0		4·2		3·8
	4·0		3·9		4·3
	3·1		4·2		4·9

Combined Bending and Direct Stresses.

A tension or compression member may be subject to bending or tranverse stress, either from its own weight, if in a horizontal position, or from some tranverse load. In all such cases the member must be calculated for both direct and cross-bending stress.

It may be remarked that it is not economical to subject a member to combined stresses of this description. The stress on the extreme fibres will be the governing factor in the calculation, while there

will be a portion of the interior whose strength will not be utilized.

The effect of a transverse load is to produce a deflection. The effect of direct loading is to increase the deflection, if acting so as to compress the member, while the reverse is the case with a tensile force. It is possible to obtain a general formula, which will take account of both the direct and transverse stresses combined.

Let M_1 = bending moment at point of maximum deflection, from cross-bending external forces, and from eccentricity of position of longitudinal loading,

v = maximum deflection of member from all causes acting simultaneously,

M_2 = bending moment from direct loading; P into its arm = Pv,

P = total direct loading on member, tension or compression,

r_1 = unit stress in extreme fibre from bending alone,

r_2 = „ „ in member from direct loading, uniformly distributed = P ÷ A,

and other symbols, I, y, E and l having the same meaning as always, then the deflection in terms of the stress in the extreme fibre is found from the general equation

$$v = \frac{n' r l^2}{E y} \text{ (see Chapter VII.).}$$

The value of n' in ordinary cases in practice may be taken as $\frac{1}{10}$.

Hence the general relation between the deflection of a beam and the stress intensity of the extreme fibre may be expressed by the equation

$$v = \frac{r l^2}{10 E y}.$$

In all cases now under consideration the M_r will be the sum of the moments of the external forces, hence $M_r = \frac{rI}{y} = M_1 \pm M_2 = M_1 \pm Pv$, the positive sign being used when the member is under compression, and the negative when under tension. Substituting for v its value as found above, we have

$$r = \frac{M_1 y}{I \pm \frac{P l^2}{10E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20),$$

where the *negative* sign is to be used for compression members, and the *positive* sign for tension members.

The application of this general formula will become evident from examples to be hereafter given.

General Deductions from the foregoing Investigations, both as regards Tension and Compression Members.

In tension members, the *net* area of the cross section of the material must be considered as resisting the stress coming upon it.

The connections in tension members must be arranged so as to produce no eccentricity of stress.

In compression members, the load which the column will carry becomes less as the length increases, other things being the same. If, however, a long column is braced at intervals, its length is virtually diminished and its strength increased.

Fixing the ends of a column has the effect of largely increasing its strength.

Hollow pillars of cast iron, if required to bear heavy weights, should not be cast with hollow projecting mouldings, or elaborate bases or capitals, as in *Fig.* 144. These are a source of weakness from the change in the stress brought on the iron, and also from the change in the crystalline formation of the metal. If more elaborate enrichments are required, they should be cast separately, and united to the column by fastenings.

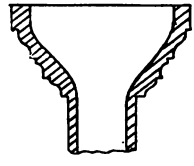


Fig. 144.

In all members, whether tension or compression, a symmetrical section is to be preferred, as tending to do away with eccentric application of the stress.

In designing a compression member, the value of the ratio $l:k$ is a most important factor, hence the larger the value of k, the less will be the area required to meet the stress, and, in general, the greater economy in construction. The general form of compression members should be approximately circular or square, so that the value of k in all directions will be as great as possible. There is obviously a limit to the disposition of a given area of metal in a form giving the largest value of k, for the principle, if carried to extremes, would result in members of very thin metal. The limits of market forms must be taken as a guide to the thickness.

The unsupported length of a strut should not be greater than 50 times the least dimension, or 150 times the value of k.

As regards the design, longitudinally, of a column, it is clear that true cylinder or square prism is not economical. Some diminution

of the dimensions of the column at the ends commends itself to the eye as suitable. Hence in Greek pillars the graceful *exstasis*, or enlargement of the shaft. In the Lansdowne Bridge over the Indus at Sukkur the enlargement of the struts from the ends to the centres is very markedly a feature in the design. The curve of the bending moments has been shown to be a parabola. Practically this curve would not differ, in the cases ordinarily considered, from a circle. It would, therefore, be quite sufficient to give a circular form to the arcs of the struts in any cases where it would be worth while to make a difference between the sections at the centres and at the points of fixing, *i.e.*, the form shown at B, *Fig.* 145, rather than A, leaving, of course, sufficient material at the ends for fixing.

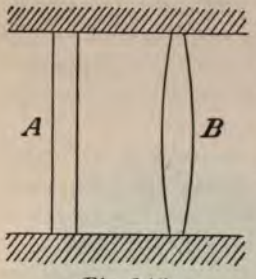


Fig. 145.

EXAMPLES.

EXAMPLE 21.—*A timber post* 10′ *high*, 6″ × 6″ *in section, is tenoned into a bressumer by a tenon* 4″ × 1½″ *in section. Find the weight the post will bear, the timber being dry fir.*

The effective section will be $6 \times 6 - 4 \times 1{\cdot}5 = 30$ square inches

The ratio of l to d is 20 to 1.

Hence from Formula (4) $P = \frac{1}{2} r_c A$.

r_c, the *safe* crushing resistance of dry fir, may be taken at 1,000 lbs. per square inch. Hence

$$P = \tfrac{1}{2} 1200 \times 30 = 18{,}000 \text{ lbs., or } 8{\cdot}04 \text{ tons.}$$

From Table XXIII. $r_c = 11{\cdot}1$ cwts., and $P = 30 \times 11{\cdot}1 = 333$ cwts., or 16·5 tons ultimate strength.

This would give the same result as before with a factor of safety of 2. As, however, Table XXIII. is for *rounded* struts, and the post in question is fixed, the low factor of safety is quite admissible.

Care should be taken that the tenon does not go to the bottom of the mortice, otherwise the timber may shrink and the whole weight come on the tenon.

As a matter of fact, the whole cross section of the post would be effective. It would not be the same as if it were hollowed, although it has been so calculated.

EXAMPLE 22.—*A principal rafter in a truss has to bear a maximum*

ressure of 10,000 *lbs. The unsupported length* (i.e., *from junction of ie beam to the first strut*) *is* 6 *feet, or* 72 *inches* (*Fig.* 146). *The timber is red fir* (pinus sylvestris). *Find suitable dimensions.*

This is an ordinary case in roof construction.

As we do not know either the breadth or depth we may proceed by trial and error.

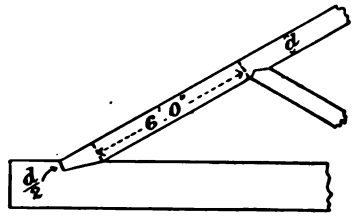


Fig. 146.

Assume the breadth to be 4". Then the ratio of l to d will be as 72 is to 4, or as 18 to 1.

Hence from Formula (4) $P = \frac{1}{2} r_c A$.

P in this case is 10,000 lbs., r_c may be taken as 1,200, and $A = 4d$.

Hence $$10{,}000 = 600 \times 4d,$$

$$d = \tfrac{10000}{2400} = 4{\cdot}18.$$

The strut may be, therefore, 4" × 5".

At the junction with the tie beam the whole pressure may be brought on the toe, *i.e.*, on half the depth as usually constructed by good carpenters. Hence an area of 4" × 2½", or 10 square inches, will have to bear the whole load of 10,000 lbs. This gives 1,000 lbs. per square inch, which is well within the crushing strength of the material.

EXAMPLE 23.—*Find the diameter of a cast-iron pillar* 12′ *high to bear a weight of* 10 *tons.*

Assuming that the thickness of the metal is $\frac{1}{12}$ of the external diameter, which is an ordinary rule, we have the area of the ring

$$= \frac{\pi d^2}{4} - \frac{\pi}{4}\left(\tfrac{5}{6}d\right)^2 = \tfrac{11}{144}\pi d^2.$$

Then in Gordon's formula

$$\frac{P}{S} = r_c \div \left(1 + a\frac{l^2}{d^2}\right)$$

we have

$P = 10 \times 2{,}240 = 22{,}400$ lbs.

$S = \tfrac{11}{144}\pi d^2 = {\cdot}24d^2$.

r_c = *safe* intensity of resistance to crushing of cast iron = 9 tons, say 20,000 lbs., per square inch.

$a = \frac{1}{800}$ from Table XXV.

Substituting values—

$$22{,}400 = \cdot 24d^2 \times 20{,}000 \div \left(1 + \frac{120^2}{800d^2}\right).$$

Whence $d = 3{\cdot}6$. Make the column 4″.

This involves a somewhat tedious quadratic, and one can obtain the required result more readily and more accurately by using Table XXVI. and by trial and error.

Assume a diameter to the column of 3″. Then the area of the ring will be $\cdot 24 \times d^2 = 2{\cdot}16$ square inches. And ratio of l to d will be 48 to 1. From Table XXVI.—

$$\frac{l}{k} = 3{\cdot}1 \times 48 = 149.$$

Hence from Table XXVI. (under heading "Cast Iron—Ends Fixed") we have 1·56 tons per square inch of *safe* stress. In this case the total safe stress will be

$$2{\cdot}16 \times 1{\cdot}56 = 3{\cdot}35 \text{ tons},$$

which is not sufficient.

Taking now 4″ and proceeding the same way, we find that the area of the ring

$$= \cdot 24 \times 4^2 = 3{\cdot}84 \text{ square inches},$$

and ratio of l to $d = 36$. Hence

$$\frac{l}{k} = 3{\cdot}1 \times 36 = 111,$$

and the corresponding safe stress per square inch is 2·80 tons. Hence the safe load on the column will be

$$2{\cdot}80 \times 3{\cdot}84 = 10{\cdot}75 \text{ tons}.$$

As the load to be borne is 10 tons, a diameter of 4″ will therefore be amply strong enough, as already proved above by Gordon's formula.

EXAMPLE 24.—*The strut of a roof is of* T*-iron* $3'' \times 3'' \times \frac{1}{2}''$, *is fastened at the ends with bolts or pins, and is* 6 *feet in length. What weight can it bear safely?*

The area of the cross section is $(3 \times \frac{1}{2}) + (3 - \frac{1}{2})\frac{1}{2}'' = 2{\cdot}75$ square inches. The ratio of l to d is 72 inches : 3 inches = 24. From Table XXVI. $\frac{l}{k} = 4{\cdot}9 \times 24$ for T-iron. Hence $\frac{l}{k} = 118$. From

Table XXVI., under column "Wrought Iron, Ends Rounded," and opposite the value of 120 we have 1·57 tons per square inch.

Hence the strut can safely bear

$$2{\cdot}75 \times 1{\cdot}57 = 4{\cdot}32 \text{ tons.}$$

To investigate this case by Professor Johnson's Formula (16), and taking $\frac{l}{k}$ as before $= 118$ we have

$$r_c = 8{,}500 - 0{\cdot}17\left(\frac{l}{k}\right)^2 \text{ for pin ends}$$
$$= 8{,}500 - 0{\cdot}17 \times 13{,}689$$
$$= 8{,}500 - 2{,}320 = 6{,}180 \text{ lbs.}$$
$$= 2{\cdot}75 \text{ tons per square inch,}$$

and the total safe stress on strut

$$= 2{\cdot}75 \times 2{\cdot}75 = 7{\cdot}5 \text{ tons.}$$

This is considerably greater than the result obtained by the first investigation, probably due to the fact that the former investigation is based on the consideration of the *most unfavourable* conditions of load and eccentricity, and the latter investigation on the *average* conditions.

If the strut were *rivetted* at the ends with at least two rivets at each end, it would be much stronger than if only fastened with one pin at each end.

EXAMPLE 25.—*The standards in a single sling spar bridge are* 28′ *long, and of an average diameter of* 9½″. *The timber is red fir* (pinus sylvestris). *The bridge bears a live load of* 560 *lbs. per foot run. The angle of the frame is* 45°, *and total span* 40 *feet.*

This is an example of combined bending and direct stress. The type is that usual in military bridging. It is shown in *Fig.* 147.

Each frame carries half the weight borne by the slings + the weight on the transom. These weights are the same, being that of 10′ in length of the bridge

$$= 560 \times 10 \times 2 = 11{,}200 \text{ lbs.}$$

The crushing weight on each standard will be half that on the frame, and will, therefore, amount to

$$\tfrac{1}{2}\ (W \sin\theta + W \sin\theta) = W \sin\theta = 11{,}200 \times {\cdot}707 = 7{,}920 \text{ lbs.}$$

As the diameter is 9·5, the area of the cross section is

$$4{\cdot}75^2 \times 3{\cdot}1416 = 71 \text{ square inches,}$$

and the ratio of l to d is 35·5.

From Formula (4) $\quad P = \frac{1}{4} r_c A.$

Hence $\quad r_c = \dfrac{4 \times 7{,}920}{71} = 450$ lbs. per square inch,

which is well within safe limits, 6,400 being the ultimate strength of the material.

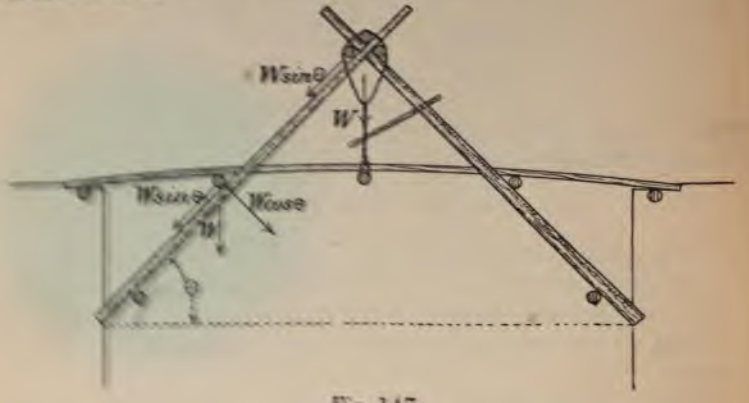


Fig. 147.

The same investigation by Gordon's formula is

$$r_c = \frac{P}{S}\left(1 + a\left(\frac{l}{d}\right)^2\right),$$

where $a = \frac{1}{1\,0\,0}$ for a fixed column (see Table XXV.). This assumes that the riveting, etc., has the effect of fixing the ends. This would be approximately true.

Substituting values—

$$r_c = \frac{7\,9\,2\,0}{71}\left(1 + \tfrac{1}{1\,0\,0}35{\cdot}5^2\right)$$
$$= 854 \text{ lbs.,}$$

which is a higher result than that given above, but is still within perfectly safe limits.

But there is the transverse stress as well.

The force tending to cause transverse rupture on each standard is

$$\tfrac{1}{2} W \cos\theta = \tfrac{1}{2}(11{,}200 \times {\cdot}707) = 3{,}960 \text{ lbs.}$$

The bending moment is

$$\tfrac{1}{4}Wl = \tfrac{1}{4}3{,}960 + 336$$
$$= 339{,}640 \text{ inch-lbs.}$$

The Moment of Resistance

$$= \frac{rI}{y} \text{ and } I = \cdot 7854R^4$$
$$= r \times \cdot 7854 \times 4 \cdot 75^3$$
$$= 84r.$$

Taking Formula (20)—

$$r = \frac{My}{I - \frac{Pl^2}{10E}},$$

we have the following values :—

$$M = 339{,}640 \text{ inch lbs.}$$
$$y = 4 \cdot 75 \text{ inches.}$$
$$I = 398.$$
$$P = 7{,}920 \text{ lbs.}$$
$$E = 1{,}440{,}000 \text{ lbs.}$$
$$l = 336 \text{ inches.}$$

Substituting these values,

$$r = \frac{339{,}640 \times 4 \cdot 75 \times 14{,}400{,}000}{14{,}400{,}000 \times 398 - 7{,}920 \times 336^2} = 4{,}804 \text{ lbs. per square inch.}$$

This is very near the limit of *ultimate* resistance (6,400) and cannot, therefore, be considered safe.

In such a bridge, therefore, stiffening is most necessary. It should be noted also that as the method of applying the load is eccentric, a liberal factor of safety should be adopted.

APPENDIX I.

To prove that the radius of gyration $= \sqrt{\frac{I}{A}}$. *

Definition 1.—The Moment of Inertia, I, of a body about any axis is defined as the sum of the products of each element of the mass into the square of its distance from the axis $= \Sigma my^2$.

* Proof kindly furnished by Captain C. F. Close, R.E.

DEFINITION 2.—The radius of gyration, k, of a body with reference to a fixed axis is defined as the distance from the axis of that point at which, if the whole mass of the body were collected, the angular motion of the point would be the same as that of the body when acted on by the same force.

Now the angular motion of a body is thus investigated:—Let $d\phi$ be an elementary angular movement about the axis, dt an element of time, then

velocity of particle m $\quad = y \frac{d\phi}{dt}$,

and since generally force = mass × velocity (where force is measured by momentum)—

Force of particle $\quad = my \frac{d\phi}{dt}$,

and moment of force $\quad = my^2 \frac{d\phi}{dt}$.

Total moment of all forces $= \Sigma\, my^2 \frac{d\phi}{dt}$

$= \mathrm{I} \frac{d\phi}{dt}$ = I × angular velocity.

But the moment of all forces $= \mathrm{M}k \times k \frac{d\phi}{dt}$ from the Definition 2 $= \mathrm{M}k^2 \frac{d\phi}{dt}$.

Hence $\quad \mathrm{M}k^2 \frac{d\phi}{dt} = \mathrm{I} \frac{d\phi}{dt}$,

or $\quad k^2 = \frac{\mathrm{I}}{\mathrm{M}}$

in an area $\quad k = \sqrt{\frac{\mathrm{I}}{\mathrm{A}}}$.

APPENDIX II.

PROOF OF EULER'S FORMULA FOR COLUMNS.

LET OQ (*Fig.* 137) represent a column free to move about its end supports, and just beginning to deflect under the load P.

The M_f at any section, distant x from O $= \mathrm{P}y = \frac{\mathrm{EI}d^2y}{dx^2}$ from the fundamental equation for deflection of beams.

Hence $\quad \frac{d^2y}{dx^2} = -\frac{\mathrm{P}y}{\mathrm{EI}}$.

Multiplying by dy, x being the independent variable, and integrating once, we have

$$\left(\frac{dy}{dx}\right)^2 = -\frac{\mathrm{P}}{\mathrm{EI}} (y^2 + \mathrm{C}).$$

When $\frac{dy}{dx}=0$, $y=V$, the maximum deflection, hence $C=-V^2$, and

$$\left(\frac{dy}{dx}\right)^2=\frac{P}{EI}(V^2-y^2), \text{ or } dx=\frac{EI}{P}\frac{dy}{\sqrt{V^2-y^2}}.$$

$$x=\sqrt{\frac{EI}{P}}\text{ arc sin }\frac{y}{V}+C,$$

when $x=0, y=0$, and $C=0$.

$$y=V\sin x\sqrt{\frac{P}{EI}}, \quad *$$

when $x=\frac{1}{2}l$, $y=V$.

Hence $$\tfrac{1}{2}l\sqrt{\frac{P}{EI}}=\frac{\pi}{2},$$

or $$P=\frac{\pi^2 EI}{l^2}.$$

But $P=Ap$, and $I=Ak^2$.

Hence $$p=\frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}.$$

APPENDIX III.

To express the value of the stress due to the bending of a column under a load P in known terms.

If v be the lateral deflection of the column under a load P, the M_{ff} from this source will be Pv. For such bending as occurs within the elastic limit of the material, the M_f at other points will be analogous to that produced by a uniform load on a beam under transverse stress, and will vary as the ordinates of a parabolic curve. The deflection in the centre for such a beam will be at the elastic limit $V=\frac{5}{48}\frac{(f-r_c)l^2}{Ey}$, f being the elastic limit stress for the material, and r_c the resistance per unit to crushing. We may without appreciable error call $\frac{5}{48}=\frac{1}{10}$.

Hence $$M_{ff}=Pv=\frac{P(f-r_c)l^2}{10Ey}.$$

But $M_{ff}=\frac{r_c''I}{y}$ where $r_c''=$ the stress produced on the outer fibres and $I=Ak^2$.

Hence $$r_c''=\frac{P}{A}\frac{(f-r_c)}{10E}\left(\frac{l}{k}\right)^2=\frac{(f-r_c)}{10E}\left(\frac{l}{k}\right)^2 r_c.$$

* This is the equation to the curve of the elastic line.

CHAPTER X.

Joints and Fastenings.

General Remarks.—Timber Fastenings.—Scarf Joints.—Mortice and Tenon.—Truss Oblique Joints, various forms.—Iron Joints.—Screws.—Pin and Link.—Gib and Cotter.—Rivets.

It is not possible in every structure to design the whole out of one solid block of the material proposed for use. The practical limits of manufacture, or natural growth, or facilities for moving, or some other cause, make it necessary that all structures, except those of a comparatively small size and restricted utility, be built up of several parts of various materials. The question, therefore, of the best method of uniting the parts, whether they be composed of the same or different materials, is one of extreme importance. The strength of a chain is proverbially that of its weakest link, and it is, perhaps, no exaggeration to say that in any design of a structure the skill of the engineer is nowhere more manifest than in the perfection of his arrangements for the joints and fastenings throughout.

In masonry structures the joints are usually formed by the tenacity of the mortar and by the skill in bonding the bricks or stones. In some cases where greater strength is required, cramps, dowels, or other similar devices are adopted. These fastenings, however, do not readily lend themselves to theoretical treatment, and it is not intended in this chapter to consider their arrangement.

With iron and steel, and with timber, the case is different. It is with these materials that we are now concerned.

In the design of a joint there are certain broad principles which should be borne in mind in any case:—

(1). The members which are connected should be weakened as little as possible by the joint or fastening. In certain cases the nature of the stress coming on the member may be such that the weakening of the part may be a matter of little importance. As a general rule, however, it is well to reduce the sectional area of the part joined as little as possible.

(2). Each part of the joint should be proportional to the stress it as to bear.

(3). Abutting surfaces in compression should be placed as nearly s possible at right angles to the direction of the pressure coming on hem, so as to avoid resolved stresses in other directions.

(4). The joint should be arranged so that it is affected as little as ossible by expansion or contraction of the materials.

(5). The joint should be designed so that the stresses are *uniformly* ransmitted from one member to the other.

Timber Joints.

The timber joints which we shall consider are :—(1). Scarf joints. (2). Mortice and tenon joints. (3). Oblique joints in trusses.

It may be said that investigation of these is unnecessary, as long experience has taught carpenters the best proportions to give in every case. Although this may be true in a country where the properties of the timber used for construction are accurately known, yet the proportions thus arrived at by practice will not hold good where the material is different. And, as a matter of fact, trade customs are not always based upon scientific principles.

(1). *Scarf Joints.*—These are either for tension members, or for transverse stress. *Fig.* 148 shows such a joint designed to bear tension. In this case the net area only of the cross section of the beam, viz., $ac \times cd$, is available to bear the tensile stress on the beam.

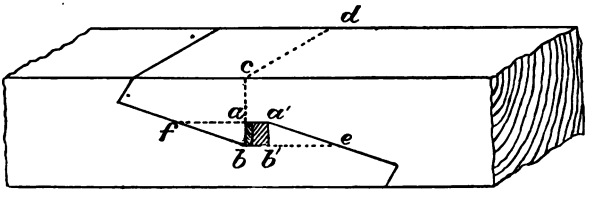


Fig. 148.

The joint may fail in one of three ways :—

(1). The part $ac \times cd$ may be so reduced that the beam may be torn at *ac*.

(2). The hard wood wedges at the centre may crush into the fibres of the wood at *ab* or *a'b'*.

(3). The part *b'e* or *af* may shear longitudinally.

Hence, in order that all the parts of the joint may be of equal trength, $ac \times$ resistance per unit to tension should $= ab \times$ resistance

per unit to crushing, and $= af \times$ (longitudinal) resistance per unit to shearing.

The proportions of the depths of the various parts of this joint must, therefore, vary inversely as the resistance to tension, compression, and shearing, longitudinally, of the timber used.

A scarf joint to bear transverse stress, as in *Fig.* 149, is a weak form of joint, and must necessarily be supplemented on the tension side by an iron plate. As the central wedges are situated near the neutral axis, there is little direct strain on them or on the adjacent timber.

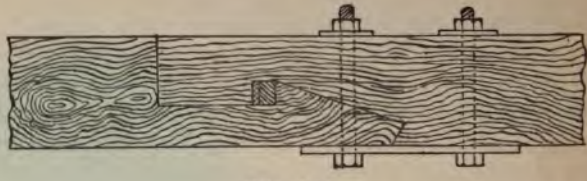


Fig. 149.

(2). *Mortice and Tenon Joints.*—In these joints the tenons have to bear shearing stress when the two beams joined together are both horizontal. The beam in which the mortice is cut is very little weakened by it, as it usually comes near the neutral axis. It is better to join beams by a mortice and tenon joint than by notching, or by cogging, for the beam so notched or cogged is more weakened than it would be if it were morticed. The reason of this is that the notching or cogging necessarily takes away material from the layers of the beam which are subject to the greatest stress, while the mortice is made at that part of the beam where the stress is least (*Figs.* 150 and 151).

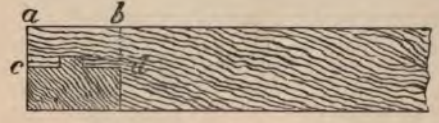


Fig. 150.

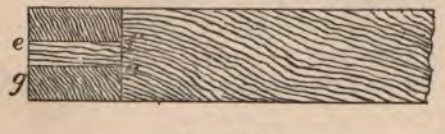


Fig. 151.

Of course, this reasoning applies only to beams which are supported or fixed at the ends; not to such beams as wall plates, which are supported through the whole of their length.

o find the necessary depth of the tenon, we may take the owing :—

EXAMPLE 26.—*The floor joists of a room* 18′ *span are* 2″ *broad,* ′ *deep, and are tenoned into a trimmer at one end. The material is red* (pinus sylvestris). *Find the depth of the tenon, the breadth being the ıole width of the joist,* 2 *inches.*

From Table XIV., Chapter III., the shearing resistance of red fir :ross the grain is 5,000 lbs. per square inch. Allow a factor of afety of 5 ; this gives 1,000 as the working value.

The stress intensity at the neutral axis will be, as we have seen n Chapter VI., $\frac{3}{2}$ of the average intensity.

The total shearing stress coming upon the tenon will be half the :otal weight coming on the joists (Chapter VI.). The joists of a floor are generally calculated to bear 140 lbs. per foot super, and are 12″ apart. Hence half the weight coming on the joist will be $140 \times \frac{1 \cdot 8}{2} = 1{,}260$ lbs.

Hence if d be the required depth of the tenon,

$$2'' \times d \times 1{,}000 = 1{,}260 \times 3 \div 2.$$

Hence $$d = \cdot 97''.$$

This is far less than is made in practice. As a rule, a tusked tenon is made, which combines a maximum depth to resist shearing with a minimum weakening of the trimmer. This will be evident from *Fig.* 152.

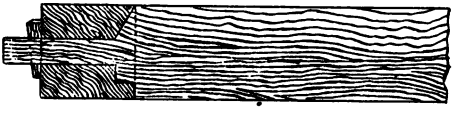


Fig. 152.

In ordinary construction the maximum shearing in floors comes on the tenons joining trimmers to trimming joists (*aa*, *Fig.* 153). These have to bear half the total weight on the whole of the trimmed joists A, B, C, etc.

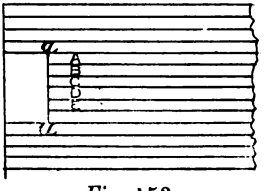


Fig. 153.

(3). *Oblique Joints in Trusses.*—These are very important because of the magnitude of the stresses they are frequently called upon to sustain and to transmit to other parts of the structure.

(i.). *Example* 27.—As one of the most usual cases of such joints, let us consider the joint in *Fig.* 154, which represents the junction of principal and tie-beam in an ordinary king-post truss, the same

as has been already considered in Example 22, Chapter IX. The compressive stress on the principal is 10,000 lbs., the breadth of the beam is 4″, and the depth 5″. We have already seen that the depth of the part *ab* should be half the depth of the beam, *i.e.*, $2\frac{1}{2}$″, and that the angle *cab* should be a right angle. This joint may fail by the shearing of the end of the tie beam along the layer *bd*. As this shearing will be in the direction of the fibres, the resistance per square inch for red fir will not be more than 500 lbs. ultimate, and 100 lbs. safe. Hence the unknown length *bd* × the breadth of the beam 4″, and × 100 lbs., must be equal to the thrust of the principal resolved horizontally. Let x be the unknown length *bd*.

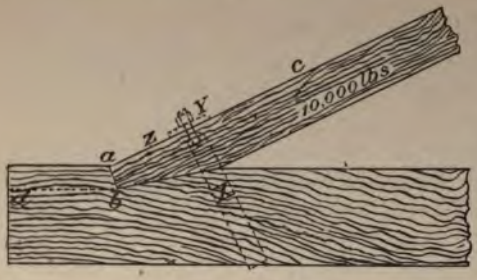


Fig. 154.

To find the resolved component of the thrust, draw AB = the thrust on any scale (*Fig.* 155) and parallel to the principal. Draw BC horizontal, and AC vertical. Then BC on the same scale = the horizontal thrust = 8,945 lbs. Without drawing a figure this result might be obtained from a table of natural cosines, for BC = AB cos ABC.

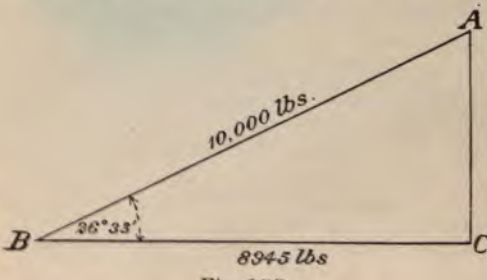


Fig. 155.

Then $4 \times x \times 100 = 8{,}945$, and $x = 22''$.

It would be impracticable in most instances to give so great a length to the end of the tie beam, hence some form of fastening is usually adopted to prevent the joint giving way with a moderate length of tie beam, say 10″ beyond the junction.

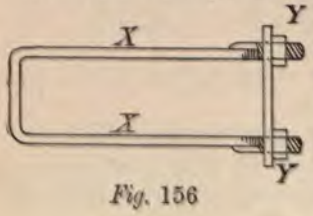


Fig. 156

The form of fastening usually adopted in England is shown in *Fig.* 156,

i., a strap encircling the principal and tie beam, and placed, as nown in dotted lines in *Fig.* 154, at right angles to the principal.

This strap may fail in the following ways :—

(*a*). The sectional area of the iron at XX may be insufficient for he stress brought upon it.

(*b*). The sectional area of the bolts at the ends may be similarly nsufficient.

(*c*). The depth of the nuts YY may be insufficient to prevent the screws stripping.

(*d*). The breadth of the strap below, and of the plate Z above, nay be insufficient to prevent the fibres of the wood crushing at the points where they come in contact above and below.

We have first to find the stress on the strap. This can be done, as before, by drawing a stress diagram. We have already found that the horizontal component of the thrust is 8,945 lbs. Of this thrust the 10″ projection of the tie beam can bear $10 \times 4 \times 100 = 4{,}000$ lbs., leaving 4,945 to be borne by the strap (*Fig.* 157). Draw DE on any scale horizontal = 4,945, EF vertical, and DF parallel to the direction of the strap. Then DF = 11,000 lbs. = the tension on the strap, half of which, or 5,500 lbs., has to be taken by each side of the strap.

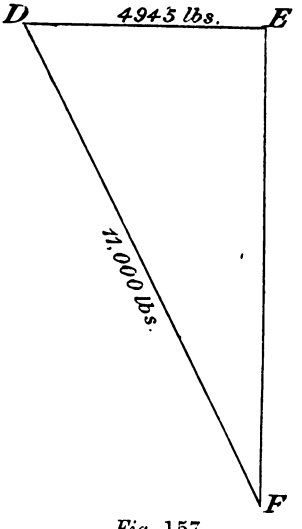


Fig. 157.

(*a*). The section of the iron must be capable of sustaining a safe tension of 5,500 lbs. As wrought iron can stand a safe tensile stress of $22 \div 5 = 4{\cdot}5$ tons per square inch, we have as the section of each strap $5{,}500 \div 4{\cdot}5 \times 2{,}240 = {\cdot}54$″. Make $2'' \times \frac{5}{16}''$.

(*b*). The section of the screws at the bottom of the threads must also be capable of sustaining a stress of 5,500 lbs. each. If d be the diameter of each screw, then $\frac{\pi d^2}{4} \times 4{\cdot}5 = 5{,}500 \div 2{,}240$. Hence $d = \frac{7}{8}''$.

(*c*). The depth of the nut, to prevent stripping, as we shall see presently, should be equal to the diameter of the bolt, *i.e.*, $\frac{7}{8}''$.

(*d*). The breadth of the strap is taken above at 2″, and the width of the tie beam is 4″. Hence there is an effective area to resist

crushing of $2 \times 4 = 8$ square inches. The total crushing force coming upon the timber, communicated by the strap, is 11,000 lbs. This gives 1,375 lbs. per square inch. As red fir can bear a crushing load of 5,300 lbs. per square inch, this gives a factor of safety of nearly 4, which, under the circumstances, is probably sufficient, but it must be remembered that timber is weaker to resist crushing across the grain than in the direction of the grain. Hence a bearing plate is desirable.

The above investigation results in a section of iron which is larger than that usually adopted in practice. It has also been based upon the consideration of a definite length of tie beam being available for part of the stress. This may not be always possible.

Again, a very high factor of safety (5) has been taken for the iron, and it is reasonable to consider that, as the metal will be subject to no fatigue, a high factor of safety is unnecessary. Instead, therefore, of the above investigation, we might consider that the end of the tie beam took *no part** of the resistance, and we might assume a higher value of resistance, say 7 tons per square inch to the iron, instead of 4·5 tons as above. On these assumptions we may draw ED = total stress on the principal (*Fig.* 158) and parallel to it, DF parallel to the direction of the strap, and EF vertical. Then DF will represent the stress to be borne by the whole strap, and may be either measured from the diagram to scale, or when the strap is at right angles to the principal, may be calculated, since DF = ED cot EFD. In this case DF = 20,000 lbs. Taking a factor of safety of 3, the area of each strap will be $10{,}000 \div 2{,}240 \times \frac{2}{3} = 0{\cdot}6'$, so that $2'' \times \frac{3}{8}''$ will be sufficient. The other dimensions will be as before.

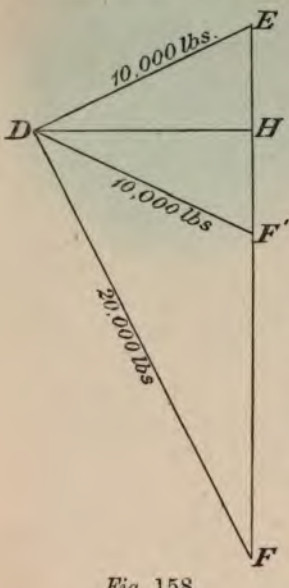


Fig. 158.

* Experiments at Chatham on full-sized trusses have shown that either the strap takes the whole stress or none at all. In the latter case failure occurs by the shearing of the end portion of the tie beam. In the former case failure almost invariably occurs by the strap crushing into the lower fibres of the tie beam.

(ii.). *Example* 28.—When we come, however, to consider the best *inclination* to give the strap, we see from *Fig.* 158 that if it be inclined as DF', the stress is considerably diminished as compared with DF, and that if we could only take it horizontal as DH, we should have a minimum stress. The practical difficulty presents itself, however, how to attach the lower end, for if made very oblique to the tie beam it would not make a satisfactory joint. Hence we come to the form of fastening shown in *Fig.* 159, where the strap is secured by a bolt passing through the tie beam.

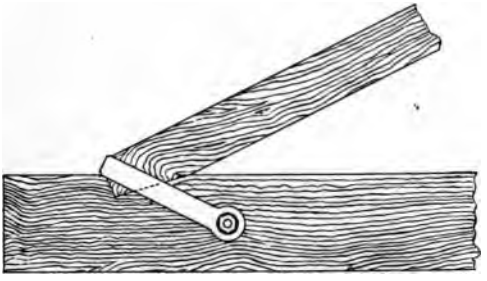


Fig. 159.

In this case the tension on the strap tends to shear the bolt passing through the tie beam, and to cause that bolt to crush the fibres of the wood adjacent to it. In *Fig.* 159 let the inclination of the strap to the horizontal be the same as that of the principal, so that there are equal stresses on the principal and strap, then each limb of the strap will have to bear 5,000 lbs. Make $1\frac{1}{4}'' \times \frac{1}{4}''$ (since this = ·312", and multiplied by $\frac{2}{3}$ tons gives 5,140 lbs., which is slightly more than the 5,000 lbs. required). The shearing stress will be found easily, for, if d = the diameter of the bolt to resist shearing, then $\pi \times \frac{d^2}{4} \times 7 = 5{,}000 \div 2{,}240$, whence d = ·65", say $\frac{3}{4}''$.

But for bearing, or resistance to crushing, if d' be the diameter, then $d' \times 4 \times 1{,}800$ (factor of safety of 3) = 10,000. Hence $d' = 1{\cdot}39''$, say $1\frac{1}{2}''$.

This is an important result, for it shows that the weakness of this form of joint depends on the quality of the timber. If the tie beam be made of a very hard timber, it is possible that the determining factor for the diameter of the thickness of the bolt would be not the thickness required to resist crushing of the adjacent timber, but the shearing resistance of the metal. However, in the majority of cases, it is necessary in this form of joint to consider the crushing resistance of the wood, and to give a large diameter of bolt accordingly. *Fig.* 160 shows the form of this junction in the type drawing of Indian arracks, calculated for sàl (*shorea robusta*), a wood which is hard

than any of the timber usually employed for barracks in Europe. There, it will be noticed, the diameter of the bolt is taken at $2\frac{1}{4}''$ in spite of the hardness of the wood.

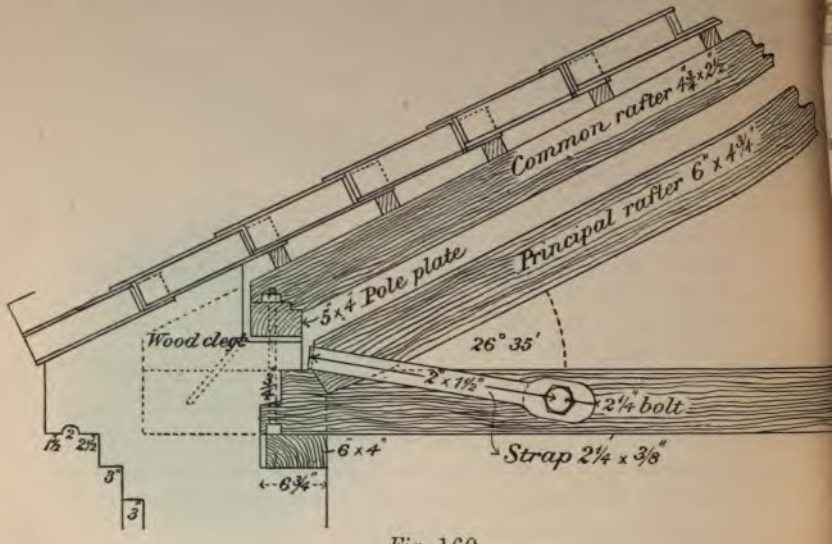


Fig. 160.

A still further development of the same fastening is shown in *Figs.* 161, 162, where the tie beam is in two parts, the principal coming between them, and the joint secured by a horizontal strap passing round the end, with a block of hard wood at the end, and the space between the parts of the tie beam kept in their position by a chock of wood. In this case it will be seen that the stress on the strap is a minimum. Taking the figures used hitherto for this joint, we find that if the thrust on the strut or principal is 10,000 lbs., the stress on the tie beam, or on the strap, is 8,945 lbs., that is, 4,472 lbs. on each limb. This would be safely borne by a section of $1\frac{1}{8}'' \times \frac{1}{4}''$.

The tie beam being made of two planks, each, let us assume, $1\frac{1}{2}''$ thick and 8" broad, we have a total breadth at the ends of $4'' + 2 \times 1\frac{1}{2}'' = 7''$. Hence the bearing area of the strap at the end is $1\frac{1}{8}'' \times 7'' = 7{\cdot}8$ square inches. The intensity of pressure at that part is, therefore, $10{,}000 \div 7{\cdot}8 = 1{,}280$ lbs. per square inch, which is well within safe limits for all kinds of timber, but if additional security is required the end block may be made of hard wood.

The length of the bolt passing through the tie beam and separating chock is also 7″. To find the diameter of the bolt passing through, we have to equate the bearing area of the bolt with the whole stress coming on the tie beam. As the whole strength of the joint depends largely on this bolt, it will be safe to take the factor of safety as 5, and not 3, as heretofore. Hence

$$7 \times d \times 5{,}300 \div 5 = 8{,}945.$$
$$d = 1{\cdot}2''.$$

Make $1\frac{1}{4}''$ diameter.

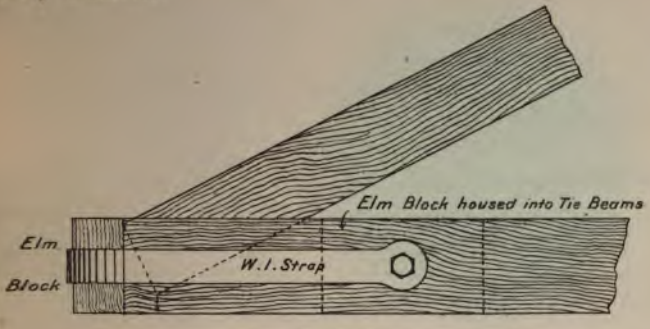


Fig. 161.

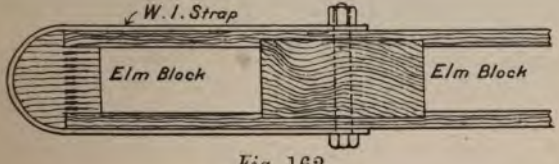


Fig. 162.

In this case the value 5,300 is the minimum crushing resistance of red fir. If harder wood be used the diameter of the bolt can be reduced.

For shearing we may take 5 tons per square inch as the safe resistance of the iron, and as the bolt would shear in two places, we have

$$\frac{1}{4} \times 3{\cdot}1416 \times d^2 \times 5 \times 2{,}240 = 4{,}472 \text{ lbs.}$$

Hence $$d = {\cdot}71, \text{ say } \tfrac{3}{4}''.$$

The bolt will, therefore, require to be $1\frac{1}{4}''$, as this is the larger size. To prevent the chock sliding between the parts of the tie beam, it will be necessary to cut a slot $\frac{1}{2}''$ deep, as shown in *Fig.* 162.

This will weaken the tie beam, no doubt, but will leave a net area of $(8 - 1\frac{1}{4})\ 1 = 6{\cdot}75$ square inches to resist a tensile stress of 4,472 lbs., or 700 lbs. per square inch, which is much less than the working resistance of even the weakest timber.

Finally, to see whether the slots are sufficient to bear the thrust brought upon them, we may equate the area of each slot with the stress, *i.e.*, $8 \times \frac{1}{2} \times p = 4{,}472$, or $p = 1{,}118$ lbs. per square inch, which is perfectly safe.*

Some time has been spent in the consideration of this joint, because it is a typical one for joints in timber trusses, both for roofs and bridges, and also because it is one which is often constructed in the least economical manner.

Any other cases of timber joints would be dealt with on some of the foregoing principles.

Iron Joints.

The iron joints which are commonly employed are :—(1). Screw joints. (2). Pin and Link. (3). Gib and Cotter. (4). Rivets.

(1). *Screw Joints.*—In structures, screws are frequently found in the form of bolts and nuts. In this form they are available either to resist stress in the direction of the bolt or at right angles to its axis.

The pitch of a screw is the distance from centre to centre of the threads, hence the pitch in inches = 1 ÷ number of threads to the inch.

From statics we learn that if P be the power moving in a circle concentric with the circumference of the bolt, and W the force parallel to the axis of the bolt, then W : P : : circle described by P : distance through which W would be raised. Hence W × pitch = P × circle P describes.

This, however, is only true for square threaded screws, and it ignores friction. Where a V-shaped thread is used (as is usually the case in structures) part of the power is dissipated in an outward direction, as indicated by the horizontal arrow in *Fig.* 163. On the other hand, the whole depth of the screw *bc* is available for resistance to tension or stripping, whereas in a square thread only half of the depth *ad* (*Fig.* 164) is available. Square threaded screws are, therefore, much used in *machines* where the object is to utilize to the fullest extent possible the power available, and where there is not

* This form of joint has been the subject of a series of experiments at the S.M.E., Chatham, on full-sized trusses.

much tensile stress in the direction of the axis of the bar. These conditions are generally reversed in the case of *structures*, for there the object is usually to resist a force in the direction of the axis of the bar with the least amount of material. In this investigation, therefore, attention will be paid only to V-shaped screws. As the thread is usually cut into the bolt, the effective section of the bolt or rod is that of the sectional area at the bottom of the threads.

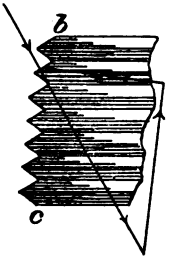


Fig. 163.

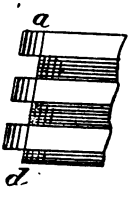


Fig. 164.

The thread on the bolt is called the "male" thread, and that on the nut into which the bolt fits is called the "female" thread. If D be the diameter of the bolt taken at the bottom of the threads, and d the diameter of the bolt before the thread has been cut in it, the ratio between the values of D and d depends on the pitch and the kind of thread employed. The screw threads employed in this country are those known as Whitworth's, and in their case

$$D = 0{\cdot}9d - 0{\cdot}06.$$
$$d = 1{\cdot}1D + 0{\cdot}07.$$

If l be the length of the female screw, or the depth of the nut, the shearing area against which the screw will act is $\pi \times D \times l$, and if r_s and r_t be the resistances per square inch to shearing and tension respectively, we can find a value for l in terms of r_s, r_t, and d, all of which are known quantities.

For r_s it is safe to take a low value, as the metal has been weakened by cutting.* We may take 2 tons per square inch. For r_t take 5 tons. Then the total tension on the bolt should be equal to the

* From General Wray's *Instruction in Construction.*

total resistance to shearing, in order that the various parts of the joint should be of equal strength. Hence

$$\tfrac{1}{4} \times \pi \times D^2 \times r_t = 3{\cdot}1416 \times D \times l \times r_s.$$

Hence $$l = r_t\,(0{\cdot}9d - 0{\cdot}06) \div 4r_s.$$

Substituting the values of 5 and 2 for r_t and r_s, we find that l must be rather more than $\frac{1}{2}d$. The usual practice is to make $l = d$, which is a useful and easily-remembered rule.

The width of nuts and bolt-heads may be generally taken as $1\frac{1}{2}$ times the diameter of the bolt. With wood a washer of iron is placed between the bolt and the wood, so as to distribute the pressure. The size of this washer is generally as follows:—For hard woods the diameter or width is three times, and for soft wood four times, the value of d, while the thickness is $\frac{1}{2}d$ (*Fig.* 164).

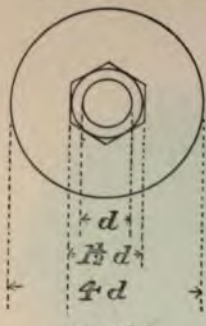


Fig. 165.

No example is given of a screw connection, as the rule, above given, of making the depth of the female screw = the diameter of the connecting bars, is of universal application, giving an ample margin of safety.

(2). *Pin and Link.*

This form of fastening is employed with "eye-bars," where tension bars are flattened out at the ends, and formed with an opening through for a transverse connecting pin. This forms a very efficient form of joint in many cases, and it is largely employed in the connection of roof members, in suspension bridges, and sometimes in the connection of the members in framed girders for bridges. In America it is largely used in bridge work, where English practice would use rivetted joints. Formerly the eye was made by bending the end of the bar round, and welding the end to the main part. Weldless links are, however, usually employed for important work; the end of the bar being flattened and bored. Experiment shows that the requisite proportions of the various parts will depend largely on the mode of manufacture.

Fig. 166.

It is evident that the stresses that take place round the eye must be very complex. Most of the longitudinal fibres in the shank B (*Fig.* 166) are interrupted by the hole D. Immediately

ehind the hole there can be no direct tension at all, and the tension t B is transferred laterally to the sides by some oblique component f the stress. It would be unwarrantable to assume that the tress is uniformly distributed over the sides of the hole, and lthough the gradual formation of the shoulders of the link will end to the uniform transfer of the stress, it is reasonable to consider hat there will be a greater stress intensity at the inner sides of the ye than at their outer edges. At the back of the eye the stresses re such that the pin exerting a direct pressure on the metal next to t will tend to crush the iron and elongate the hole.

These assumptions are borne out by experiment, and as the subject has been very carefully considered both in England and America, it is sufficient if we take the best results of the experiments.

The maximum dimensions in English practice are those of Mr. Berkley :—

Width of shank, B (*Fig.* 166)	... = 1·00
Diameter of pin, D	... = 0·75
Width of metal across the eye, $b+b$ ...	... = 1·25
„ „ „ „ behind the eye, E	... = 1·00
Radius of shoulder, r	... = 1·00
„ „ neck, R	... = 1·50

According to American practice, Mr. Shaler Smith has given the ollowing rules for the ratios of the parts of tie bars :—

The proportions will depend partly upon the mode of manufacture, nd these proportions will again be modified whenever it may be ecessary to use a pin whose diameter is greater than about 0·75 the idth of the bar. When the size of the pin is a matter of uncon-trained choice, its diameter should be between 0·66 and 0·75 times he width or diameter of the bar, if the greatest efficiency is to be ought.* The best proportions for "hammered" and for "weldless" yes are shown in *Figs.* 168 and 170. *Figs.* 167 and 169 show the ltered proportions where the bar is relatively narrow. In hammered yes the width E behind the pin = B, but the ratio $\frac{b}{B}$ depends on the iameter of the pin. As compared with Mr. Berkley's rules, the oportions (for ratios of D = 0·75 B) give more metal at the side of e eye, and if we compare *Fig.* 169 with *Fig.* 168, we see that it tainly appears as if the metal were better distributed in the

* Mr. Claxton Fidler, from whose book much information is derived.

former than in the latter. Each side of the head is formed by a circular arc described about a centre situated in a vertical line passing through the centre of the pin ; if the radius of this arc = r, the radius of the neck $R = 1\frac{1}{2}r$.

The heads of the weldless eye bars (*Figs.* 167, 168) are made circular, concentric with the pin, so that $E = b$. The radius of the neck $R = 1\frac{1}{2}r$, as before, but in this case r is described from the centre of the pin.

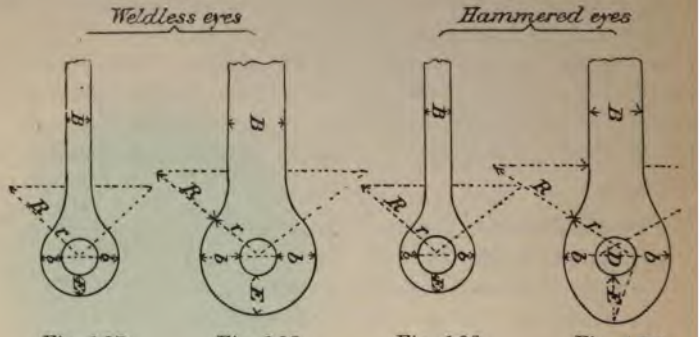


Fig. 167. *Fig.* 168. *Fig.* 169. *Fig.* 170.

TABLE XXVII.

Proportions of Eye Bars as Determined by Mr. Shaler Smith.

Ratio of diameter of Pin to width of Bar or diameter of Bar if round.	Hammered Eyes.	Weldless Eyes.
	Ratio of area of Metal at side of Eye to area of Bar section. (Metal section at back of Eye = that of Bar).	Ratio of area of Metal at side Eye to area of Bar section. (Metal section at back of Eye = that at side of Eye).
0·67	0·66	0·75
0·75	0·67	0·75
1·00	0·75	0·75
1·25	0·76	0·80
1·33	0·79	0·85
1·50	0·83	0·92
1·75	0·84	1·00
2·00	0·88	1·12

So far we have considered the design of the link only. There are yet two important points to be considered, viz., the *thickness* of the eye and the method of calculating the *diameter of the pin.*

As regards the first of these, it is sufficient if we make the bearing area of the eye at the back of the pin = the stress coming on the link. That bearing area will be the diameter of the pin × the thickness of the link, and this area × the safe resistance of the metal to crushing must be = the stress on the link. To ascertain this area, therefore, we must first find out the diameter of the pin.

This diameter will depend on the arrangement of the links. The pin may be regarded as a short beam under transverse stress, and under shearing stress. Now if there are several links connected by the one pin, and if these links radiate in various directions, the arrangement of the links on the pin will make a very great difference in the shearing stresses, and in the M_{ff} which the pin has to resist. This fact is evident if we take an example. Let us suppose that the pin in *Fig.* 171 has to bear stresses indicated by the figures, and that the directions of those stresses are indicated by the arrows, then it is evident that the arrangement in *Fig.* 171 is less economical than that in *Fig.* 172, inasmuch as the maxima shearing stresses and the

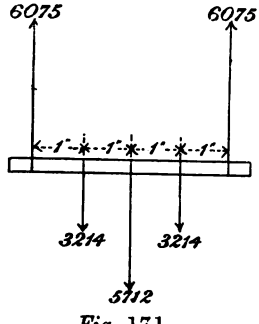


Fig. 171.

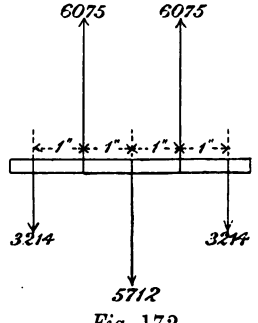


Fig. 172.

maxima moments in the one are much greater than in the other. Hence it is advisable to arrange the links so that the stresses shall come alternately in different directions on the pin. The shearing stresses are usually considered as uniformly distributed over the cross section of the pin, and in ordinary cases it is sufficient to reckon upon this. But strictly speaking, this is not the case, for reasons stated in Chapter VI., and in very important joints this matter should

not be lost sight of. To find the maximum shearing stress on any section of a pin connecting several radiating bars, it is necessary to find the sum of the vertical and horizontal components of the forces acting upon it. The maximum shearing stress will be the square root of the sum of the squares of these components.

As regards bending moments, the pin may be regarded as a beam with loads brought on at the centres of the members, as long as there is no deflection of the pin. If it bends at all, this method of computation would be erroneous, because the forces and the leverage would both be altered.

From the foregoing remarks it will be seen that the true design of a pin and link joint, where many radiating bars are united, is a somewhat complex problem. In the ordinary cases of its application to roofs, it is not, perhaps, necessary to go into all the minute particulars, but in cases of bridge construction there is no doubt that every detail of this useful form of joint should be most thoroughly investigated. Practically, the steps to be taken may be summarized as follows :—

1. Diameter of pin is determined by its shearing resistance to greatest force coming on it. If P be the maximum force in tons tending to shear the bar, then $P = \frac{\pi d^2}{4} \times 4$, 4 being the safe resistance of wrought iron to shearing in tons per square inch.

Hence $$d = \sqrt{\frac{P}{\pi}}.$$

2. To find thickness of the eye, t, make $t = \frac{P}{d \times r_c}$, where r_c = safe resistance to crushing = 5 tons per square inch for wrought iron.

3. Find width at side of eye by proportions of areas of bar and metal at side of eye as in Table XXVII. If the bar is flat, the table gives the width of section ; if the bar is round, it gives area of section.

4. Width at back of eye is found by making section at back of eye = section of bar. If this gives a very elongated eye, the thickness t may be slightly increased.

American practice allows 8,000 lbs. per square inch safe intensity of shearing stress for wrought iron, and 10,000 lbs. for steel.

For bearing, the same authorities allow 12,000 lbs. for railway bridges, and 15,000 lbs. for highway construction, if either the pins, or the plates, or both, are of wrought iron. If the whole is of steel,

then 15,000 and 18,000 lbs. per square inch are the corresponding values.

As an example of this form applied to an ordinary roof we may take :—

EXAMPLE 29.—The strut and tie-bars in an iron roof are shown in *Fig.* 173 where the directions of the bars, and the stresses are indicated.

The strut B is to be formed of two parallel bars connected at intervals by distance pieces, and the remaining members, being tension rods, are to be formed of round bars.

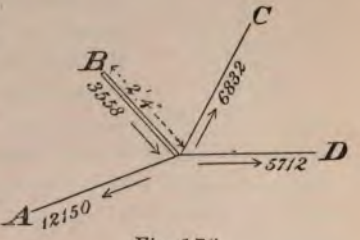


Fig. 173.

It is required to design the members and the joint, which is to be a pin and link connection.

The problem is both one in the design of tension and compression bars, and in joints, but both are taken together, because, as was pointed out in the preceding chapter, the design of such members often is entirely governed by the design of the fastening.

(i.). *The design of the strut.*

The length of this member is given at 2′ 4″. It is necessary to know this in order to calculate the dimensions of the bars. Then from Table XXVI., Chapter IX., we have for this form of cross section, $\frac{l}{k}=3\,\frac{l}{d}$. But if we take one dimension of the bars as 1″, $l:d::28:1$. Hence $\frac{l}{k}=3\times 28=84$. From the Table XXVI. this corresponds to a safe stress per square inch for ends rounded of 2·5 tons. It has actually to bear 3558 lbs. $\div 2=0{\cdot}8$ tons. Hence the section of each bar might be $0{\cdot}8\div 2{\cdot}5=0{\cdot}32$ square inch.

Make each bar $2''\times\frac{1}{4}''$. It might be of less breadth, but 2″ will be required to give room for the bolt, which is 1″ diameter.

To find the number of distance pieces required in the total length of 28″, we have to consider each bar as a rectangular strut of a given section, under a given load, and to estimate from the tables the greatest length that such a bar ought to be. The given section is $2''\times\frac{1}{4}''=0{\cdot}5$ square inch, and the given stress is 0·8 ton. This gives a stress intensity of $8\div 5=1{\cdot}6$ tons per square inch. From Table XXVI. this corresponds to a ratio of about 120 for ends rounded.

In this case the ends are not rounded, but as the fixing may be imperfect, it will be on the safe side if we consider the ends as rounded. Since $n = 3{\cdot}5$ for a rectangular section, the maximum safe length for a rectangular strut with one side $\frac{1}{4}''$ wide under the above stress, is $\frac{1}{4}'' \times 120 \div 3{\cdot}4 = 8{\cdot}5$ inches. As the length of the strut is 28″, we may divide it into 4 parts, each of 7″, using three distance pieces, as shown in *Fig.* 174. It would have been almost sufficient if it were divided into 3 parts each 9 inches long, but the other is more accurate.

Fig. 174.

(ii.). *Design of the tension bars.*

These bars being circular their section is easily found.

We may take 5 tons per square inch as the safe intensity of resistance of wrought iron to tension, as none of these bars will be subject to fatigue.

Bar A has stress of 12,150 lbs. = 5·44 tons.

Then $\frac{1}{4}\ 3{\cdot}1416\ d^2 \times 5 = 5{\cdot}44,$

$d = 1{\cdot}16$

Make $1\frac{1}{4}''$.

Bar C has a stress of 6832 lbs. = 3·05 tons.

Then $\frac{1}{4}\ 3{\cdot}1416 \times d^2 \times 5 = 3{\cdot}05,$

$d = {\cdot}88.$

Make 1″.

Bar D has a stress of 5712 lbs. = 2·55 tons

Then $\frac{1}{4}\ 3{\cdot}1416\ d^2 \times 5 = 2{\cdot}55,$

$d = {\cdot}8.$

Make 1″.

(iii.). *Design of the joint.*

We have to consider: (*a*), the arrangement of the bars upon the pin; (*b*), the diameter of the pin; and (*c*), the dimensions of the eyes.

As regards (*a*), we see in *Fig.* 173 that the tendency of the bar A is to draw the pin to the left, and of the members B, C, and D to thrust or pull it to the right. In arranging the members on the pin, we must endeavour to bring these forces alternately. By putting D in the centre, halving A and placing it next, then halving C, and

putting it next, with the bars of B on the outside of all, we get an arrangement which is the most satisfactory (*Fig.* 175).

With this arrangement we have to calculate the shearing stress on the pin, which will determine (*b*) its diameter. In most cases it will be sufficient to take the greatest force coming on the pin, and reckon it only; but it is more accurate to find the square root of the algebraic sum of the vertical and horizontal shearing stresses at the section which appears to be that of maximum stress. As an example, we may follow the latter method.

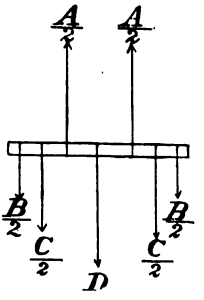


Fig. 175.

From *Fig.* 176 we see that the components of the various stresses are :—

		lbs.		lbs.	Tons.
Vertical components		A = 3500.	Therefore	$\frac{1}{2}A_v = 1750$	= 0·78.
		B = 2800.	„	$\frac{1}{2}B_v = 1400$	= 0·625.
		C = 6000.	„	$\frac{1}{2}C_v = 3000$	= 1·34.
Horizontal	„	A = 10750.	„	$\frac{1}{2}A_h = 5375$	= 2·4.
		B = 2400.	„	$\frac{1}{2}B_h = 1200$	= 0·29.
		C = 4400.	„	$\frac{1}{2}C_h = 2200$	= 0·98.
		D = 5712 = 2·55 tons.			

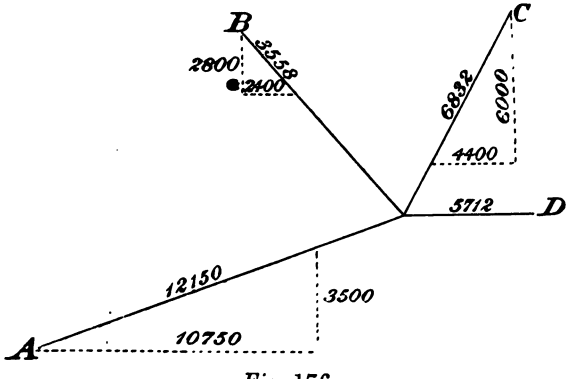


Fig. 176.

From an inspection of *Fig.* 175, it appears that the maximum shearing stress would come at the points where the halves of A are linked to the pin.

Here the square of the stress = the algebraic sum of the squares of the vertical and horizontal components of the stresses coming on the pin between that section and the end of the bar.

Hence

$$\text{stress}^2 = (\tfrac{1}{2}A_h)^2 + (\tfrac{1}{2}A_v)^2 + (B_v)^2 - (\tfrac{1}{2}B_h)^2 - (\tfrac{1}{2}C_h)^2 - (\tfrac{1}{2}C_v{}^2).$$

Substituting values,

$$(\text{Stress})^2 = 5{\cdot}76 + 0{\cdot}60 + 0{\cdot}40 - 0{\cdot}084 - 0{\cdot}96 - 1{\cdot}79.$$
$$= 3{\cdot}93, \text{ say } 4,$$

hence the shearing stress = 2 tons.

Taking 4 tons as the safe shearing intensity, we have

$$\tfrac{1}{4}\, 3{\cdot}1416 \times d^2 \times 4 = 2,$$

whence $d = 0{\cdot}8$ say $1''$.

After calculating the thickness of the various eyes coming on the pin, we must consider whether this thickness is sufficient to withstand the bending moment.

(*c*). To calculate the dimensions of the eyes and the thicknesses of the bars at the eyes, we may refer to Table XXVII.

Take D first, as it is in the centre. Its diameter is $1''$, hence ratio of diameter of pin to width of bar is = 1. From the table we see that the area of the section at the sides of the eye should be 0·75 area of the bar, *i.e.*, $0{\cdot}75 \times \frac{\pi}{4} \times 1 = {\cdot}59$ square inches. As we may consider the eyes to be hammered, the metal at the back of the eye should be the same as the original bar; hence, area of metal, *i.e.*, width × thickness, will = $\frac{1}{4} \times 3{\cdot}141 \times 1 = 0{\cdot}78$ square inches. To find the thickness, *t*, we have to take the bearing area of the bar on the pin, of which the diameter has been ascertained to be $1''$. Hence, if *t* = thickness required, $t \times 1 \times 10{,}000$ (safe resistance to crushing) = 5712 (stress on bar; hence $t = 0{\cdot}57$ say $\frac{5}{8}''$. Then, if *a* be the width at the back of the eye, $a \times \frac{5}{8} = 0{\cdot}78$, and $a = 1{\cdot}28$. As this would result in a thin elongated bar, make thickness $\frac{3}{4}''$, and width at sides = ·59 ÷ ·75 = $\frac{13}{16}''$. The eye will, therefore be as shown in *Fig.* 177.

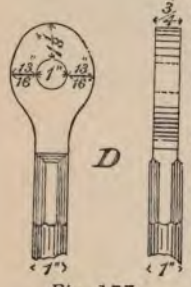


Fig. 177.

Next taking the bar A. This is divided into two, each having a stress of 6,075 lbs. The thickness of the bar at the back is found by equating the diameter of the pin × the required

thickness × safe resistance to crushing, with the stress coming on each fork of the divided bar, or half the total stress on the bar.

Thus $t \times 1 \times 10{,}000 = 6075$, whence $t = 0{\cdot}6$, say $\frac{5}{8}''$.

The diameter of the bar being 1·25, the ratio of diameter of pin to width of bar is 0·8. The corresponding ratio for the area of metal at the sides of the eye is about 0·7. Hence area at eyes

$$= 0{\cdot}7 \times \frac{\pi}{4} \times 1{\cdot}25^2 = 0{\cdot}86 \text{ square inches.}$$

Hence breadth at sides of eye

$$= {\cdot}86 \div {\cdot}62 \times 2 = {\cdot}7, \text{ say } \tfrac{3}{4}''.$$

The width at the back of the eye, so that the metal shall be the same as the original bar (half being in each fork), is found as before—

$$a \times \tfrac{5}{8} = \tfrac{1}{2} \times 0{\cdot}78 \times 1{\cdot}25, \text{ whence } a = 0{\cdot}8. \quad \text{Make } a = 1''.$$

Working out the bar C in the same manner, we find that the breadth at the sides of the eyes is $\frac{7}{8}''$, that the thickness of the eye is $\frac{3}{8}''$, and that the width at the back of the eye is $1\frac{1}{8}''$.

The bars are, therefore, as shown in *Figs.* 178 and 179.

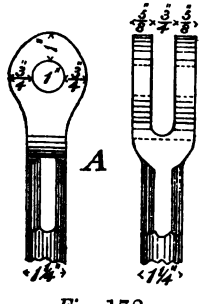


Fig. 178.

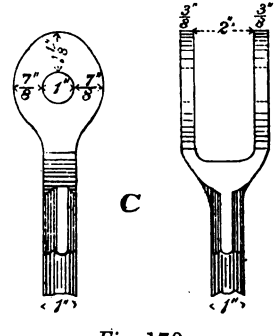


Fig. 179.

Finally, to see if the pin can bear the transverse stress coming upon it, we have to take moments at the position of greatest stress and calculate whether the stress intensity produced is greater or less than the safe modulus of rupture for wrought iron.

In *Figs.* 180 and 181 the position of greatest stress will be at the

centre of the pin. Considering all the forces as acting at the centres of the bearings of the links, we have as the M_{II}

$$-6075 \times \tfrac{5}{8}'' + 3416 \times \tfrac{9}{8}'' - 1779 \times \tfrac{11 \cdot 5}{8}'' = 353 \text{ inch-lbs.}$$

Then $M_r = rI \div y$, where r is unknown, $I = \frac{\pi R^4}{4} = \frac{1}{4} \times 3 \cdot 1416 \times 0 \cdot 5^4$, and $y = 0 \cdot 5$.

Equating the above, we find that $r = 3{,}638$ lbs. per square inch, which is well within the limits of safety for wrought iron.

The joint as constructed will be as shown in *Figs.* 180 and 181.

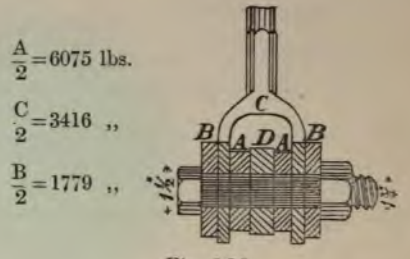


Fig. 180.

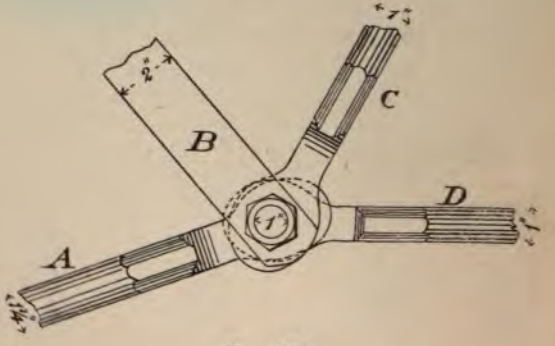


Fig. 181.

This investigation has been a lengthy one, and it would not be necessary to go through all these steps in the usual cases met with. Yet the joint is such a useful one, and its application so economical, that it is desirable to be acquainted with the true principles of its design. It will be evident from the foregoing investigation that accurate fitting is essential to the proper working of this joint. The

trade of the fitter is not one usually associated with structures, but here is an instance where this trade has its proper place in construction.

The pins are kept in their places by nuts. These may be either solid, or they may, in important joints, be of the shape shown in *Fig.* 182, and known in America as Lomas nuts. As there is no tension in the direction of the pin, there is no objection to cut the screw thread in the thickness of the bolt, and diminish the section. In some cases in America the end of the screw is protected during construction by an arrangement shown in *Fig.* 183.

Fig. 182.

Fig. 183.

(3). *Gib and Cotter.*

This form of joint is frequently used instead of bolts or rivets in roof construction. Its chief advantage is that by means of the cotter, which is wedge-shaped, the bar can be tightened or slackened slightly. *Figs.* 184 and 185 show the method of connecting a bar such as the tie bar of a roof, to two plates, such as might be bolted or

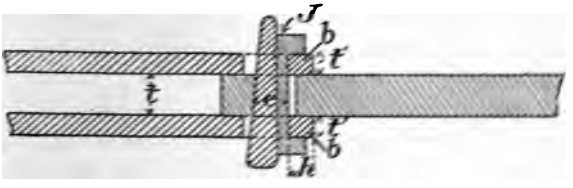


Fig. 184.

Fig. 185.

rivetted to the heel of a principal rafter, by a gib and cotter joint. The end of the round tie rod is flattened, and a slot cut in it, while corresponding slots are cut in the plates. A key or gib J is inserted

in the slots—this is not an essential part of the joint, for the principle might be carried out without the gib at all, but it is generally introduced to make up for the exact correspondence in the position of the slots which may not be quite coincident. The cotter bears against the surfaces of the slot in the tie bar, and against the gib alternately, while the gib bears against the sides of the side plates at *i*, *i*.

The joint may fail:—

1. By the shearing of the gib and of the cotter (double shear).
2. By the cotter cutting into the tie rod.
3. By the gib cutting into the plates at *i*, *i*.

If t = the thickness of the tie-rod, w = the width of the cotter, c = breadth of gib and cotter, and t' = the thickness of each plate, and T = tension in the tie bar, then for equality of strength in all parts of the joint, as the cotter is in double shear—

$$T = 2w \div c \times r_s = w \times t \times r_b = w \times t' \times r_b \times 2,$$

where r_s and r_b are the resistances per square inch of the material to shearing and crushing respectively.

By taking any convenient value for w, the other amounts can be very easily obtained.

Taking, for example, the tie-rod under investigation in the last example, we have as follows:—

EXAMPLE 30.—*A tie bar* $1\frac{1}{4}$″ *diameter has to bear a maximum tensile stress of* 12,150 *lbs., and is connected to the heel of the* T*-iron principal by plates, at one end riveted to the tongue of the* T*-iron, and with slots for a gib and cotter joint at the other end. Find the dimensions of the various parts of the joint.*

We may start by assuming that the flattened end of the bar is 1″ wide, *i.e.*, $t = 1$″. Then the width t' of the plates is clearly $= \frac{1}{2}$″. Assuming that the width of the cotter w is $\frac{1}{2}$″, we have to see from the equations above whether the thickness $t = 1$″ is sufficient.

$$T = 12{,}150 \text{ lbs.} = 5{\cdot}44 \text{ tons} = w \times t \times r_b = \tfrac{1}{2}'' \times 1'' \times r_b,$$

whence $r_b = 10{\cdot}88$ tons per square inch. This is rather excessive, so we must increase the value of both w and t. Try $w = \frac{3}{4}$″, $t = 1\frac{1}{4}$″, then, repeating the same process as before, we have $r_b = 5{\cdot}8$ tons, which is near enough to safe requirements.

Hence $$t' = \tfrac{1}{2} \times 1\tfrac{1}{4} = \tfrac{5}{8}'',$$

and $$c = T \div 2w \times r_s = 5{\cdot}44 \div 2 \times \tfrac{3}{4}'' \times 4 = 1'' \text{ nearly.}$$

To find the depth a (*Fig.* 185) of the plates, we have the net area of the plate × the safe resistance per square inch of wrought iron to tension (5 tons) must = the total tensile stress, *i.e.*—

$$(a - w)\, t \times 5 = \mathrm{T},$$

i.e.—

$$(a - \tfrac{3}{4})1\tfrac{1}{4}'' \times 5 = 5{\cdot}44,$$

whence $a = 1{\cdot}64$. Make $1\frac{3}{4}''$ to allow for any error in manufacture.

The plates at each side should be the same depth at least. The slots must be sufficiently far from the ends to prevent the portion at the ends from shearing off. Calling h (*Fig.* 184) the length of such a piece, then

$$2 \times h \times t' \times 4 \text{ tons} = \frac{5{\cdot}44}{2},$$

and substituting values—

$$h = 0{\cdot}54'', \text{ say } \tfrac{3}{4}''.$$

The ends of the bar will also be $\frac{3}{4}''$ beyond the slot. A taper of 1 in 24 to 1 in 48 is usually given to cotters.

(4). *Rivets.*

Rivets are a modification of the bolt and nut* connection, the difference being that whereas in the one case the fastening is secured by the mechanical arrangement of the screw, in the other the same end is attained by taking advantage of the plasticity of the metal at a high temperature, when it is, therefore, capable of being hammered down so as to grip the plates which are to be connected. Thus in the rivet it may be said that the head, shank, and nut are in one piece. Inasmuch, however, as the temperature at the time of fastening has much to do with the grip in the direction of the axis of the rivet, and as the amount of grip thereby produced is a very indeterminate quantity, rivets are not a form of fastening to be recommended in the case of stresses coming in the direction of, or parallel to, their axes. In such cases the rivet heads are very liable to fly off. So also, when the length of the rivet becomes more than a certain amount, say 4 times its diameter, it is preferable, because more reliable, to use bolts and nuts. This is a very important point, and should never be overlooked.

The diameters of rivets used in structural work vary from $\frac{1}{2}''$ to $1''$,

* The records of the St. Louis tornado prove conclusively that where there is bending or twisting, a rivetted joint stands much better than a bolt and nut connection.

the most usual sizes being $\frac{3}{4}$" and $\frac{7}{8}$". Smaller sizes than $\frac{3}{4}$" are only used in unimportant details, and larger ones are used only where the metal is of great thickness.

The proportions of rivets are shown in *Fig.* 186.

Rivets may be either fastened by hand power, or by some form of machine. The latter is by far the better way of securing a good joint, but it has the disadvantage that if carelessly done the head may be eccentric to the axis of the shank; it is also difficult to apply in certain positions when the work has to be built up *in situ*. Portable riveters, worked by steam or by hydraulic power, are, however, frequently used with work thus built up.

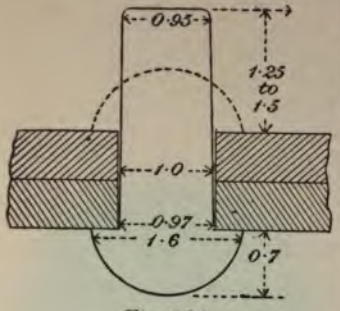


Fig. 186.

The holes for the rivets are either punched or drilled. Punching has an injurious effect on the metal round the hole. A molecular change in the metal is produced in a narrow ring round the hole, rendering it harder and more brittle. The harder the material the greater the injury from this cause, and steel is more affected than iron. It is also more difficult to space the rivets accurately when the holes are punched, and when two or more plates are used the holes do not correspond, so that they require to be rhymered before the rivets can be inserted. This rhymering cuts away the zone of the metal affected by the punching, and so tends to remedy the injury above alluded to. In the process of punching the plates stretch a little, and so cause inaccurate fitting. Hence where several plates are used, drilling should be resorted to. The sharp edge round the drilled hole, however, should be rounded off, for if it is left it may assist the shearing of the rivet.

In the process of rivetting the plates are temporarily bolted together until the rivets are inserted. This bolting should not be done loosely, especially with machine rivetting, as it may tend to cause "collars," *i.e.*, portions of the metal may be squeezed between the plates, keeping them asunder (*Fig.* 187). In cooling, the rivets exert considerable pressure on the plates, and no doubt a large amount of

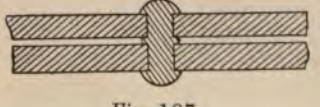


Fig. 187.

frictional resistance is thereby exerted to stresses coming on the plates. The amount of this frictional resistance is so indeterminate that it is neglected in calculation.

Loose rivets may be detected by tapping them on one side with a light hammer, and holding a finger on the other. Such rivets should be cut out and replaced with properly fitting ones.

In punched work the holes are sometimes 10 to 20 per cent. larger than the sizes shown on drawings of the joint, hence the shearing resistance of the rivets is in excess of the calculated amount. This makes up for the inequality of stress caused by unequal spacing, which, as we have seen, is liable to be caused in punched work.

Usually, the size of a rivet, before it is heated for insertion in the work, is the same as that calculated and shown on the drawings.

In considering the strength of any given riveted joint we have to consider :—

(1). The strength of the rivets in resisting the shearing stress.

(2). „ „ plate across the line of easiest fracture.

(3). The bearing area of the rivets.

(4). Any combination of the above which would tend to weakness.

Other important points such as the arrangement of the rivets, thickness of cover plates, etc., will be considered separately.

As regards (1), it is evident that if all the rivets are doing their share of resistance, the shearing stress on each rivet will be equal to the total stress on the member divided by the number of rivet sections that must be sheared. If the joint is a single cover, as in *Fig.* 188, the number of rivet sections will be the number of rivets; if *in* double shear, as in *Fig.* 189, the number of rivet sections will be twice the number of rivets.

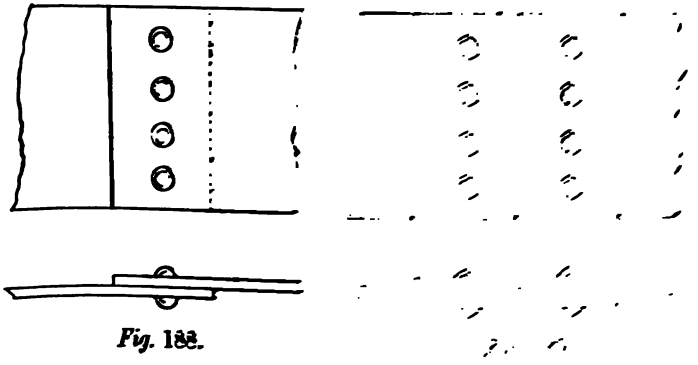


Fig. 188.

The shearing strength of the rivets, if of wrought iron, is nearly, but not quite, equal to the tensile strength of iron of the same quality. But as rivet iron is generally better than plate iron, the difference in strength is to a great extent compensated for. On the other hand, it is not sound to assume that all the rivets are under the same intensity of stress, for many causes may contribute to produce unequal distribution. Hence it is considered by the best authorities on the subject that if we take 5 tons as the tensile strength of the iron, we may take 4 to 4½ tons as the safe strength of the rivets, according to the nature of the joints and the workmanship. For steel rivets, 5 to 5½ tons may be taken. The following table may be useful in calculations:—

TABLE XXVIII.

Showing the Value of the Pull T *in tons that can be taken up by the Shearing Strength of one Rivet, and also the decimal co-efficient* N, *representing the number of Rivet Sections required to resist a Pull of* 1 *ton.*

Nominal Size.		Wrought Iron r_s=4 tons.		Steel r_s=5 tons.	
Diameter.	Area.	T	N	T	N
⅝″	0·3068	1·38	0·725	1·534	0·652
¾″	0·4417	1·987	0·503	2·208	0·453
⅞″	0·6013	2·706	0·370	3·006	0·333
1	0·7854	3·534	0·283	3·927	0·255

The *pitch* of rivets is the distance from centre to centre. The longitudinal pitch is the distance from centre to centre in the direction of the stress (*a*, *Fig.* 190).

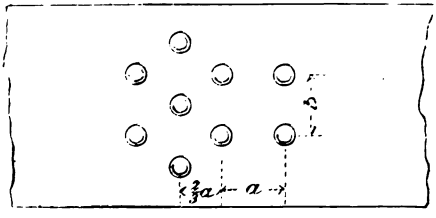


Fig. 190.

The transverse pitch is that at right angles to the stress (*b*, *Fig.* 190). The distance between one row of rivets and another should not be less than ⅔ *a*.

The rules for the pitch can be calculated from the consideration

of equality of strength of the joint at all parts, *i.e.*, that the shearing strength of the rivets should = the tensile strength of the net area of the plate. This gives a value to the pitch = twice the diameter of the rivet, *i.e.*, the metal left between the rivet holes must be at least as broad as the diameter of the rivet. This leaves very little margin for imperfect workmanship, hence it is safer to take the pitch as = 3 diameters of the rivet.

In compression members the maximum longitudinal pitch should not exceed 12 times the thickness of the thinnest outer plate. In general a pitch of 6″ should not be exceeded, or water is apt to get in between the plates, and cause oxidation.

The distance of the outer rivets from the edge of a plate depends on the direction of the fibre of the metal. In the *direction* of the fibre, the distance from the end to the centre of the rivet should not be less than $\frac{1}{2}'' + t + \frac{1}{2}d$, t being the thickness of the plate. Transversely, *i.e.*, at right angles to the fibres, it may be $\frac{1}{8}''$ less than this. A safe rule is to make the *end* distance from edge of hole = 2 diameters of the rivet, and the *side* distance $\frac{1}{4}''$ less.

Diameter of Rivets for Given Plates.—This may also be calculated from the consideration of equality of strength in the parts of the joint; making the shearing strength of the rivets = the bearing resistance of the metal of the plates, and thus deducing a ratio of the thickness of the plate to the diameter of the rivet. This ratio, however, would vary with circumstances, and it is generally safe and sufficiently accurate to take the following rule (Professor Fairbairn's) :—

$d = 2t$ for plates under $\frac{1}{2}''$, $d = 1\frac{1}{2}t$ for plates of $\frac{1}{2}''$ and over, d and t being the diameter of the rivet and thickness of the plate, respectively, in inches. Professor Unwin's rule is $d = 1{\cdot}2\sqrt{t}$.

Where angle or *shaped* iron has to be rivetted to plates, etc., it is sometimes necessary to consider the available space for giving the necessary clearance for the tool forming the head. This may make a smaller rivet necessary than that found by the rules above.

It is desirable in all cases to use one size of rivet throughout any one structure.

The classification of rivetted joints may be made both according to the arrangement in section and in plan.

I. *Arrangement in Section.*—(i.). Lap joint. (ii.). Butt joint with single cover. (iii.). Butt joint with double cover. (iv.). Butt joint grouped.

II. *Arrangement in Plan.*—(i.). Single rivetting. (ii.). Double, treble or chain rivetting. (iii.). Zig-zag rivetting.

I. (i.). *Lap Joints* (*Fig.* 188) are faulty in that they necessarily cause unequal transmission of stress, and as the calculations for rivetted joints are based on the principle of equality of distribution, it is evident that such joints should not be used where a better form can be substituted. It has been pointed out in the last chapter that equality of distribution is important both in tension and compression members.

(ii.). The disadvantage of unequal distribution is not overcome in *single cover joints* (*Fig.* 191). The only advantage here is that the inconvenience of overlapping the plates is obviated. The number of rivets is doubled, as compared with the last instance, and the thickness of the cover plates must be at least equal to the thickness of the plates joined.

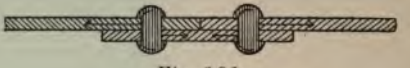


Fig. 191.

(iii.). In *double cover joints* (*Figs.* 189, 192), however, the stress is transmitted uniformly. Before the joint can fail by shearing, each rivet would have to give way at two sections (*Fig.* 194), hence this joint is said to be in "double shear," the resistance to shearing being doubled. The covers need only be half the thickness of the plates, though, as a matter of practice, they are generally made the same thickness as the plates.

Fig. 192.

(iv.). It will be seen, however, that when there are two or more lines of plates to be joined, the butt joint shown above (iii.) will *not* be economical, both because it would involve each cover being at least equal to half the combined thickness of all the plates, and because the number of rivets on each side of the joint would require to be equal in shearing and bearing area to the combined stress on all the plates. Hence for economy of material and rivets, a

grouped joint, as in *Fig.* 193, is introduced. This may be regarded as a bringing together of several butt joints with covers. In this joint the rivets are in single shear.

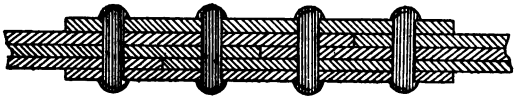


Fig. 193.

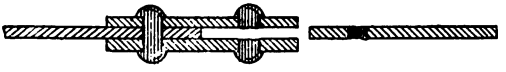


Fig. 194.

The considerations for ascertaining the thickness of the covers, and the number and distribution of the rivets in this form of joint, will be considered presently.

II. As regards *arrangement in plan*, (i.), single rivetting (*Fig.* 188), requires no explanation. For a given number of rivets it involves the least length of cover or lap, but it is obviously limited by considerations of the breadth of the plate, and the rules for pitch transversely.

(ii.). Double or treble, or chain rivetting, is where each rivet lies directly in a line with those in the next row (*Fig.* 195). The plate and cover is here weakened by the number of rivets in one row. In *Fig.* 195 where six rivets are used, double rivetting being adopted, with $\frac{1}{2}''$ plates, the length of the cover is 12", taking the rules for pitch as given above. The net area of the plate is $(8 - 3 \times \frac{3}{4})\ \frac{1}{2} = 2{\cdot}87$ square inches. In *Fig.* 196, where the rivetting is treble, the length of the cover is $16\frac{1}{2}''$, but, if the width

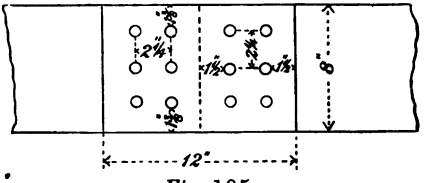


Fig. 195.

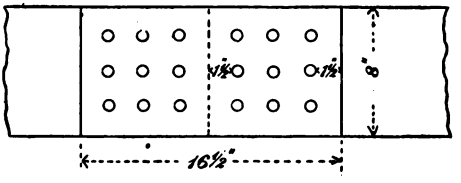


Fig. 196.

be, as before, 8″, the net area of the plate is 3·25 square inches. Hence if we take 5 tons as the safe tensile resistance per square inch of wrought iron, we have the strength of the joint and the bar in *Fig.* 196 16·25, as compared with 14·35 in *Fig.* 195, for the same width of bar, 8″.

From this we see that where there is little room to spare in the width of a member, *i.e.*, where the stress coming on it is nearly equal to its safe resisting power, it is better to put few rivets in a row, and several rows. Where, on the other hand, there is a large margin of safety in the member, double or even single rivetting tends to economy of length of cover.

(iii.). By adopting zig-zag rivetting, however, the greatest economy is generally attained. Taking the same width of bar as formerly, we find that by arranging the rivets as in *Fig.* 197 we have a length of cover 13·5″, and a net area of plate $= (8 - 1 \times \frac{3}{4}) \times \frac{1}{2}''$ $= 3{\cdot}625$ square inches. This gives a strength of 18·1 tons as compared with 16·35 and 14·35 in the other cases.

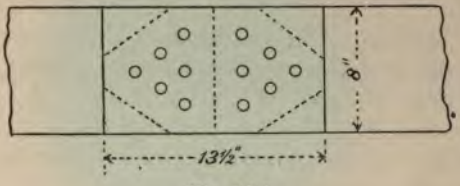


Fig. 197.

Here the strength of the plate is weakened by one rivet hole only. This may not appear self-evident, and may be explained as follows:—

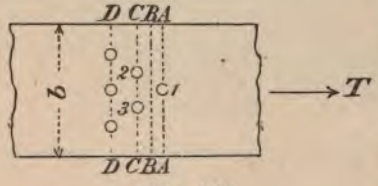


Fig. 198.

T = the tensile pull coming on the plate, p = resistance to tension per square inch, d = diameter of one rivet, t = thickness of the plate, and b its breadth, all in inches. Then at the section AA (*Fig.* 198), for equilibrium, T must $= btp - dtp$. After passing AA, the rivet a having done its work, the stress tending to pull apart the plate at BB is $T - dtp$. At CC, for equilibrium, this must = the resistance of the net area, *i.e.*, $T - dtp = btp - 2dtp$. Hence $T = btp - dtp$ as before. At DD the stress tending to pull apart the plate is $T - 3dtp$, for three rivets (1, 2 and 3) are now doing their share of resistance. The net area resisting this pull is $btp - 3dtp$. So we see that the plate is

actually stronger at DD than it is at AA, and that in any case its strength is only weakened by one rivet section, *dtp*.

The same fact may also be explained by considering that before tearing can take place at DD, the rivets at 1, 2 and 3 must be sheared.

The cover plate is generally cut in a diamond or lozenge shape, parallel to the outer line of rivets (see dotted lines *Fig.* 197).

Calculations for a Grouped Joint.

We shall now investigate the calculations necessary for a grouped joint. The same principles will apply to the investigation of other joints, but there are certain peculiarities in grouped joints which do not occur in others.

We have to find: (1), the thickness of the covers; (2), the number of the rivets; and (3) the distribution of those rivets.

(1). Where there is only one joint with several plates, it is clear from *Fig.* 199 that the stress on the covers is inversely proportional to the distance from the joint, and that the total stress in the two covers combined is equal to the stress in the plate which they cover. The other plates in the joint simply act as distance pieces. On the same principle we see that the theoretical thickness of the covers in a grouped joint would be somewhat as shown in *Fig.* 200, and that the thickness at most would be found from the expression $\frac{n}{n+1} \times t$, where n = number of plates and t = thickness of one plate.

Of course, a cover plate would, in any case, be made of uniform thickness throughout, and in most cases the covers are made of the same thickness as the plates they cover.

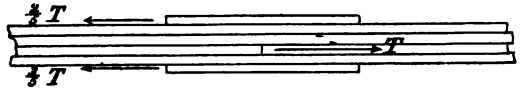


Fig. 199.

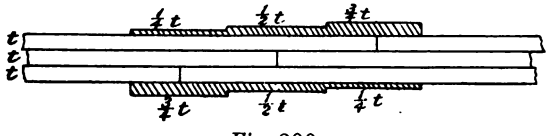


Fig. 200.

(2). As regards the total number of the rivets, we must ascertain

the diameter from considering the ratio of diameter : thickness of plate (already investigated), and then find out how many rivets are required to resist shearing and bearing. Let n' be the number of rivets in each end group, n'' the number in each intermediate group, and for the sake of example let there be three plates, as in *Fig.* 201. Let t and t' be thickness of one plate and cover respectively.

1. Then, for shearing—

$$R = \frac{N\pi d^2}{4} \times r_s = 2(n' + n'')\ 3{\cdot}1416 \div 4 \times d^2 \times r_s.$$

The general equation, where p = number of plates, is

$$R = (2n' + (p - 1)n'')\ {\cdot}78d^2 r_s \ldots\ldots\ldots\ldots\ldots\ldots (\alpha).$$

2. For bearing, as in *Figs.* 201, 202—

From A to B resistance $= n' \times 3tdr_c$.
„ B to C „ $= n'' \times 2tdr_c$.
„ C to D „ $= n'' \times tdr_c$.

Hence total R $= 3\ (n' + n'')\ tdr_c$,

and the general equation, as before, is

$$R = (pn' + \tfrac{1}{2}p\ (p - 1)\ n'')\ dtr_c \ldots\ldots\ldots\ldots\ldots (\beta).$$

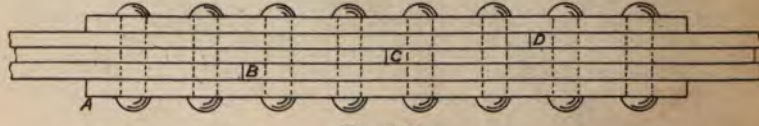



Fig. 201.

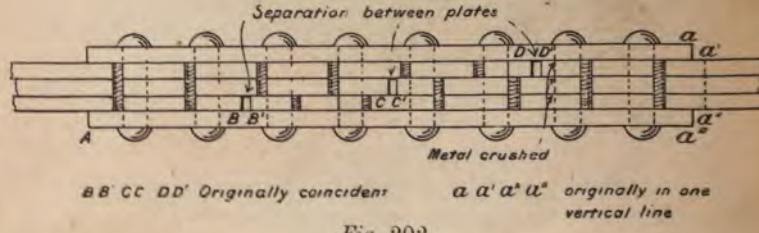


Fig. 202.

From these two equations (α) and (β) we can ascertain

$$N = \{2n' + (p - 1)\ n''\}$$

the total number of rivets. We must, of course, take the larger value as obtained from these equations. We have to find out now their distribution, *i.e.*, the values of n' and n''.

(3). From the known breadth of the plate, and the rules for pitch, we can easily ascertain the number of rivets *across* the plate, which will take us one step further into the investigation. Call this known number m. In order to find out the values of n' and n'' we must consider what possible ways there are for the joint to fall, other than those already indicated. There are two such ways, viz. :— (i.). By the rivets in the central groups shearing, and, at the same time, by the covers tearing, as in *Fig.* 203. (ii.). By the rivets cutting into the covers in the end groups, and at the same time cutting into the plates, on one side or the other of the joint, in the central groups.

Fig. 203.

In (i.) the resistance of the covers to tearing is

$$2\,(b - md)\,t' r_t,$$

t' being the thickness of the cover, and the shearing resistance of the rivet is

$$2n'' \times 0{\cdot}78 d^2 r_s.$$

Hence $$R = 2\,(b - md)\,t' r_t + 2n'' 0{\cdot}78 d^2 r_s,$$

and the general equation is

$$R = 2\,(b - md)\,t' r_c + (p - 1)\,0{\cdot}78 d^2 r_s \ldots\ldots\ldots\ldots\ldots (\gamma).$$

(ii.). The resistance of the covers is

$$2n' \times t' \times d r_c.$$

The resistance of the plates marked between B and D (*Fig.* 201) is

$$2n'' \times t \times d \times r_c + n'' t \times d r_c = 3n'' \times t \times d \times r_c.$$

Hence $$R = d r_c\,(2n't' + 3n''t),$$

and the general equation is

$$R = d r_c \left\{ 2n't' + \frac{p}{2}\,(p - 1)\,n''t \right\} \ldots\ldots\ldots\ldots\ldots\ldots (\epsilon).$$

From these equations we can ascertain the value of the numbers n' and n''.

As a matter of fact, the number of rivets is practically found from (α), and the distribution from (γ), the other equations giving results which are less than those found by the solution of the former two.

EXAMPLE 31.—*Design a ground joint for four* $\frac{1}{2}''$ *plates, each* 12″ *wide, the plates to be weakened as little as possible. Each plate may have to bear a tensile stress of* 20 *tons.*

The size of the rivets will be $1\frac{1}{2} \times \frac{1}{2}'' = \frac{3}{4}''$. As the plate will have to be weakened by at least one rivet section, the net area left will be $12 - \frac{3}{4}'' = 11\frac{1}{4}'' \times \frac{1}{2}'' = 5{\cdot}62$ square inches. This will have to bear a stress of 20 tons, hence the intensity will be $20 \div 5{\cdot}62 = 3{\cdot}56$ tons per square inch, which wrought iron is quite able to bear safely.

Let p = number of plates = 4.

n = number of rivets in the end groups.

n'' = number of rivets in each central group.

m = number of rivets across the plate in direction of the width, b, d, t, r_t and r_b having the usual significance of breadth, depth, etc.

Then for shearing from equation (α)—

Total stress

$$= (2n' + 3n'') \times \text{area of } \tfrac{3}{4}'' \text{ rivet} \times 4 \text{ tons per square inch,}$$

i.e.—

$$4 \times 20 = (2n' + 3n'')\, 0{\cdot}4417 \times 4 \quad (0{\cdot}4417 \text{ being taken from Table XXVIII.})$$

$$2n' + 3n'' = 46,$$

i.e., the total number of rivets.

For bearing from equation (β)—

$$\text{Stress} = \tfrac{1}{2}p\,(2n' + (p-1)\,n'')\,dtr_b,$$

i.e.,

$$80 = 2\,(2n' + 3n'')\,\tfrac{3}{4} \times \tfrac{1}{2} \times 5.$$

$$2n' + 3n'' = 21.$$

Hence 46, being the greater number, are required.

As regards distribution, we see from considerations of pitch (*Fig.* 190) that the greatest number that we can put in the breadth of the plate is 5. Hence $m = 5$ and $b = 12$. From equation (γ)—

$$\text{Total stress} = 2\,(b - md)\,tr_t + (p-1)\,n'' \times 0{\cdot}78d^2r_s,$$

i.e.,

$$80 = 2\,(12 - 5 \times \tfrac{3}{4})\,\tfrac{1}{2} \times 5 + 3n'' \times 1{\cdot}7668,$$

$$n'' = 7{\cdot}3, \text{ say } 8.$$

This gives eight rivets in each of the central groups, of which there are three, hence the number of rivets in each end group is $\frac{1}{2}(46 - 3 \times 8) = 11$.

To find what bearing strength this arrangement will have, we apply the values thus found to equation (δ).

$$\begin{aligned} R &= (2n' + \tfrac{1}{2}p\,(p-1)\,n'')\,dtr_b \\ &= 2 \times 11 + \tfrac{1}{2}4 \times 3 \times 8)\ \tfrac{3}{4} \times \tfrac{1}{2} \times 5 \\ &= 132 \text{ tons.} \end{aligned}$$

As the total stress required is only 80 tons, it is evident that there is no fear of the joint giving way by the fourth method of failure.

The joint may therefore be constructed as in *Fig.* 204. In the fourth row 5 rivets are placed. As there might be 8 rivets without weakening the plate more than by the diameter of one rivet hole, this is quite safe.

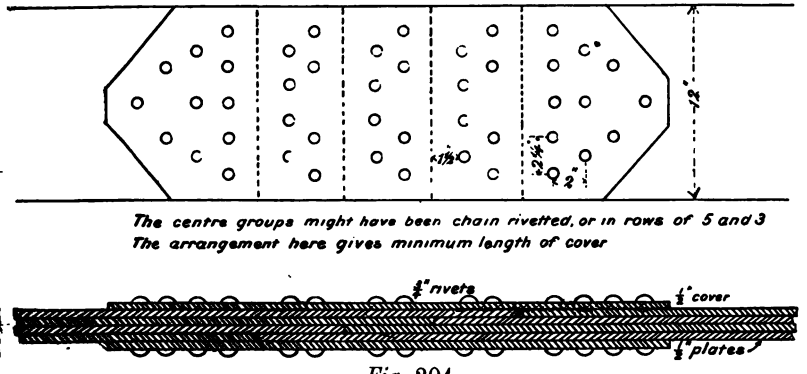

The centre groups might have been chain rivetted, or in rows of 5 and 3
The arrangement here gives minimum length of cover
¾" rivets
½" cover
½" plates

Fig. 204.

In the above example the thickness of the covers are taken as each $\frac{1}{2}''$, *i.e.*, the same thickness as the plates. It might have been $\frac{4}{5}$ of $\frac{1}{2}''$, but practically it would be $\frac{1}{2}''$.

Rivetted joints are frequently used for the fastening of the members of iron roof trusses, lattice girders, etc. The calculations of these joints are very simple; all that is required is to ascertain the number of rivets necessary to transfer the stress coming on the member from one to the other, the calculation being worked out both for shearing and bearing, and the greater value taken. Then

the requirements for pitch have to be met, and finally the rivets have to be arranged so that the resultant stress borne by all of them comes symmetrically on both the members thus united, so as to produce equality of stress throughout the member. To fulfil the last condition properly, it may be necessary to find out the position of the neutral layer of the member, and to arrange the rivets so that they fall equally as regards their distance from that neutral layer.

To illustrate this subject it may be well to take the following:—

EXAMPLE 32.—*A tension bar has to bear a stress of* 10 *tons, and it is proposed to use a section of* $5'' \times \frac{1}{2}''$. *Find the number of rivets required to fasten it to a girder boom built up of* $\frac{1}{2}''$ *plates with angle irons, as in* Fig. 205.

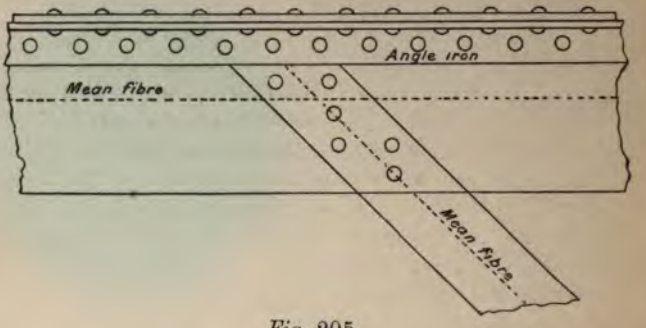


Fig. 205.

As the thickness of bar and plates is $\frac{1}{2}''$, we may use $\frac{3}{4}''$ rivets Deducting one rivet section from the bar we have $(5 - \frac{3}{4}) \times \frac{1}{2}'' = 2{\cdot}25$ square inches. The stress intensity will therefore be $10 \div 2{\cdot}35 = 4{\cdot}4$ tons per square inch, which is safe enough.

Then for shearing $T = n \times 0{\cdot}78d^2 \times r_s$,

i.e., $10 = n \times 1{\cdot}7668$,

$n = 5{\cdot}6$, say 6 rivets.

For bearing $T = n \times d \times t \times r_b$,

$10 = n \times \frac{3}{4} \times \frac{1}{2} \times 5$,

$n = 5{\cdot}3$, say 6 rivets, as before.

These 6 rivets should be grouped round the centre fibre of the bar, as in *Fig.* 205, one rivet being in the first row, as we have gone on the assumption that the bar is weakened only by one rivet section.

If there is found to be not enough room on the plate of the boom to allow all the 6 rivets to be placed with due regard to the requirements of pitch, we may use what are called *gusset plates*, *i.e.*, subsidiary plates of iron of sufficient size to admit of all the rivets being placed at either end, and thus acting as a single cover plate to the two members. Another alternative, where the conditions of pitch are almost, but not quite, fulfilled, is to use a slightly larger size of rivet, and thus reduce the number. In the foregoing example, for instance, it would have been possible to use 5 rivets of $\frac{7}{8}$" diameter instead of 6 $\frac{3}{4}$" rivets.

General Remarks on the Design of Rivetted Joints.

In conclusion, it may be said that **the most careful attention should be paid in rivetted joints to the arrangement, so as to secure uniformity of stress throughout the whole of the rivets, and uniformity of transmission to the members which the joint unites.** It has been pointed out in a previous chapter that the formulæ for both tension and compression bars are really based upon the assumption of concentric loading, and hence we are basing our calculations upon wrong premises if we fail to secure at the ends, by means of the joints, that uniformity. In joining bars to the ordinary trough section of girders, it is often impossible to arrange that the rivets shall be arranged symmetrically on the mean fibre, not only of the oblique bar, but also of the T-shaped boom, because that mean fibre is so near the angle that it is not practicable to drive the rivets. Instead, therefore, of having the trough section, as shown in *Fig.* 206, a more scientific form has been

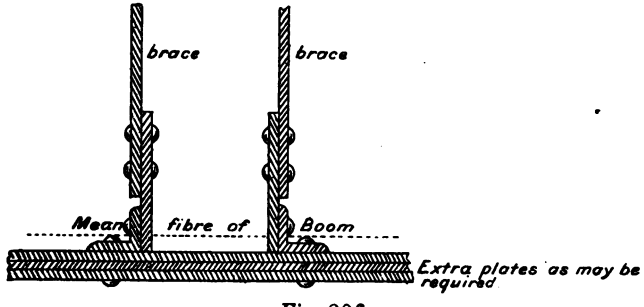


Fig. 206.

adopted to some bridges, viz., that shown in *Fig.* 207, where the

beam consists of two vertical bars, with distance pieces
or with some fir
stiffening.* This
economical from
point of view, b
distinct practical
that water will n
the trough, as in

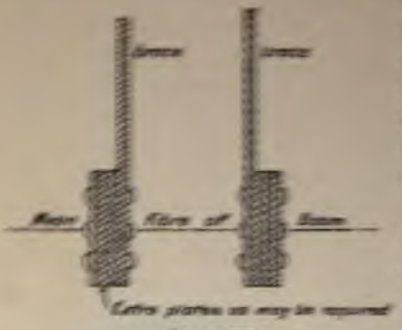


Fig. 207.

For the same
better, where pos
two flat bars, w
pieces, for struts in
than T or angle
the latter it is ver
arrange the joint so there shall be uniformity of stress in

The above principles apply to all forms of joints, bu
sity is more apparent in the case of rivets than in any o

Plates III. and IV. show various forms of joints
examples.

APPENDIX.

Professor Warren gives the following rules for grouped joint

Let N = total number of rivets in each joint.
p = number of plates.
b = breadth of each plate less rivet holes.
t = thickness of each plate.
m = number of rivets in each end group.
r = number of rivets in each central group.
d = diameter of one rivet.
a = area of one rivet $= .7854d^2$.

$$N = \frac{0.628}{a} pbt \text{ for iron} = 2m + (p-1)r \quad \ldots\ldots$$

$$N = \frac{0.8}{a} pbt \quad \text{,, steel} = 2m + (p-1)r \quad \ldots\ldots$$

$$\text{Iron } pbt = 2bt + \frac{(p-1)ra}{0.628} \quad \ldots\ldots$$

$$\text{Steel } pbt = 2bt + \frac{(p-1)ra}{0.8} \quad \ldots\ldots$$

Find value of r from (3) and (4), and substitute in (1) and (
Thus in (4)—

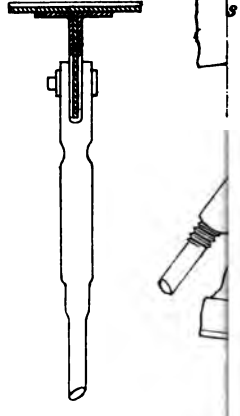
$$r = \frac{0.8\,bt}{(p-1)a}\,(p-2).$$

* This form has been adopted, among other places, in the
constructed by the M.S. and L.R. at Nottingham, Leicester, e

FASTENING

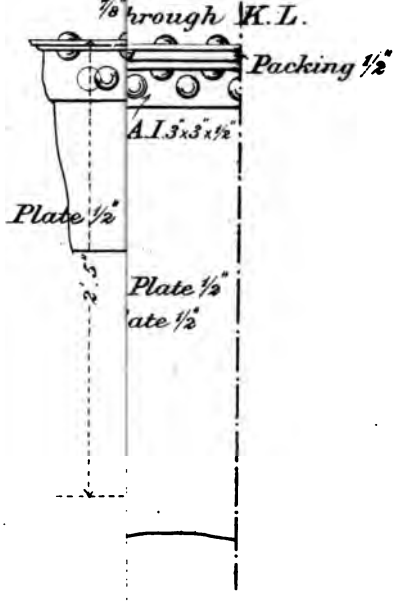
PLATE III.

Fig. 8.
Section A-B.

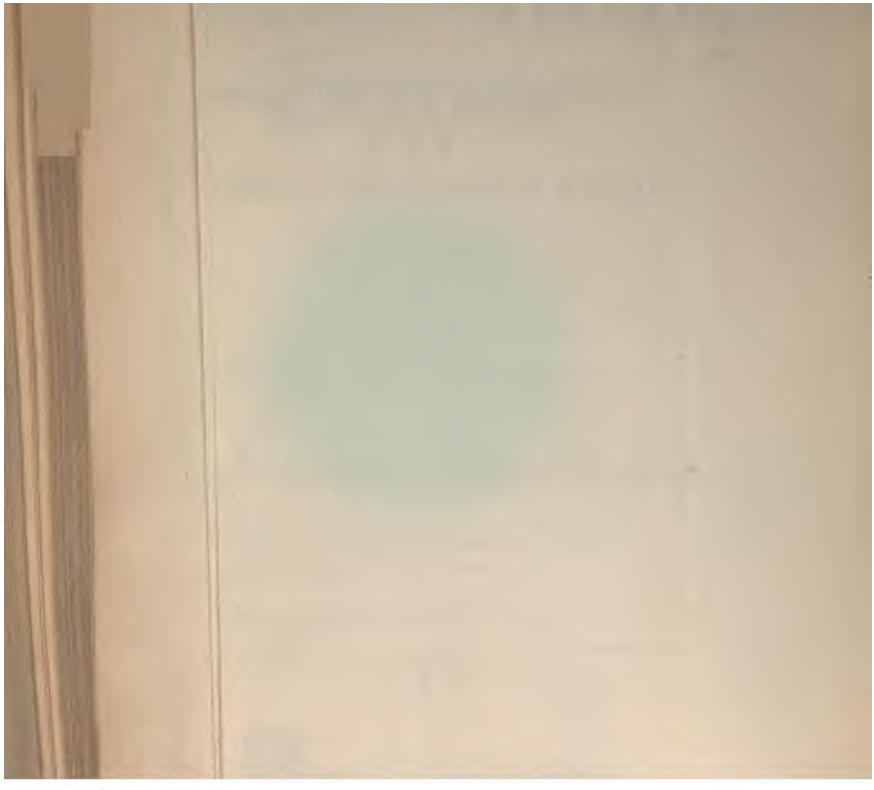


struts,
s.

g. 14.

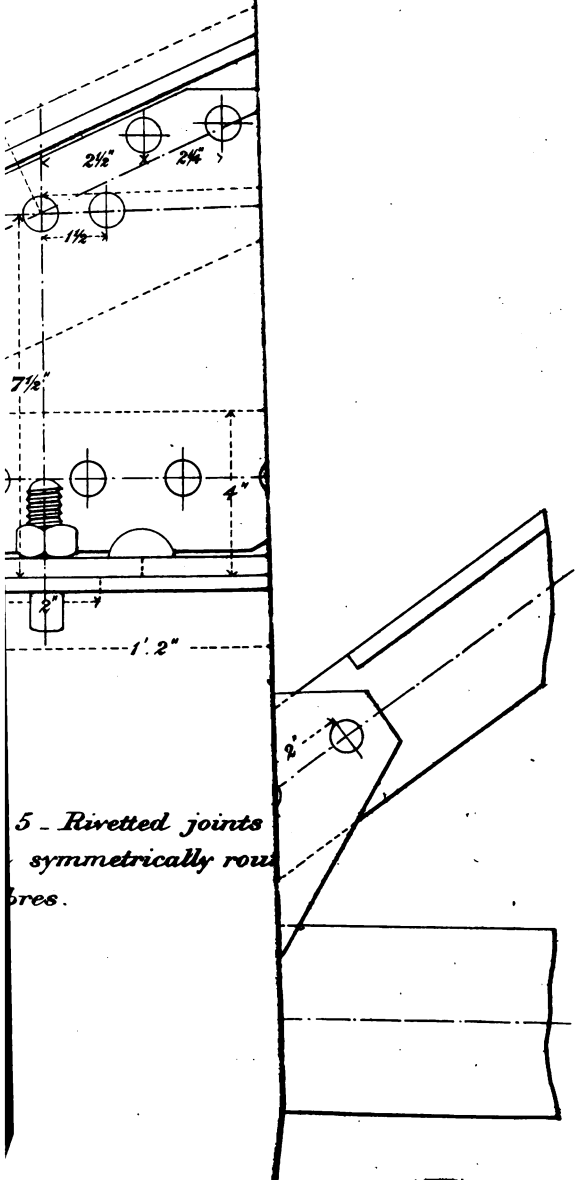


link
screw



FASTENIN PLATE IV.

Fig. 4.



5 - Riveted joints
symmetrically rou
bres.

CHAPTER XI.

Common Timber Roofs.

Importance of Subject to Military Engineers.—Loads on Roofs.—Wind Pressure.—Couple Roofs.—Couple-close Roofs.—Collar Beam Roofs.—King-post Roof.—King-post Roof without Common Rafters.—Queen-post Truss.—Hip and Valley Rafters.

The design of common wooden roofs has not received much attention in books dealing with the design of structures.

The reason of this is not far to seek. Civil engineers, as a rule, are more occupied with iron or steel for permanent structures than with timber, and architects are naturally more occupied with the artistic treatment of their designs than with a detail that does not in most cases lend itself to "the poetry of construction," nor, except in large buildings, to considerable economy.

On the part of the military engineer, who has to construct public buildings in various parts of the world, most of which are roofed with timber procured locally, and which, though of no great span, are repeated again and again, such inattention would be inexcusable.

It is fitting that in this work special attention should be paid to this important matter, for it is obvious that although engineering economy applied to one roof, or to one truss in a roof, may be a matter of little importance, yet when that roof is repeated in scores of similar buildings (as in a barrack, or cantonment) the economy becomes very appreciable.*

Loads on Roofs.

The loads on roofs are partly permanent, partly temporary.

The permanent load consists of the roof covering, the common rafters, purlins, and trusses, the ceiling, if there is one, and any other load, such as a lantern skylight, that may occasionally be constructed

* Tredgold points out that in practice "roofs are generally made too heavy, and that he will do a most acceptable service to his profession, who shall show how to retrench and execute the same roof with the smallest quantity of timber. He will by this take an unnecessary load off the walls, and a large and useless expense from the owner."

as an accessory. The temporary loads are workmen going up for repairs, snow in certain latitudes, and (for some few cases, comparatively speaking) wind pressure. Each of these loads must now be examined.

Roof Covering.—The following table shows the weight of various kinds of roof covering :—

TABLE XXIX.

Weights of Various Forms of Roof Covering in lbs. per square foot (Exclusive of Workmen).

Bermuda slates or slabs	*24
Corrugated iron, 16 W.G.	3¼
,, ,, 18 ,, ...	2¼
,, ,, 20 ,, ...	2
Ceilings, ordinary lath and plaster	10
Lead, including laps, but not boarding or rolls	5½ to 8½
Felt (asphalted) without boarding, each layer	¾
Sheet iron, 16 W.G.	2
,, laid on the Naini Tal system with battens, etc. ...	4 to 6
,, 20 W.G.	1
Shingles (used in Barbados, Mauritius, Halifax, and in some cases in India)	2 to ¾
Slates, laid with 3″ lap, including nails:—	
Ladies, 16″ × 8¼″	1¼
Countesses, 20″ × 10″	1
Duchesses, 24″ × 12″	3½
Slate boarding, fir ¾″ thick	2½
,, 1″ ,, ...	3¼
,, 1¼″ ,, ...	4¼
Slating, Morar, inclusive of battens	4 7
Tiles, ordinary plain, pointed with mortar, including laths ..	18 to 2 0
,, Algeciras, as used at Gibraltar, laid complete, excluding laths	2 3
,, Italian, not including boarding	1 4
,, Indian country	1 4
Allahabad tiling (used in N. India) single (wet), including battens	1 6½
,, ,, ,, double, ,,	3 3½
Mangalore tiles (used in S. India and Ceylon) ... about	1 0
Thatch, including battens	6½
,, ,, bamboo frames	1 0
Zinc, including laps, 14 to 16 zinc gauge	1¾ to 2½

(Obtained from General Wray's *Instruction in Construction*, the M.W. Handbook, and information sent by C.R.E.'s at foreign stations).

* This is as reported by C.R.E., Bermuda. There seems to be, however, some ambiguity as to whether the 24 lbs. is the weight of *one slab*, or the weight of the completed roof per square foot. The latter is reported by other officers as 17 lbs. 24 lbs. has been taken as being the heavier weight, and therefore on the safe side.

The slope, or pitch, as it is sometimes called, of a roof varies with he material used for covering. The usual slope given for slate roofs n England,* and for tiled roofs in India is 26° 33′, or rise = ¼ span. it should, however, be noted that, if there is any doubt as to the slope, it is better to give a steep rather than a gentle slope, because the wind tends to drive rain beneath the overlapping covering more easily with gentle slopes, and also tends to strip that covering with greater readiness when the slope approaches the horizontal. The thrust on the principal rafters is also greater (other things being equal) with roofs of gentle slope.

As regards the estimating of the load of common rafters, etc., it is best to begin calculating the thickness of all members in regular sequence from the roof covering downwards, and thus to ascertain accurately the weight coming on the trusses. Thus in a slate roof, having first obtained the weight of the slates from Table XXIX., we may calculate the thickness of the boarding or battens, on which the slates are to be fastened, then calculate the common rafters, then the purlins, and thus we shall know accurately the weight coming on the supporting trusses. For approximate calculations the tables in the usual engineering pocketbooks may be used (Hurst, Molesworth, etc.), or the following rule may be taken :—For every square foot of roof allow 2 lbs. for boarding and framing of the truss, but excluding the tie beam, for which 20 lbs. per foot run may be allowed. This rule, however, is only approximate, and is not recommended.

In may be said, and often is said, that there is no necessity for calculating the scantlings of timber in ordinary roofs, when there are scantlings for various spans given in the pocketbooks referred to. Such an argument is so fallacious as hardly to require an answer, yet it is so frequently made that it is necessary to allude to it. The scantlings in question are quoted from Tredgold ; they are for trusses (*a*) made of red fir (*pinus sylvestris*) ; (*b*), at a distance apart of 10 feet ; (*c*), with a roof covering of countess slates, on boarding ; (*d*), a slope of 26° 33′ ; and (*e*) with a ceiling attached to the tie beam. *If any one of these five conditions be altered, the pocketbook scantlings are inapplicable.*

It must be also remembered that Tredgold's rules are empirical,

* It is doubtful whether any slate roof should be given a less slope than 30°. The writer's experience at several large military stations induces him to think that slopes of 26° 33′ are a very false economy.

and sometimes result in roofs which are unnecessarily heavy. An instance of a roof which was actually constructed at Bombay in 1839, with a saving of one-third of the timber which would have been used if Tredgold's scantlings had been used, is quoted in Barlow's edition of Tredgold (1870), proving that the recognition of the uncertainty of applying these scantlings in all cases has long been recognized.

Temporary Loads.

Snow is usually taken in this country at a maximum of 5 lbs. per square foot of surface covered. In America it is considered that at least 12 lbs. should be allowed. Snow weighs much less when freshly fallen than it does when partly melted and saturated with water. In the latter case it may weigh 20 to 30 lbs. per cubic foot, but long before this has taken place, it will have disappeared off a roof of the ordinary slope, partly by sliding, partly by melting. This will not be the case, however, in valleys, which should be avoided in roof construction as much as possible in all climates, but especially where snow is likely to fall.

Allowance for Men repairing Roofs.—This subject has generally been ignored in textbooks, because the allowance for wind pressure has been more than sufficient to cover all that is required to enable workmen to go on the roof with safety. It is, however, necessary to consider this load. The writer has known cases where a roof has been designed with so little regard for this matter that workmen laying slates in one part damaged slates in another part, owing to excessive deflection of the timbers, and other instances where the roofs were not safe for working parties who had gone to clear away snow. As will be presently seen, the rules ordinarily in use for estimating wind pressure (and which constitute the greatest factor in the total pressure hitherto ordinarily allowed for), are based upon erroneous assumptions, hence it is unsound to assume that if a roof is safe as regards wind it is also safe for any other occasional load that may be brought to bear upon it.

The weight of workmen repairing a roof may be considered to be about, *as a maximum*, half that which would be brought upon a road bridge. Men working on a roof move cautiously, and do not crowd together. The load on a bridge is not more than 70 lbs. per foot super, hence if we take 35 lbs. per foot super for allowance for workmen we shall be well on the safe side, for the nature of the framing is to distribute the load, and it is not likely that there will be many men together at one time on one part of the surface. In

I.W.D. Handbook 15 lbs. is allowed for workmen, but it is ps safer to allow more, to admit of men clearing snow, etc.

*Wind Pressure.**

is is such an important subject, and the effect of recent tigation has been so contrary to ideas formerly held, that it is sary that the matter should be entered into in some detail.
edgold held that an allowance of 40 lbs. for "wind and other ional loads" was sufficient. This view was objected to by writers on the ground that the wind could not blow on both of the roof at the same time, and that, in estimating wind ure, the effect of the wind should be considered as acting on one nly, that it should be treated as a uniform load on that side, that its amount should be found by considering the maximum ontal wind force as ascertained by observations upon a plane hen resolving that force normally to the direction of the slope of oof. These rules were held as applicable to roofs of any height the ground, and built upon any kind of walls. It would be necessary, in order to ascertain the maxima stresses upon any ber of a roof truss, to consider it under three distinct conditions, to draw stress diagrams for each condition, viz.: (1), with anent load alone; (2), with wind blowing from one side; and with wind blowing from the other side. When there was a on roller under one end of the roof (as in the case of iron roofs), natter was still further complicated. The results of these were tabulated, and the maxima stresses on each member estimated. made the calculation for any roof laborious, and tended to urage the easier but extravagant aid of pocketbook tables, out due discrimination of data.
ne disaster to the Tay Bridge in December, 1879, however, drew ntion of engineers to the matter of wind pressure, and although knowledge of the subject is not yet complete, the experiments h have been made during the last ten years have been such as ad to a much more exact realization of the actual effect of the l on structures.†

*This section is based generally on the views of Professor Unwin, as essed in a lecture delivered by him at Chatham in December, 1896.

†It may generally be said that the action of the wind on a structure is gous to the effect of water on obstacles in a stream. The problem is, in one of hydrodynamics.

It is proposed here, briefly, to give a summary of these experiments under the headings of

(1). Amount of actual observed pressure, and its nature.
(2). Influence of height above the surface of the ground.
(3). „ shelter.
(4). „ form of structure.
(5). Lifting action of whirlwinds.

As regards (1), it has long been recognized, and recent observations tend to confirm this view, that there is a definite ratio between the velocity of the wind and the pressure caused. This ratio may be approximately expressed by the following empirical formula:— $v^2 = 285P$, where v = velocity in miles per hour, and P = the maximum pressure caused by gusts in lbs. per square foot on a completely exposed plate normal to the wind. The maxima recorded pressures are estimated at 62·7 lbs. per square foot (Mauritius hurricane, 1892), 80 lbs. (Bidston, Lancashire). In England generally, however, the maximum is rarely more than 55 lbs. The Board of Trade standard is 56 lbs. Tornadoes in America have been estimated to exert 84 to 100 lbs. It must, however, be remembered that this maximum pressure is not all positive pressure on one side of the structure. It is made up of positive pressure on one side and negative on the other, the former being about $\frac{8}{10}$ of the total. *Fig.* 208 shows the relative amounts on a thin plane normal to the wind and with freedom from interference with the development of the negative pressure. Thus we see that if a structure be so situated that the negative pressure cannot come into play, the total pressure sustained will be only about 0·6 that due to the normal wind velocity.

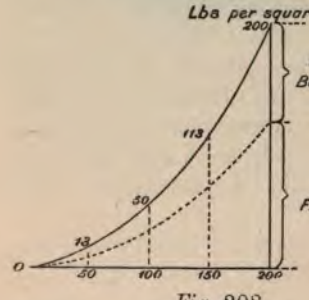


Fig. 208.

(2). The influence of height has been the subject of careful observation. The results of experiments made by Mr. Stevenson (*Fig.* 209) show that up to a height of about 15′ the wind pressure is low and much influenced by the ground friction. Above 20 feet it gradually increases as the ordinates of a parabolic

s shown in dotted lines on the figure, having its vertex 72 round. It may be inferred from this that if the pressure at be taken as 30 lbs., that at 100 feet would be 42, at 200 feet nd so on, the maximum being 100 lbs. Exposure on a high e, free from the retarding effect of ground friction, would e same effect as if the building were of a considerable height.* from this that it is on lofty structures that the necessity of ig to wind pressure is most urgent.

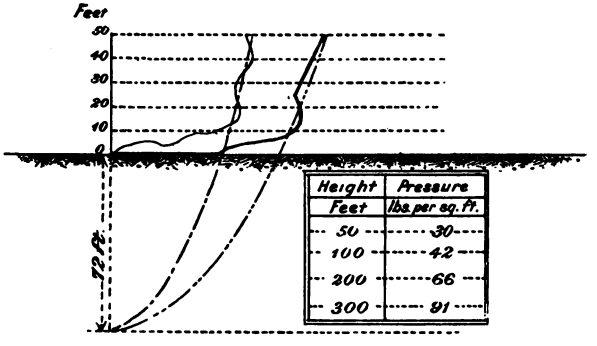


Fig. 209.

Most important of all considerations, and one which has only been the subject of accurate experiment, is *the effect of shelter.* elter is not only the protection afforded by obstacles in front ructure, but the protection of parts of the structure on other Thus, in a roof built on the usual system, with no very great on of eaves, the effect of the wind blowing against the walls oduce a buffer or cushion of air, which interferes with the e of the wind on the roof, as shown in *Fig.* 210. From ients made by Professor Kernot, of Melbourne,† it appears effect of the wall shelter is such that, with roofs up to 30° no wind pressure is practically exerted on the roof at all. opes of more than 30°, the percentage of the maximum wind is shown in the tabular statement in *Fig.* 211. It will be

Bidston observatory is 60′ high on the top of a hill 200′ facing the . Practically, the wind has in this case no retardation from ground

epted as perfectly trustworthy by the best authorities.

seen also from the same figure that the relative lengths of the roof and the wall have an appreciable influence on the subject.

When striking a roof, the most effective force in point of destruction is that produced by ricochet. The valleys of high-pitched roofs are centres of energy, and the resultant forces deal havoc among the slates in those parts. The effect of the wind current passing up the slope of a roof is to strip off the covering, but there is no recorded case of a roof having been blown directly *in*, although there are any number of instances of a roof having been blown *out*, or upwards from below.

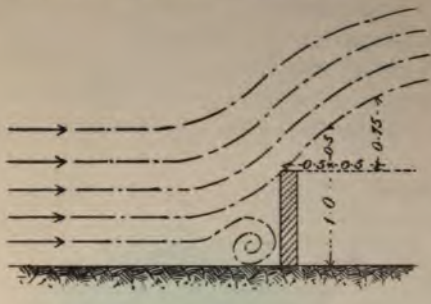
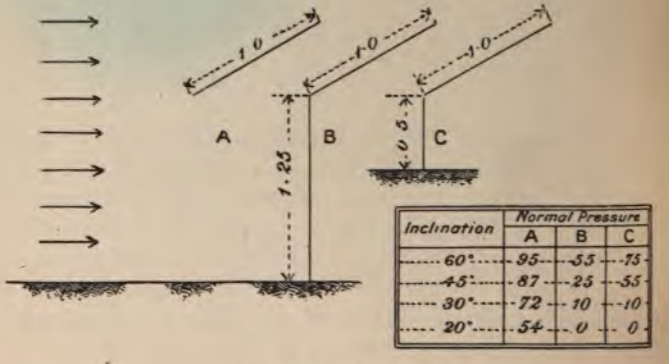


Fig. 210.



Inclination	Normal Pressure		
	A	B	C
60°	95	55	75
45°	87	25	55
30°	72	10	10
20°	54	0	0

Fig. 211.

(4). The *shape* of a structure exposed to wind pressure has a very considerable effect upon the pressure. If the face exposed to the wind is not a plane, the *front* or positive pressure is modified, and if the form of the structure interferes with the convergence of the air currents in rear, the *back* or negative pressure is modified. Thus, if we put K for the ratio of the pressure on a body to the pressure on

a thin plate of area = its projected area on a plane normal to the wind, then :—

For a sphere, K = 0·31.
„ cube, K = 0·80, wind normal to face.
„ „ K = 0·66, wind parallel to diagonal of face.
„ cylinder, K = 0·47. Height of cylinder = diameter, and wind normal to axis.
„ cone, where height = diameter of base, K = 0·38, where the wind is parallel to base.

On a cylindrical chimney the wind pressure would only be about half that on a thin plate of area = the diametral plane. If we were to take the positive front pressure only, it may easily be proved that the ratio would be 0·78, but the modifying effect of the negative pressure is to reduce it further to about 0·5.

As regards the distribution of pressure on an oblique surface, it has been proved that when a plane surface is opposed in an oblique direction to the action of the wind, the pressure is not equally distributed, but is greater near the forward edge, and less towards the leeward edge. If a plane makes an angle θ with the direction of the wind (*Fig.* 212), the resultant of the normal pressure R divides the surface into two unequal segments a and b, the values of which are considered to be as follows :—

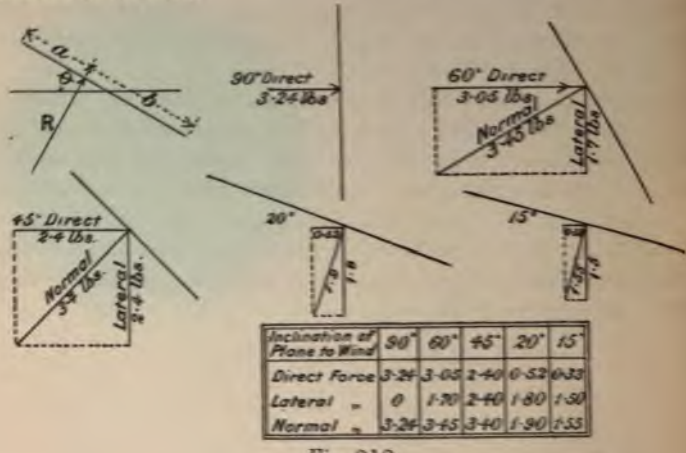
When $\theta = 72°$ to $75°$, $\frac{a}{b} = 0{\cdot}9$.
„ „ $= 57°$ „ $60°$, „ $= 0{\cdot}8$.
„ „ $= 43°$ „ $48°$, „ $= 0{\cdot}7$.
„ „ $= 25°$ „ $29°$, „ $= 0{\cdot}6$.
„ „ $= 13°$ „ $= 0{\cdot}5$.

Fig. 212 also shows the value of forces normal to the surface of a plane at various inclinations. These are from experiments made in 1868 for the Aeronautical Society at Greenwich.

Sir B. Baker made a series of experiments on wind pressure on structures of various forms during the construction of the Forth Bridge. On bodies of simple form, cubes, cylinders, etc., his experiments confirm those of other observers. His experiments on the sheltering effect of one girder in a bridge on another will be referred to (in Part II.) when considering the forces acting on bridges.

(5). This leads on to another consideration, viz., the lifting effect of whirlwinds. This is unquestionably the most destructive of all

the wind effects. The amount of lifting force has been *known* to be at least as much as 136 lbs. per square foot.* Nor is this lifting effect confined to countries subject to hurricanes, for the following case (among others) is related by Mr. Scott, of the Royal Meteorological Society, as having occurred in England:—At Walmer, Kent, a storm passed over a 2-storied house with the roof overhanging about 18". The two windows of the upper story were provided with common linen blinds. The wind lifted the roof bodily, and sucked one blind along with it. It fell outwards in the direction in which the storm was going, and when the roof dropped into its place again, it *shut the blind into the space between the roof and the top course of brickwork in the wall.*



Inclination of Plane to Wind	90°	60°	45°	20°	15°
Direct Force	3·24	3·05	2·40	0·52	0·33
Lateral "	0	1·70	2·40	1·80	1·30
Normal "	3·24	3·45	3·40	1·90	1·55

Fig. 212.

This subject is of special importance in connection with suspension bridges.

From these facts, so far as they bear upon the question of roofs, and the stresses produced, we may conclude:—

(*a*). That the method of considering that the wind pressure acts on one side only of a roof, and is uniformly distributed normally over that side is erroneous.

(*b*). That, up to a slope of 30°, roofs which are built with continuous walls below may be considered safe from wind pressure as long as the walls are safe.

(*c*). That sheds or other open buildings, as long as they are not a

* *Minutes of Proceedings, Institution of C.E.*, Vol. LXIX.

ry great height from the ground (say within 20′), are sufficiently sure against downward wind pressure.

(*d*). That all sheds, and roofs with very deep eaves, and verandahs, pecially of buildings with several stories, should be secured against ıe upward effect of the wind by being firmly tied down to the walls the building, or to the supporting pillars.

(*e*). That valleys should be avoided as much as possible. We have lready seen that this is desirable because of the accumulation of now that is apt to take place, but it is also advisable on account of he ricochet of wind force.

It is, however, most desirable to stiffen roofs laterally by wind ies or by some diagonal bracing, as there may be otherwise a endency for the trusses to heel over under the influence of a lateral rind. Hipped ends tend to lateral stiffening.

On the whole, therefore, if we take 35 lbs. as the load for workmen, nd 5 lbs. for snow, we shall arrive at the 40 lbs. originally stated by Fredgold as the allowance to be made in roofs for temporary loads, nd in the roofs considered in this chapter may omit the onsideration of downward pressure from wind altogether.

Couple Roofs.

The simplest form of roof is that known as the *couple roof*, where the rafters, placed usually in this country at 12″ central intervals, are notched over the wall plates at one end and butt against the ridge at the other, as shown in *Fig.* 213.

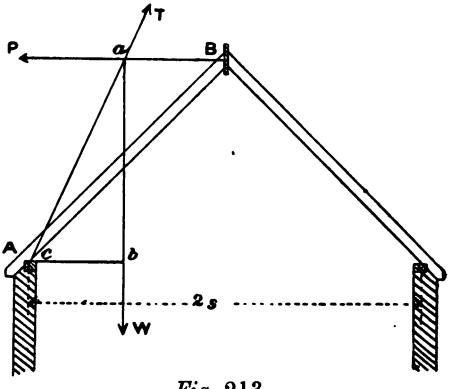


Fig. 213.

In this case there is no transverse stress in the ridge board. Its function is to afford a convenient position for uniting the upper ends of all the rafters, and for transmitting the pressure from the rafters at one side down those at the other. The rafters are under a combined stress, viz., under the transverse stress of the weight, and under the compressive stress from the corresponding rafter at the other side. It is, however, sufficient to consider them as beams supported,

and uniformly loaded, without taking into account the compressive stress as well, as it is usually small.

There is a lateral thrust on the wall, equal in amount to the horizontal pressure at the ridge. The amount of total pressure per unit on the wall will be presently investigated.

If the ridge piece be, as in *Fig.* 214, so that the ends of the rafters rest on the bevelled sides, and are fastened thereto, then there will be a transverse stress on the ridge, but the rafters will be relieved of compressive stress, and the wall will have but little lateral thrust.

Fig. 214.

The lateral thrust brought on the walls by this form of roof restricts its use to small spans, 12′ being the maximum under ordinary circumstances. Sometimes this lateral thrust is taken by iron tie rods at intervals, connecting the walls, and preventing the thrust from tending to push out the walls; and, in certain classes of buildings, this is a suitable form of roof.

To find the amount and direction of the oblique thrust coming on the wall is very easy. For if we know the amount of the weight coming on the rafter AB (*Fig.* 213), we know that it is kept in equilibrium by three forces, viz., W, acting vertically through the centre of the beam; P, the horizontal reaction at the ridge; and a reaction at the wall plate, which must, for equilibrium, pass through the intersection of the other two. The magnitude of these forces can be obtained by laying off to any scale $ab = W$, and drawing bc horizontal. Then bc = horizontal reaction at the ridge, and ca = the oblique reaction at the wall.

Without drawing a diagram, this amount and the angle of its direction can be expressed analytically. It may be here remarked that it is desirable in all roofs to have both graphic and analytical methods for ascertaining the stresses. The one method acts as a check on the other.

In the case in point, if $2s$ be the span, and a be the rise,

$$P : W :: \tfrac{1}{2}s : a.$$

Hence
$$P = Ws \div 2a = \tfrac{1}{2}W \cot \theta,$$

and
$$T = \sqrt{W^2 + (Ws \div 2a)^2}$$
$$= W\sqrt{1 + \tfrac{1}{4}\cot^2 \theta}.$$

The inclination of T to the horizontal is $\tan^{-1}\frac{2a}{s}$. The application of these principles will be evident from the following :—

EXAMPLE 33.—*A couple roof, 16′ span, with rise = 4′, rafters at 12″ intervals, and covering countess slates on $\frac{3}{4}$″ boarding, is built of ordinary red or Scots fir. Find the scantlings of the rafters and the magnitude and direction of the thrust on the walls.*

The half-span is 8′. It is quite safe to consider this as the span for calculation, as only the part of the load resolved normally to the rafters tends to transverse rupture. If W be that load, and θ the angle of the roof (*Fig.* 215), then the resolved portion normal to the roof surface is W cos θ. But the half-span is to the slant length of the rafters as cos θ:1. Hence it is exactly the same if we take the whole weight × the half-span, or take the normal component of the weight × the slant length.

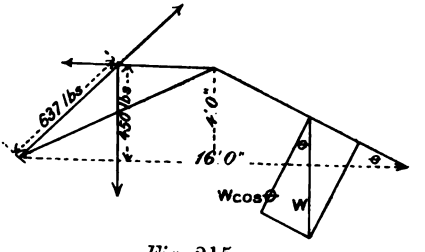


Fig. 215.

It is desirable that a roof of this form be stiff as well as strong, so that the slates may not break by the rafter bending. Hence we may use the deflection formula (2) from Chapter VIII., as well as that for strength.

In the formula $bd^3 = L^3Sw \div 133$ we have the following values :— L = span in feet = 8, S = spacing of the rafters in feet = 1, w = the weight per square foot of the load = 10·5 (permanent) + 40 (temporary) = 50·5 lbs., say 50 in round numbers.

Then, substituting values—

$$bd^3 = 8 \times 8 \times 8 \times 1 \times 50 \div 133 = 192.$$

From Table XXI., p. 152, Chapter VIII., we see that 2″ × 4$\frac{3}{4}$″ would be a suitable scantling.

For strength—

$$bd^2 = \frac{L^2SW}{140} = 64 \times 1 \times 50 \div 140 = 23.$$

This would be satisfied by 2″ × 3$\frac{1}{2}$″.

We take the larger value, viz., 2″ × 4$\frac{3}{4}$″.

To find the thrust on the walls, we have $s = 8'$, $a = 4'$, and slant length of rafter $= 8 \times \sec 26° 33' = 9'$, hence the amount of the thrust T

$= \sqrt{W^2 + (Ws \div 2a)^2} = \sqrt{(50 \times 9)^2 + (50 \times 9)^2 \times 8 \div 4 \times 2} = 637$ lbs.

The angle of the thrust is $\tan^{-1} 8 \div 8 = 1$, hence the angle is 45°. This may be also seen from the diagram (*Fig.* 215), where the stresses are drawn to scale.

Some may consider that it is not worth while to devote so much space to the consideration of so elementary a form of roof, but as the same investigation would be applicable to the hip rafters in an ordinary hipped end, and as it has been found that there is often difficulty in understanding the action of the stresses in such ends, it is considered advisable to give a case of the simplest solution before going to the treatment of more complex forms. In a hip rafter the stresses are transferred to the walls at the angle, and have a tendency to thrust out the walls. This tendency is, to a certain extent, counteracted by the use of "angle braces" and "dragon beams," which bind together the walls, but in any case the masonry should be so bonded that there will be no chance of the thrust pushing it out of plumb.

The consideration of the design of walls to bear outward thrust will be considered in a later chapter.

Couple Close Roofs.—To counteract the tendency of the rafters to cause lateral thrust on the walls, a tie beam is given in the couple close roofs. This tie beam also serves to support a ceiling, if such be required.

This is the simplest form of truss, or framed structure, and as such demands attention.

If the truss be as in the previous case, *i.e.*, where the rafters butt against a vertical ridge board, it is evident that the stress on the tie will be the same as the horizontal thrust against the ridge, and expressed as follows:—The thrust $P = W \times s \div 2a = \frac{W}{2} \cot \theta$.

For instance, in the last example, let it be intended to relieve the thrust on the walls by using $\frac{1}{2}''$ iron tie rods. Find how many tie rods will be required.

A $\frac{1}{2}''$ iron bar has a sectional area of about 0·2 square inch, and the safe tensile stress of wrought iron is 5 tons. Hence the safe stress on a $\frac{1}{2}''$ bar is about 2,000 lbs. The weight on each rafter in the last example is 450 lbs., $s = 8$, and $a = 4$. Hence the horizontal thrust at the ridge for each rafter $= 450 \times 8 \div 2 \times 4 = 450$. There-

ore, $2000 \div 450 = 4{\cdot}4$, so it would be quite sufficient to put the ie rods at 4′ intervals.

If, however, the couple close roof is of the form shown in *Fig.* 216, the case is different. In this case it is a true truss, the principal rafters of which are under a combined stress, of transverse load and compression. Such a case would occur where the trusses are supported on pillars, and the roof covering on horizontal battens, such as are generally required with corrugated iron, sheet iron, or some form of tiles. In such cases we have to investigate the stresses coming directly on the members from the direct compressive or tensile stresses, and then to consider the bending moment from the transverse load, combining the two as indicated in Chapter IX.

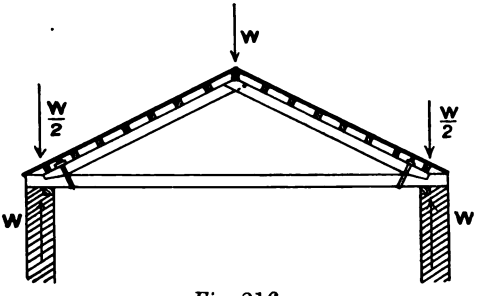


Fig. 216.

(i.). *The Direct Stresses.*—To find these we may draw a stress diagram. Let W represent on any scale the total load coming on one principal rafter (*Fig.* 217), then the reactions at each wall will = W, there will be a weight W at the apex, and $\frac{1}{2}$W resting at the eaves. Starting from the left-hand corner, and working round in the direction of the hands of watch, we draw the olygon of forces for the ›ft corner of the frame—› = W, *bc* = $\frac{1}{2}$W—then *cd* arallel to CD, and *ad* arallel to DA, complete ıe polygon. Marking the arrows (as indicated in Chapter II.)

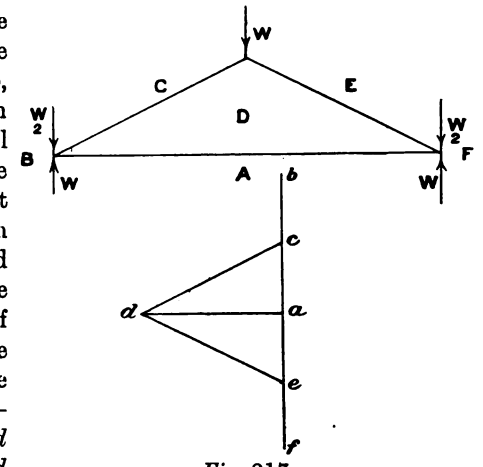


Fig. 217.

we see that the arrows* point up in *ab*, down in *bc*, down and to the left in *cd*, and horizontally to the right in *da*. That is to say that the direct stress acts in CD towards the joint, and is a thrust, while in DA it acts away from the joint, and is a pull. Passing now to the joint at the apex we know DC from the investigation just completed, and from the ends *c* and *d*, if we draw *ce* and *dc* parallel to the forces CE and ED, we have the polygon for the apex. From this we see that ED is also a strut. The force polygon for the point at the right-hand corner can be similarly drawn.

(ii.). This stress diagram gives us the direct stresses only. The transverse moments are found by principles with which the reader is perfectly familiar, as they have been stated in Chapters IV. and VIII.

EXAMPLE 34.—*A couple-close roof is to be constructed, span* 18′, *distance apart of trusses* 8′, *roof covering sheet iron on battens at* 2′ *intervals, roof slope* 30°, *timber deodar. Let it be further assumed that it may be necessary to lift a weight of half a ton by a block and tackle from the tie beam (centre).*†

Weight on battens = 6 lbs. (permanent) + 30 lbs. (temporary) = 36lbs. per square foot. Without going into the calculations of these battens we may take it that they are 4″ × 4″, and add 2 lbs. per square foot to the load. This will be very nearly, if not absolutely correct. The total load W coming upon one principal will be 9 × sec 30° × 8 × 38 lbs. = 3,142 lbs. (*Fig.* 218).

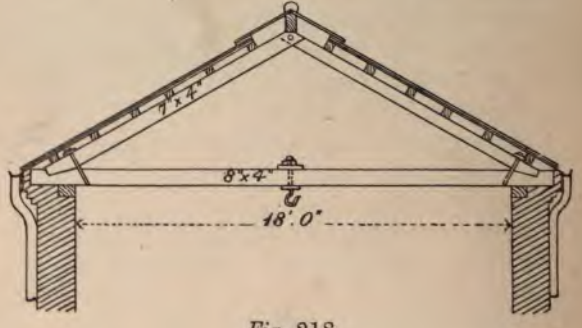


Fig. 218.

* Shown in lower part of *Fig.* 219.

† The roof of a horse infirmary at a hill station in India. An English dray horse has been known to weigh a ton, but an Indian troop horse or battery mule is not half that weight.

The reactions at the walls are equal to this 3,142 lbs. + ¼ a ton 560 lbs. (which may be required to be raised from the tie beam) = 3,702 lbs.

Starting from the left corner, as before, we lay off (*Fig.* 219) $ab = 3702$, $bc = \frac{1}{2}\,3142 = 1571$, and draw cd and da. We find $cd = 4268$, and $da = 3680$.

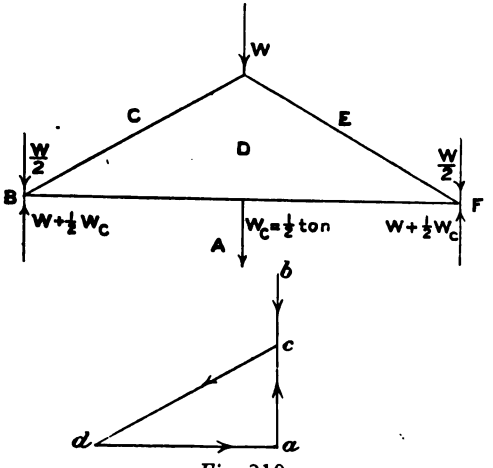


Fig. 219.

We need go no further, for the stress in DE = stress in DC.

This gives us the direct stresses. To find the combined stress we we must find the M_{ff} in DC and DA respectively.

The total weight on DC is 3,142 lbs., and the length 124″. As the $M_{ff} = Wl \div 8$* the value of this is $3,142 \times 124 \div 8 = 48,800$ inch-lbs.

Then, in the equation

$$r = My \div I - \frac{Pl^2}{10E},$$

$M = 48800$, $y = \frac{1}{2}d$, P = direct stress = 4268, $l = 124''$, $E = 1,500,000$ (see Table XIV., Chapter III.), and $I = \frac{1}{12}bd^3$, r may be taken as = 2000 (safe).

Making $b = \frac{1}{2}d$, and substituting the above values, we have $d = 6{\cdot}8$. Make 7″ × 4″.

As regards the tie beam, the M_f is $\frac{1}{4}Wl$, where $W = \frac{1}{2}$ 2240, and $l = 18' = 216''$. Hence $M_f = 60,480$ inch-lbs.

* That is considering it as a supported beam. It is really partially fixed, but considering it as supported only is on the safe side.

The equation

$$r = My \div I + \frac{Pl^2}{10E},$$

then becomes

$$2000 = 60480 \times \tfrac{1}{2}d \div bd^3 + 6400 \times 216^2 \div 15{,}000{,}000.$$

Taking $b = 4''$, so as to have the tie beam and principals of the same width for convenience of framing, we have in this equation only one unknown quantity, viz., d. By trial and error we see that a value of less than 8″ quite satisfies the equation. Make 4″ × 8″.

In this case we have assumed that the roof is loaded to the greatest extent. As this will not always be the case, and as such loading tends to reduce the compression in the upper fibres of the tie beam when loaded, it may be as well to ascertain what is the greatest compression or tension per square inch in the tie beam from transverse load alone. The M_f is 60480, which must be $= \frac{rbd^2}{6}$, where $b = 4$, and $d = 8''$. Hence $r = 60480 \times 6 \div 4 \times 64 = 1{,}410$ lbs. which is well within the safe limits of the material.

The joint at the foot of the principal would be made as described in the last chapter. The roof is shown in *Fig.* 218.

Collar-beam Roofs.—As it is often inconvenient to bring the tie beam so low as is done in the case of the *couple-close* roof, the tie is sometimes put at some distance up the rafters. This form of roof, known as the *collar-beam roof*, is suitable for spans up to about 18′. It does not relieve the walls of lateral pressure, but that lateral pressure is comparatively small, and the collar affords an intermediate point of support for the rafters*, and thus tends to economy in their scantling. The collar beam is usually placed at about one-third of the height from the wall. If not required to carry a ceiling, its scantling may be quite light, for all that it has to do is to take the tensile stress, similarly to the tie beam in the previous example. But if it has to take a ceiling it must be calculated for deflection under a uniform transverse load.

If constructed as in *Fig.* 220 (which is the usual method) the tensile stress on the collar, and the inclined pressure at the junction, is calculated as in the couple roof.

* If the joint were a perfectly smooth pin, it is conceivable that the collar would become a strut. This, however, could hardly ever happen in practice.

The pressure on the walls is made up of the components resolved along the rafter, of the weights on the rafter, both above and below the collar. If W_1 be the weight above the collar and W_2 that below (*Fig.* 220) the angle of the roof being θ, then the thrust along the rafter is $(W_1 + W_2) \sin \theta$.

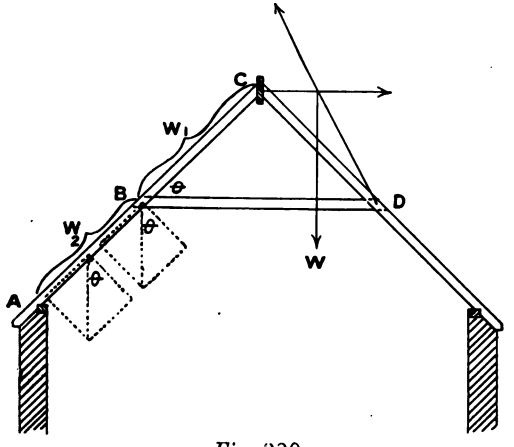


Fig. 220.

If the collar be placed low, the scantling of the rafter will be determined from the consideration of the weight W_1 acting transversely on a span = distance BC between collar and ridge. But if the collar be very high, then the scantling required for this purpose may be less than required for the lower part, which has to be sufficient to resist the M_f produced by $W_1 \cos \theta \times AB + W_2 \cos \theta \times \frac{1}{2}AB$. At the point B the rafter is exposed to the maximum transverse stress, and it is evident that the joint at that point should weaken the rafter as little as possible.

If there be a ceiling on BD, half its weight must be added to W_1 in estimating the pressure on the walls. If there be a ceiling, it is clear that the higher the collar the less the scantling required, whereas without a ceiling the scantling of the collar differs but little, whether it be placed high or low.

The following example is given of a collar-beam roof taken from a building at Bermuda :—

EXAMPLE 35.—*Design a collar-beam roof* 18′ *span, rafters at* 12″ *intervals, roofed with Bermuda slates, slope* 30°. *Timber pitch pine* (pinus

rigida), *a ceiling to be given under the collar, which is to be one-third of the way up* (Fig. 221).

From the figure we see that the collar divides the rafter into two portions, the upper 7′ and the lower 3′ 6″ long. The collar has a clear span of 12′.

For pitch pine the ultimate value of $r = 7{,}000$, hence with a factor of safety of 5 the value may be taken at 1,400. $E = 1{,}745{,}000$.

As the rafters are 1′ apart, and the weight of Bermuda slates is 24 lbs. per square foot, we have, allowing 30 lbs. for occasional loads, a total weight per square foot of 54 lbs., and the weight on the rafter from A to C is $7 \times 1 \times 54 = 378$ lbs. The joint at A has to bear, in addition to this, half the weight of the ceiling, *i.e.*, $\frac{1}{2}(12 \times 10) = 60$ lbs. Hence total $W = 378 + 60 = 438$ lbs. $= W_1$.

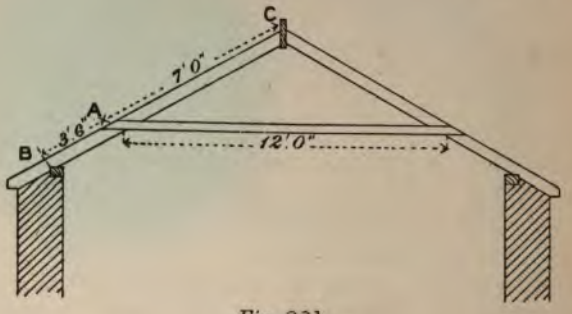


Fig. 221.

The weight from A to B is, similarly, $3{\cdot}5 \times 1 \times 54 = 189$ + ceiling $3{\cdot}5 \times 10 = 35$ lbs. Total, 224 lbs. $= W_2$.

Taking moments round A we have

$$R\ (= W_1 + W_2)\ \cos\theta \times 3{\cdot}5 - W_2 \cos\theta \times 1{\cdot}75$$
$$= W_1 \cos\theta \times 3{\cdot}5 + W_2 \cos\theta \times 1{\cdot}75 = M_B.$$

This is the same as if we took moments round B. Substituting values—

$$M_{ff} = (438 \times 42'' + 224 \times 21'')\ 0{\cdot}86 = 20{,}000 \text{ inch-lbs.}$$

$$M_r = \frac{rbd^2}{6} = 1400 \div 6bd^2 = 233bd^2.$$

Hence $$bd^2 = 20000 \div 233 = 86.$$

Make $6'' \times 2\frac{1}{2}''$ (see Table XXII., Chapter VIII.).

see if the upper part AC requires a greater or less scantling—

$$M_B = \frac{Wl}{8} = 7 \times 54 \times 84'' \div 8 = 3{,}970 \text{ inch-lbs.},$$

ı is much less than the 20,000 found above. Hence we may
″ × 2½″ throughout.
r deflection of the collar the maximum deflection permissible

$$= \tfrac{1}{40} \text{ of } 12'' = 0{\cdot}3'' = V.$$
$$W = 12 \times 10 \times 1 = 120 \text{ lbs.}$$

ıe general equation

$$V = \frac{nWl^3}{EI}$$

nes
$$0{\cdot}3 = \frac{5 \times 120 \times 144^3 \times 12}{384 \times 1745000 \times bd^3},$$

ıce $bd^3 = 107$. Make 2″ × 4″.

King-post Roof.

hen the span of a roof is greater than about 18′, it becomes ısary to give intermediate support to the principal rafters by ıs of struts, and to the tie beam by a suspension tie from the ı. The king-post truss, thus formed, is one of the commonest nost useful forms of arranging roof timbers, and its investigation be thoroughly studied. The term "king-post" is, of course, a omer, for the suspension member connecting the tie beam with idge is not a *post* under compression, but a tension *bar*. It may dvantageously, in some cases, made of an iron rod instead of er.

ıese trusses may be constructed to bear the weight of the roof ıither by having purlins over the junction of principals and s, these purlins having over them common rafters, or (2) by ıg a series of purlins at intervals along the principals, with no ıon rafters. In the former case the principals are under direct ı only, whereas in the latter case they are under combined s.

e shall consider examples of trusses under both conditions. former is that most usually met with, and will be first con-ed.

ıe distribution of the weight of the roof covering has first to be ıined. The common rafters usually butt against the ridge, so

that no weight at all is brought on the king head. The pole plate is usually resting on the tie beam beyond its junction with the principal. The purlins therefore transmit, usually, all the weight that the truss has to bear, and that weight is probably not much more than *half* the whole weight of the roof, or $\frac{W}{2}$ on each side. The method usually accepted, however, is to consider part of the weight coming at the ridge and part on the heel of the principal. If W = the total weight coming on *one* principal rafter, made up of weight of roof covering on one side in the space between two adjacent trusses + the weight of the common rafters + the weight of the purlins + principal itself + a small addition for iron straps, then it would be sufficiently accurate if we considered that $\frac{1}{2}$W came on the purlin, and was transmitted thence to the strut below. But it is perhaps more accurate to consider the common rafters as beams continuous over two spans,* and the reaction at the centre (as will be explained in Part II.) is $\frac{5}{8}$W, the remaining proportions of the loads being as shown in *Fig.* 218. Similarly, if w_c be the weight of the ceiling + other members of the truss, $\frac{5}{8}w_c$ will be supported at the centre. The total reaction at each wall will be $W + \frac{1}{2}w_c$, and the *net* reaction will be $\frac{13}{16}W + \frac{5}{16}w_c$ (*Fig.* 222).

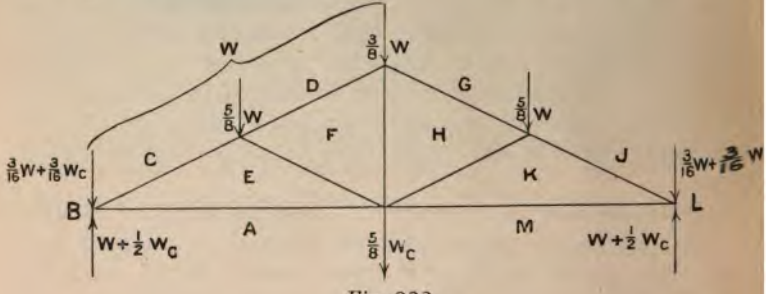


Fig. 222.

With these values we may set about drawing a stress diagra
Starting from the left hand corner, and working in the direction
the hands of a watch, we have (*Fig.* 223) $ab = W + \frac{1}{2}w_c$ upwar

* In order that this may be true, it is necessary that the three points
support should be on the same plane. This is not always the case in practic
It has been so assumed, however, in this investigation, so as to follow the sa
lines as the M.W.D. Handbook and General Wray.

and $bc = \frac{3}{16}(W + w_c)$ downwards, known forces. To find the values of the two unknown forces CE and EA, we draw from c and a, ce and ae parallel to CE and EA respectively, meeting at e. Thus ce = stress on CE, and ea that on EA. Marking the arrows in regular sequence from the known directions of AB and BC, and causing the remainder to follow round the polygon, we see that CE is in compression and EA in tension, and that the value analytically expressed of the stress on CE is $(\frac{13}{16}W + \frac{5}{16}w_c)$ cosec θ, and of EA $(\frac{13}{16}W + \frac{5}{16}w_c)$ cot θ.

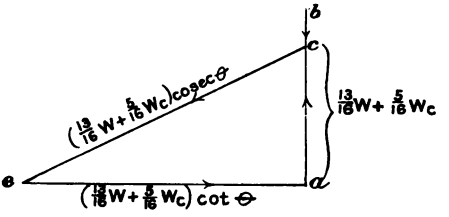


Fig. 223.

Going on now to the joint where the strut EF supports the principal, we have the known forces EC (just found) and CD. Draw ec, cd parallel to the direction of these known forces, and equal to them, on the same scale of forces as formerly. From d draw df parallel to DF, and from e draw ef parallel to EF, the two meeting in f. This completes the polygon for the point in question, and from this polygon we see that DF and EF are also in compression, since the direction of the arrows is towards the joint (*Fig.* 224).

The next joint to be considered is that at the apex. We cannot take that at the foot of the king post, as there are six forces meeting there, of which at present we only know three. At the apex there are four forces, of which we know two, viz., FD and DG. The stress diagram for this joint is as shown in *Fig.* 225, where fd and dg represent the known forces and gh and fh, drawn parallel to GH

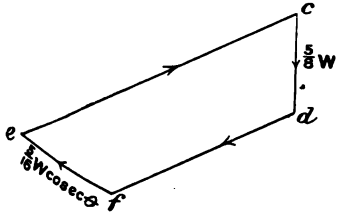


Fig. 224.

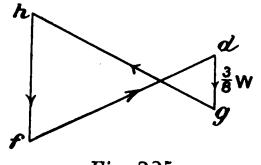


Fig. 225.

and HF respectively and meeting in h, represent the stresses unknown hitherto. From the arrows we see that GH is in compression and HF in tension.

In like manner the polygon of forces for the joint at the foot of the king post is shown in *Fig.* 226. MA, AE, EF, FH are the known forces, and HK, KM the unknown. From the polygon we see that the stress on $hf = \frac{5}{8}(W + w_c)$.

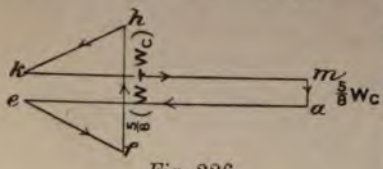


Fig. 226.

Instead of drawing each polygon of forces separately, we might have combined them in the stress diagram (or *reciprocal diagram*, as it is sometimes called) shown in *Fig.* 227.*

If w_c be neglected, which is quite permissible where there is no ceiling, the stress diagram would be as in *Fig.* 228.

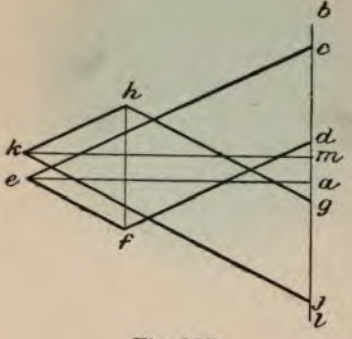


Fig. 227.

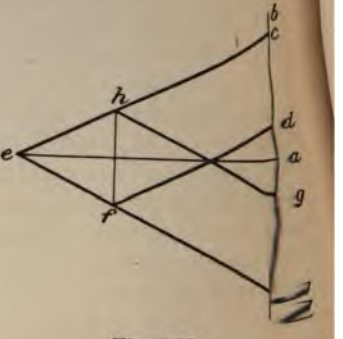


Fig. 228.

Without drawing stress diagrams, it is possible to arrive at the maxima direct stresses on each member of the truss from the values as noted above, and which are as follows :—

Maximum stress on principal (compression)

$$= ce = cg \text{ cosec } \theta = (1\tfrac{3}{16}W + \tfrac{5}{16}w_c) \text{ cosec } \theta \quad \ldots\ldots\ldots\text{(i.)}.$$

Maximum stress on tie beam (tension)

$$= ae = cg \cot \theta = (1\tfrac{3}{16}W + \tfrac{5}{16}w_c) \cot \theta \quad \ldots\ldots\ldots\text{(ii.)}.$$

Maximum stress on strut (compression)

$$= ef = \tfrac{5}{16}W \text{ cosec } \theta \quad \ldots\ldots\ldots\ldots\ldots\text{(iii.)}.$$

Maximum stress on king-post (tension)

$$= hf = \tfrac{5}{8}(W + w_c) \quad \ldots\ldots\ldots\ldots\ldots\text{(iv.)}.$$

* In all these diagrams struts are shown in thick, ties in thin, lines.

Of course, these apply only to trusses of this form, and loaded in ıe manner which we have been hitherto considering.

If we consider the weight on the truss as transmitted solely from he purlin, and, if further, we consider the purlin to bear about $\frac{3}{4}$ of he weight of the roof covering on one side (it probably bears ıbout $\frac{5}{6}$), and if the weight on the pole plate rest on the tie beam ɔutside the truss, we then have the load and reciprocal shown on *Figs.* 229, 230. This is to the same scale, and for the same loads, as

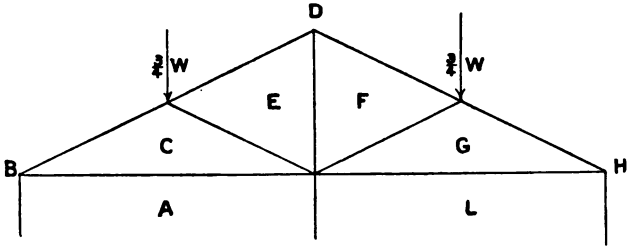


Fig. 229.

ig. 227. From it we see the stresses on the principal and tie beam 'e less, being $(\frac{3}{4}W + \frac{3}{16}w_c)$ cosec θ ıd $(\frac{3}{4}W \times \frac{3}{16}w_c)$ cot θ respectively, ıd the stresses on the king post ıd struts slightly more, than ɔrmulas (iii.) and (iv.). The rut has $\frac{3}{16}W$ cosec θ and the ng post $\frac{3}{4}W + \frac{5}{8}w_c$.

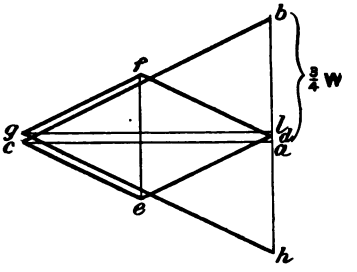


Fig. 230.

EXAMPLE 36.—*Design a king-st truss* 28′ *span from centre to ıtre of wall plates. Slope* 26° 33′. *mber, Scots fir* (pinus sylvestris). *ɔof covering, countess slates on* $\frac{3}{4}$″ *boarding. Snow,* 5 *lbs. per square foot be allowed for. Distance apart of trusses,* 10 *feet. A ceiling is to be ng from the tie beams, weighing* 10 *lbs. per square foot.*

Common Rafters.—

Weight of covering, countess slates on $\frac{3}{4}$″ boarding = 10·5 lbs.
„ „ workmen and snow ..... = 40 „

Total w (per square foot)..... = 50·5 lbs.

The rafters are usually placed, in England, at 12″ central intervals, ınd clear span from pole plate to purlin, or from purlin to ridge is 8′.

We might consider the rafters as beams fixed at the ends, but it is usual, and on the safe side, to consider them as supported only.

Calculating for strength—

$$bd^2 = \frac{L^2 S w}{140} \text{ becomes } 64 \times 1 \times 50{\cdot}5 \div 140 = 23.$$

$1\frac{1}{2}'' \times 4''$ would do, but if the wood is of inferior quality, make $2'' \times 4''$.

It is not necessary to calculate for deflection, although this would give higher values. The reason that the calculation for deflection is not necessary in this case is that the amount of deflection produced in a beam $2'' \times 4''$, with the weight, in this case, loaded uniformly, and supported at the ends of a span of 8′, would be only 0·35″, an amount that would not cause the slates to break or crack. As the rafters are not supported, but fixed, it is evident that even this small amount of deflection would not take place.

Purlins.—The increment of weight caused by the common rafters, as just ascertained, is $1 \times 2'' \times 4'' \times 36$ lbs. (weight per cubic foot of the timber) = 2 lbs. Hence the value of $w = 50{\cdot}5 + 2 = 53$ lbs. approximately.

The total weight on one side between two adjacent trusses is $16 \times 10 \times 53 = 6{,}480$ lbs., of which the central purlin bears $\frac{5}{8}$, *i.e.*, 4,050 lbs. Then $bd^2 = L \times 4050 \div 140 = 320$. Make $5'' \times 8''$ (see Table XXII., Chapter VIII.).

The calculation for deflection is again in this case unnecessary, for the same reasons before stated.

Main Truss.—The total loads coming on one principal are :—

Temporary load $16 \times 10 \times 40$	= 6400 lbs.
Roof covering $16 \times 10 \times 10{\cdot}5$	= 1680 „
10 common rafters $16 \times 10 \times 2'' \times 4'' \times 36$ ...	= 320 „
Purlin $10 \times 5'' \times 8'' \times 36$	= 100 „
Principal* (say) $1 \times 16 \times 4'' \times 6'' \times 36$	= 96 „
Ironwork (say)	= 24 „
Total W	= 8620 lbs.

Weight on tie beam—

Tie beam* (say) $28 \times 6'' \times 12'' \times 36$	= 504 lbs.
2 struts* (say) $2 \times 8 \times 4'' \times 4'' \times 36$	= 64 „
King post* (say $4'' \times 4''$) $1 \times 4 \times 4'' \times 4 \times 36$ ...	= 16 „
Ceiling at 10 lbs. per square foot $28 \times 10 \times 10$	= 2800 „
Total w_c	= 3384 lbs.

* These can only approximately be estimated as yet. Pocketbook tables are useful for this purpose.

It is worth noting that in the above value w_c the weight of the ceiling is a very large proportion of the whole. Hence, when there is no ceiling, w_c may be neglected.

From Formula (i.)—

$$P = (\tfrac{13}{16}W + \tfrac{5}{16}w_c) \text{ cosec } \theta,$$

we get—

Stress on principal

$$= (\tfrac{13}{16} \times 8620 + \tfrac{5}{16} \times 3384)\ 2{\cdot}245 = 18{,}200 \text{ lbs.}$$

Tension on tie beam

$$= (\tfrac{13}{16} \times 8620 + \tfrac{5}{16} \times 3384) \cot \theta = 8093 \times 2 = 16{,}186 \text{ lbs.}$$

Compression in struts

$$= \tfrac{5}{16}W \text{ cosec } \theta = \tfrac{5}{16}8620 \times 2{\cdot}245 = 6{,}100 \text{ lbs.}$$

Tension in king post

$$= \tfrac{5}{8}\ (W + w_c) = 5 \div 8\ (8620 + 3384) = 7{,}500 \text{ lbs.}$$

We have thus found the direct maxima stresses in the various members. We must now arrange for suitable scantlings.

Principal.—Assume one dimension as 4″. The length between heel and junction of strut is 8′, or 96″. Hence $l : h :: 96 : 4 = 24 : 1$.

Hence $P = \frac{1}{2}r_c A$ (see Formula (4), page 160).

P = direct stress on strut = 18,200 lbs., and $r_c = 1{,}200$, *i.e.*, safe stress per unit of area against crushing.

$\therefore A = 18200 \div 600 = 30$. Make 5″ × 6″.

Tie Beam—Tension = 16,186 lbs. Make 5″ wide to frame into the principal. Then r_t for red fir = 12000 (Table XIV., Chapter III.). Safe value of $r_t = 12000 \div 5 = 2400$ lbs. per square inch. Area of cross section = $16186 \div 2400 = 7$ square inches. As breadth = 5, depth = $7 \div 4 = 1{\cdot}4$. As about 3″ would be taken up with framing, the tie should be 5″ × 5″. *If there were no ceiling, it would be ample to make it the same as the principal*, viz., 5″ × 6″.

But the deflection of the tie beam under the weight of the ceiling must be taken into account. For deflection L = 14 (distance from wall to central supporting strap), S = 10 (distance apart of trusses, and w (weight of ceiling) = 10 lbs.

Hence $bd^3 = 196 \times 1400 \div 133 = 2062$. Taking $b = 4$″, d might be $7\frac{1}{2}$″. Make 5″ × 8″.

If a double tie of two planks had been used, each $1\frac{1}{2}$″ thick, the depth might have been $2062 \div 3 = 8{\cdot}75$, say 9″.*

* The economy of this is evident. The sectional area in the one case is 40, in the other 27 square inches. Tredgold gives 69 square inches !

Strut.—With one dimension assumed to be 4", we have ratio of length to least dimension = 96 ÷ 4 = 24. Hence $I = \frac{1}{24}A$. Formula $I = \frac{1}{30}W$. $\frac{1}{24}A = \frac{1}{30}W$, or $A = \frac{24}{30}W$. Hence $A = 6000 \div 600 = 10$ sq. in. If we make $T = 4$, this gives a least dimension of $2\frac{1}{2}$ and ratio $\frac{96}{2\frac{1}{2}}$ or 38. This would be too narrow; 4" would be ample, but in order to make it into symmetrical angle make it 5".

Tension Flange.—This is a tube bar, which at a safe value of f_t taken at 2400 will give a sectional area of about 2". The least practicable dimensions for making good joints are 5" × 5", the former dimension being adopted as equal to the width of all the other members of the truss.

Other Chord.—Here $L = 120$, $b = 12$, $r = 1.155$, so $\frac{Lb}{r} = 1000 \div 120 = 8.3$. Make $\frac{Lb}{r} = 9$.

The straps and other parts of the joints should be calculated for as these have been considered in Chapter III., they need not here be repeated. It must be remembered, however, that the joints form a most important part of the design, and the consideration of them should not be neglected. The strut at the foot of the king-post need only be of sufficient size to enable the fastening to be made securely to the struts coming on it; it only takes on the king-post in the present instance such load as the strap would have to bear if the bar broke. This would require a sectional area of only $\frac{1}{2400} = \frac{1}{240}$ square inches. It is not advisable to have straps so thin, as any vibration of the truss would have a most weakening effect. Before arriving at $\frac{1}{4}$" the value suffers, yet it would be best to give $\frac{1}{4}$" the smallest thing given as it is the same room for the other joints and the thickness is about the possible minimum.

The strap at the head of the king-post is not always given. Its function is to prevent any sliding of the post from shrinkage of the timber. It bears only $\frac{1}{2400}$ and fastened by $\frac{1}{4}$" bolts.

The above rules of the truss have been made on the assumption that the rules for tension and compression bars are applicable. But it must be remembered that these rules only apply properly when the stresses are exerted on the members concentrically with their axes. In the roof many cases of trusses this is far from being the case. Fig. 111 shows a common but faulty form of construction. It will at once be seen that the stresses are not applied concentrically. Fig. 112 shows the truss with these defects remedied.

It must be noted that in the calculations of this truss we have considered an almost excessive load distributed, too, in a way which is

economical than is usually the case. Yet it works out to a much re economical roof than that given by Tredgold.

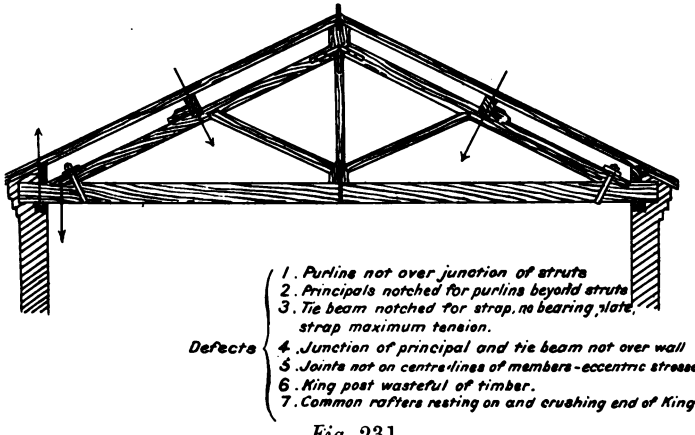

Defects
1. Purlins not over junction of struts
2. Principals notched for purlins beyond struts
3. Tie beam notched for strap, no bearing plate, strap maximum tension.
4. Junction of principal and tie beam not over wall
5. Joints not on centre-lines of members - eccentric stresses
6. King post wasteful of timber.
7. Common rafters resting on and crushing end of King.

Fig. 231.

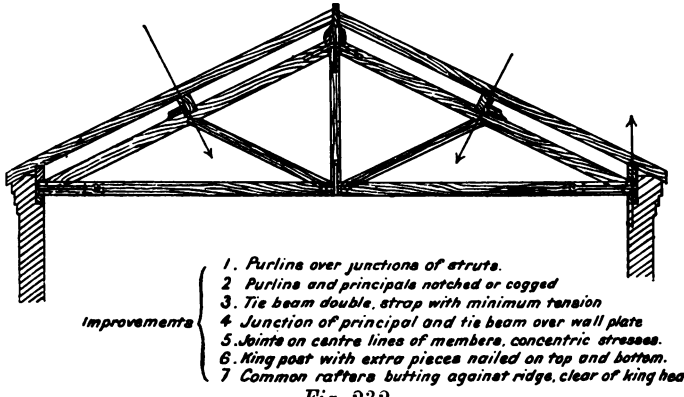

Improvements
1. Purlins over junctions of struts.
2 Purlins and principals notched or cogged
3. Tie beam double, strap with minimum tension
4 Junction of principal and tie beam over wall plate
5. Joints on centre lines of members, concentric stresses.
6. King post with extra pieces nailed on top and bottom.
7 Common rafters butting against ridge, clear of king head.

Fig. 232.

For purposes of comparison, the scantlings given for the above ıss by Tredgold are given side by side with those just calculated.

	Tredgold.	As Calculated.
Common rafters ...	$2'' \times 4\frac{1}{2}''$	$2'' \times 4''$.
Purlins	$5\frac{1}{2}'' \times 8\frac{3}{4}''$	$5'' \times 8''$.
Principals ...	$3\frac{1}{2}'' \times 6''$	$5'' \times 6''$.
Tie beam ...	$6'' \times 11\frac{1}{2}''$	$5'' \times 8''$ or $3'' \times 9''$.
Struts	$4\frac{1}{2}'' \times 2\frac{3}{4}''$	$5'' \times 4''$.
King post ...	$4'' \times 6''$	$5'' \times 2''$.

It is not clear how Tredgold proposes practically to frame the members he gives together. For instance, it would not be possible to make a good joint between a principal $3\frac{1}{2}'' \times 6''$ and a strut $4\frac{1}{2}'' \times 2\frac{3}{4}''$.

EXAMPLE 37.—*To show how the covering and timber cause variation, let us take the same span, slope and spacing of trusses, but let the timber be sâl* (shorea robusta) *and the roof covering double Allahabad tiling. No ceiling.*

The common rafters support the tiles on $1'' \times 1\frac{1}{4}''$ battens, the rafters themselves being at 2′ intervals. For strength $bd^2 = L^2Sw \div 166$.* The weight of the roof covering and workmen = 52 lbs. per square foot. Hence $bd^2 = 64 \times 2 \times 52 \div 166$. Make $2'' \times 4\frac{1}{2}''$.

Purlins.—As sâl weighs 55 lbs. per cubic foot, the increment of weight per square foot for the common rafters

$$= 5 \times 16 \times 2'' \times 4\tfrac{1}{2}'' \times 55 \div 10 \times 16 = 1{\cdot}7, \text{ say } 2 \text{ lbs.}$$

Hence w = about 55 lbs., allowing 1 lb. extra for nails, etc.

For strength—

$$bd^2 = L^2Sw \div 166 = 100 \times 8 \times 55 \div 166 = 268.$$

Make $5'' \times 7\frac{1}{4}''$ (see Table XXII., p. 155).

Principal.—Weight on one principal :—

Roof covering, battens, and common rafters	$16 \times 10 \times 55$ =	8800 lbs.
Purlin $10 \times 4\frac{1}{2}'' \times 6\frac{1}{2}'' \times 55$ ..... =		111 „
Add for principal 120 lbs. (say) and ironwork 20 lbs. ... =		140 „
	W =	9051 lbs.

Hence thrust on heel = $1\frac{3}{8}$W cosec $\theta = 7520 \times 2{\cdot}28 = 16{,}800$ lbs.

Assume 4″ as one dimension, then $l : h :: 96 : 4$, *i.e.*, $24 : 1$.

Hence $P = \frac{1}{2}r_cA$ (Formula 4, Chapter IX.).

From Table XIV. $r_c = 6{,}000$ ultimate = 1,200 safe lbs. per square inch.

Hence $16800 = 600 \times A$, *i.e.*, $A = 28$.

Make $7'' \times 4''$.

Make tie beam $4'' \times 6''$, struts $4'' \times 4''$, king post $4'' \times 3''$.

It is unnecessary to show the full calculations for these members, as there is no difference in principle from the example previously worked out.

* See page 139.

King-post Truss without Common Rafters, and several Purlins.

In this case the principal rafter is under a combined compressive and direct stress. It is not sufficient to calculate it for one and for the other, and take the larger of the scantlings thus found: it must be considered on the same principles as Example 34.

As the load is no longer transmitted by common rafters, continuous over two spans, the net reaction at the heel of the principal is not $\frac{1}{16}$W, but $\frac{3}{4}$W, hence the direct stress on the principal is $\frac{3}{4}$W cosec θ, the tension on the tie beam is $\frac{3}{4}$W cot θ, and the compression on the struts is $\frac{1}{2}$W cosec θ.

EXAMPLE 38.—*Design a truss for span of* 24′, *distance apart of trusses* 8′, *slope* 30°, *timber spruce* (abies excelsa), *roof covering corrugated iron* 18 *W.G. on purlins* 2′ *apart.*

The values for r and E for spruce are 5,000 and 1,575,000 respectively. The weight of corrugated iron 18 W.G. = 2 lbs. per square foot. Total weight per foot on roof is

$$35 \text{ lbs. (workmen)} + 5(\text{snow}) + 2 = 42 \text{ lbs.} = w.$$

Purlins.—

$l = 96$ inches, *i.e.*, distance apart of trusses.

$$W = 8 \times 2 \times 42 = 672.$$

$Wl \div 8 = rbd^2 \div 6$, *i.e.*, $672 \times 96 \div 8 = 5000bd^2 \div 5$ (factor of safety) $\times 6$.

$\therefore bd^2 = 48$. Make 3″ × 4″.

Principal.—Length of one rafter = $\frac{1}{2}$ span × sec 30° = 14 feet nearly. Weight on one rafter—

Roof covering and load $14 \times 8 \times 42$	= 4704	lbs.
Purlins $7 \times 8 \times 3'' \times 4'' \times 30$ lbs. (weight of spruce)	= 140	,,
Principal (say) $6'' \times 4'' \times 14 \times 30$	= 70	,,
Ironwork (say)	= 20	,,
Total W	= 4934 lbs., say 5000	

$P = \frac{3}{4}W \text{ cosec } \theta = 7{,}500$ lbs.

Now the M_{ff} in a continuous beam of two spans = $W'l \div 8$, where W' = weight on one of the spans = $\frac{1}{2}$W, as found before = $\frac{1}{2}5000 = 2500$. $l = \frac{1}{2}14 \times 12'' = 84''$. Hence $M_{ff} = 2500 \times 84 \div 8 = 26{,}200$ inch lbs. If r' = stress intensity from the transverse load = $M_{ff} \times 6 \div bd^2$ and

[illegible] stress intensity from the direct compression $= \frac{1}{A} + \frac{Mc}{I}$, then [illegible] must not exceed the safe intensity for stress $= 1000 - 3 = 1000$.

$$\therefore 1000 = \frac{\frac{1}{4} \times 25200}{4^2} - \frac{7500}{4}.$$

This will be satisfied if we make the principal $\frac{5}{8}$".

We might have taken the more accurate formula

$$= M_1 + \frac{1}{2} - \frac{Pr}{3IE}.$$

which would have given the same result practically.

The stress in the tie beam, struts, and king post would be found in the same way as in the last example. These are under direct stress only, unless there is a ceiling, or other load on the tie beam. It is not necessary to investigate these as the principles involved have been already illustrated.

Queen-post Truss.

When the span exceeds 30', the half-span 15' becomes inconveniently long for a ceiling, or with moulded trusses, when the span exceeds 35', the span of the common rafters become nearly 10' between supports. Under those circumstances it is better to use a queen-post truss. This span can be used up to 50' without necessitating very heavy timbers in ordinary cases. This form of truss is also convenient when it is desired to put an attic room above the level of the tie beam, or when it is considered desirable to have a flat part on the top.

The truss is shown in *Fig.* 233, and the corresponding stress

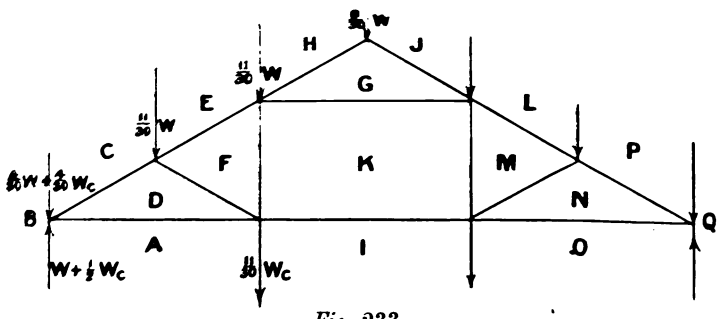


Fig. 233.

diagram in *Fig.* 234. From the latter it will be seen that if we con-

sider, as before, the common rafter as a continuous beam, and if the weight on one principal = W, and the weight on tie-beam = w_c, the direct stresses on the various members may be found from the following values:—

Compression on principal

$$= (\tfrac{13}{15}W + \tfrac{11}{30}w_c) \operatorname{cosec} \theta.$$

Tension on tie beam

$$= (\tfrac{13}{15}W + \tfrac{11}{30}w_c) \cot \theta.$$

Tension on queen posts

$$= \frac{W^*}{6} + \tfrac{11}{30}w_c.$$

Compression on struts

$$= \frac{W^*}{6} \operatorname{cosec} \theta.$$

Compression on straining beam

$$= (\tfrac{11}{20}W + \tfrac{11}{30}w_c) \cot \theta.$$

Compression on straining sill

$$= \frac{W^*}{6} \cot \theta.$$

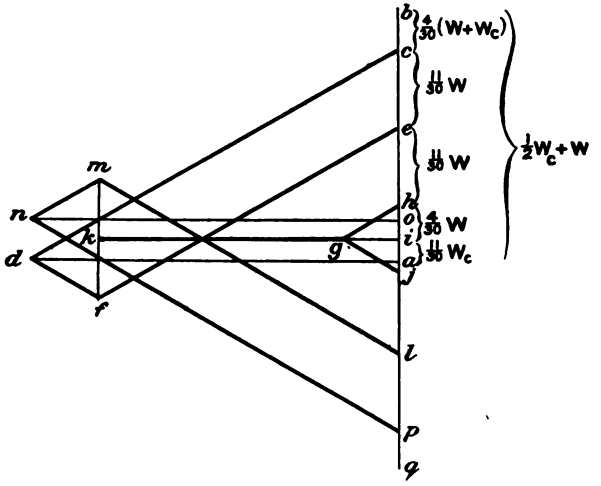


Fig. 234.

* Accurately $\frac{11}{60}W$

As in the case of a king-post truss, so, even more commonly, in the case of the queen post, the common rafters butt against the ridge. The upper part of the principal is usually omitted, and the lower part of the common rafters come on the pole plate, which rests usually on the wall or on the tie beam beyond its junction with the principal. The distribution of load in such a case is approximately as shown in *Fig.* 235, and the stress diagram in *Fig.* 236. These are on the same scale as *Figs.* 233, 234, and the resulting stresses are, it will be seen, rather less than those given above. There is no doubt, however, that they are quite as accurate.

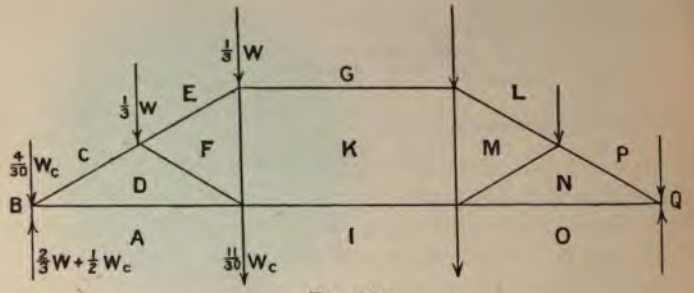


Fig. 235.

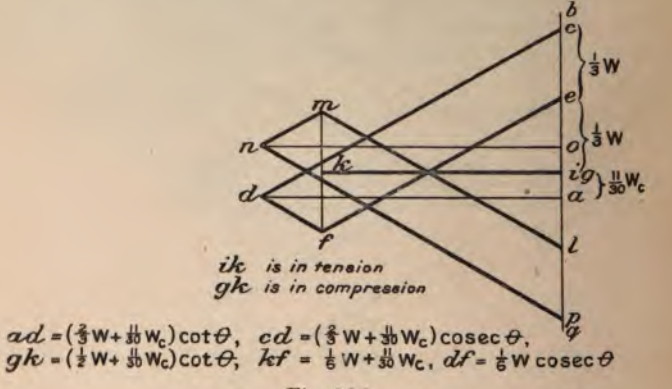


Fig. 236.

No example of a roof of this form is necessary, as the method of working out is the same as in former cases.

Other forms of timber trusses are unusual, except with curved ribs, which we shall consider in the next chapter. It is *possible* to build up trusses of very light scantling, somewhat in the form shown in *Fig.* 237, taken from an actual case. This roof can safely bear 40 lbs. per square foot. Such roofs would only be used in temporary structures, such as huts or sheds, but would be very useful in the field.

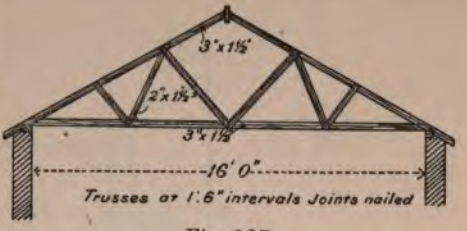


Fig. 237.

Hip and Valley Rafters.

Before leaving the subject of wooden roofs, it is necessary to say a few words on the subject of hip and valley rafters. As has already been mentioned, hip rafters are, as a rule, only inclined ridge boards under no stress. Where this construction is adopted there is an outward thrust on the walls at the angle from the jack rafters, and from the end of the hip. This is counteracted by the bonding of the masonry at the angle, and by angle braces connecting the two parts of the roof that are at right angles to each other.

With valley rafters the case is not quite so simple. There the valley rafter supports a portion of the roof indicated by the shaded part in *Fig.* 238. The valley rafter is, therefore, a beam under a uniform transverse load.

The arrangement for supporting the purlins at a hipped end is not always easy. These purlins are below the hip, and that portion of the purlin at right angles to the main line of the roof is sometimes supported by a half truss, as indicated by AA in *Fig.* 239. This half truss is often omitted, and the end purlin supported by the ends of the longitudinal purlin. This is, therefore, a cantilever loaded at its extremity with a load equal to the shaded portion in *Fig.* 239, *i.e.*, $\frac{1}{8}$ total weight of the end bay, and a uniformly distributed load = ABCD. The ordinary scantlings for purlins are generally sufficient to take this safely.

It is sometimes not very easy to arrange the trusses in a roof with many salient and re-entering angles. The trusses should be arranged so as not to come vertically over door or window openings. *Fig.* 240 shows a plan of a building of this description, and *Fig.* 241 shows

the proposed arrangement of roof timbers. The common rafters are omitted from this figure for the sake of clearness.

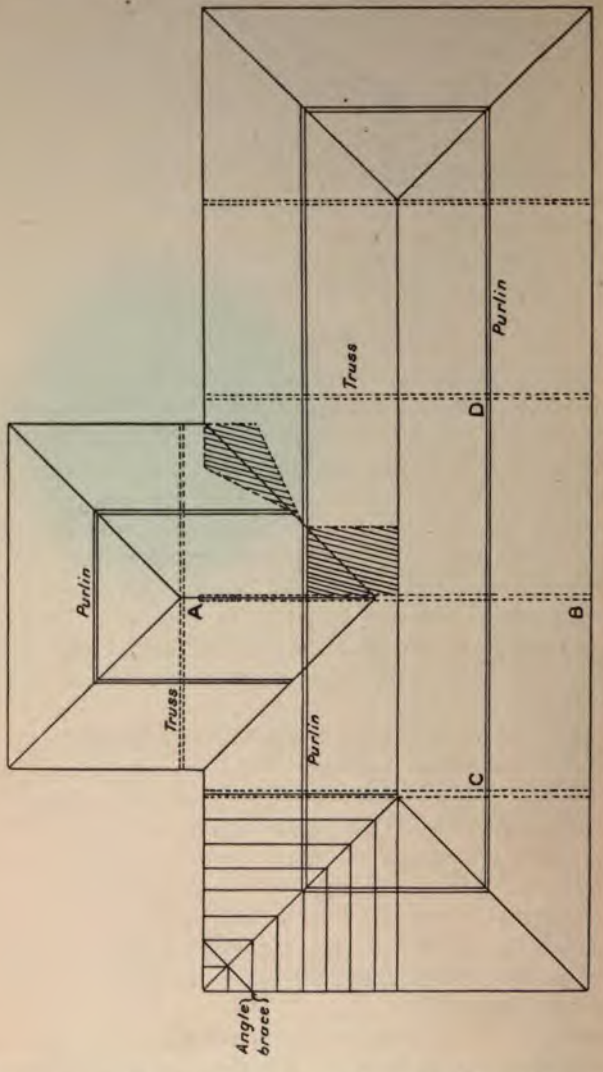


Fig. 238.

In any case the roof must be made to fit the walls, not the walls altered to suit the roof.

In a house with many partitions it is often advisable to continue the partitions, if built of sound materials, to the level of the purlins, and dispense with trusses.

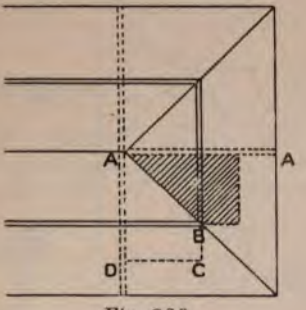


Fig. 239.

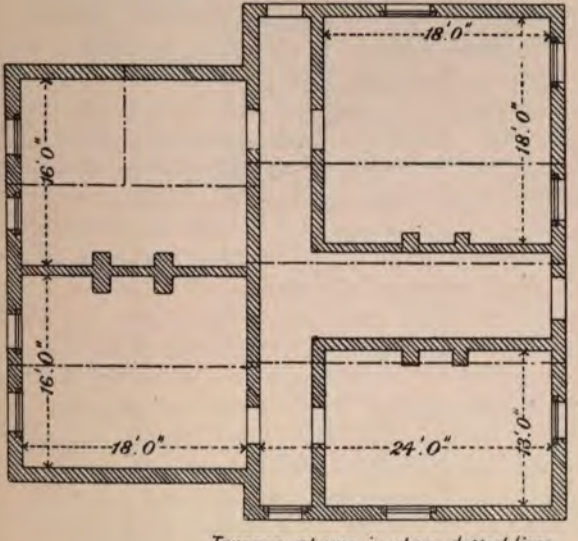


Fig. 240. *

* This is an ordinary case of a dwelling house. The plan is not selected because it is a good one. Instead of using trusses and purlins in the left portion it would have been equally good to have used collar beams. Where a place more or less square has to be roofed, valleys may be avoided by having a lead-covered flat in the centre.

On all plans of dwelling houses the portions of trusses should b indicated on the same drawing that shows door and window open ings, and chimneys.

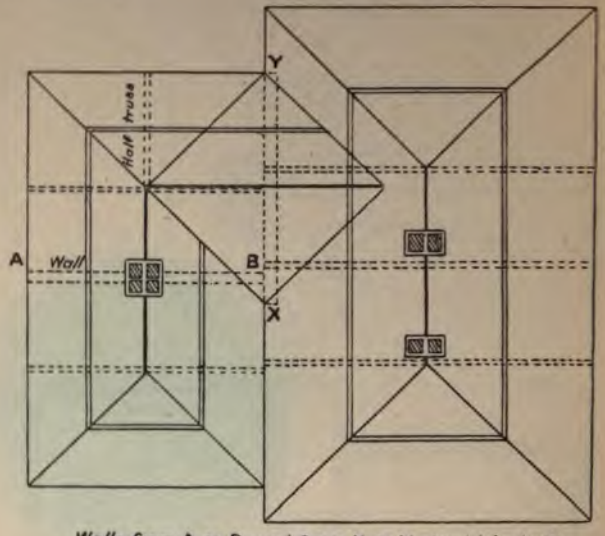


Fig. 241.

CHAPTER XII.

Roofs (Timber Ribs, Iron and Steel).

Wooden Roofs with Curved Members.—Hammer Beam Trusses.—Composite Trusses.—Iron and Steel Roofs.—Method of Sections.—Uses of Stress Diagrams in Modifying Design.—Application to Cantilevers.

In this chapter it is proposed to consider some forms of roofs other than those already investigated. Those which it is now proposed to consider are:—(1). Timber roofs with curved members. (2). Composite roofs of wood and iron. (3). Iron or steel roofs.

Wooden Roofs with Curved Members.

It is sometimes possible to build up the curved ribs of several pieces, either of vertical planks of short lengths breaking joint, or horizontal planks of considerable length bent to the required curve. In either case the method of investigation is the same, although there are practical differences in the construction. Into the latter it is not intended here to enter, nor is it proposed to consider fully the theory of loads upon curved ribs. This theory will be subsequently considered (in Part II.) in connection with suspension bridges and arches. It is sufficient for our present purposes briefly to indicate the method of investigation proposed by Prof. Rankine for ribs under any form of load. This investigation is as follows:—

Let AB (*Fig.* 242) be a curved rib of any form, and loaded with a vertical load distributed in any manner. To find the total horizontal pressure against the rib below a given point C, we may draw a vertical line *ob* (*Fig.* 243) to represent a scale of loads, taking $oh = P$ to represent the vertical load supported on the arc AC. Through *h* draw the horizontal line *hc*, cutting *oc* in *c*, where *oc* is parallel to the tangent at C, then $oc = T$ will be the thrust along the rib at C, and $hc = H$ will be the total horizontal pressure which must be exerted against CB, the part of the rib below C.

Now if C′ be a point in the rib a short way below C, and if on the diagram of forces *oc′* be drawn parallel to the tangent at C′, and on the scale of loads *oh′* be taken equal to the vertical load on arc AC′, then the horizontal line *h′c′* represents the pressure which must be exerted against the part of the rib below C′. The difference

H – H′ will represent the horizontal pressure required to be exerted along the rib from C to C′. If H diminishes in going downwards, pressure from without, or tension from within, is required, in order to preserve the equilibrium. If H increases in going downwards, tension from without or pressure from within is required.

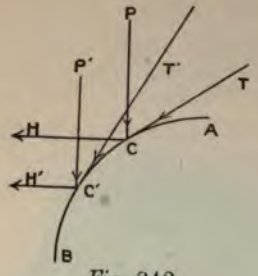


Fig. 242.

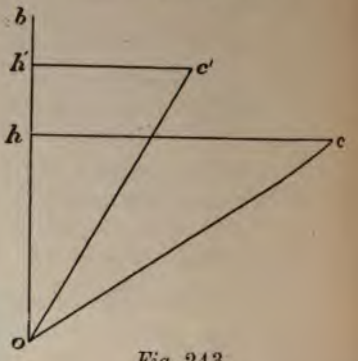


Fig. 243.

Applying these principles to the case of a rib built up of vertical or horizontal planks, as shown in *Fig.* 244, where the roof consists of purlins passing horizontally from rib to rib, we see from *Fig.* 245

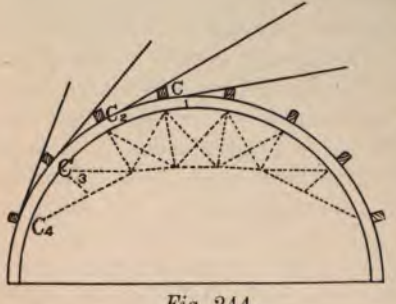


Fig. 244.

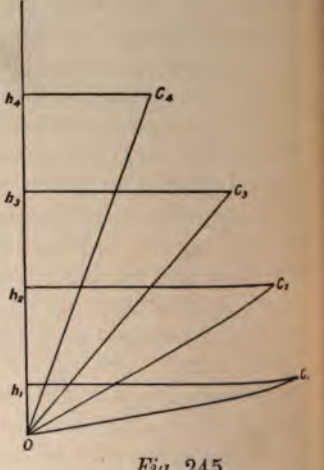


Fig. 245.

that, for equilibrium, ⌜pressure from without, or tension from within, is required, and that, although the timber itself may supply

a certain amount of that required pressure or tension, yet the structure will be in very unsuitable equilibrium, and would require bracing, as shown in the dotted lines in *Fig.* 244.

This form of structure is not uncommon, but it is not to be recommended, as it depends largely on the efficiency of the bracing, introducing a new material into the structure. A better and more common form of construction with curved ribs is that shown in *Fig.* 246, taken from an actual case. In this case the pressure is brought upon the ribs at two points equi-distant from the centre by means of two principal rafters tangential to the rib at those points. The effect of the load coming on these two principal rafters is not only to supply outward pressure, but to stiffen the whole structure, so that in the event of one side of the roof being loaded, any tendency to deformation is taken by the rafters on the other side. We may, in fact, without sensible error, consider the portion ADE as a truss, and BAC as an inclined prop in two parts BA and AC. In such a case the upper portion of the rib from A to E would be in tension, but would be prevented from sagging by the vertical tension bars from the ridge. The vertical member BC is not necessary, provided the walls of the building are sufficiently substantial to bear the lateral thrust coming upon them, but that vertical portion is frequently included for the purpose of uniting the foot of AC with AB.

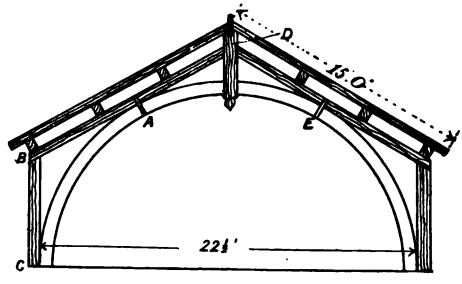


Fig. 246.

Hammer Beam Trusses.

This form of truss is frequently met with in buildings where architectural effect is a matter of importance. It is common in Gothic architecture; a well-known example being the magnificent roof of Westminster Hall, built in 1396. The absence of a tie beam enables a window, pictures, or other ornament at the end of the building to be fully seen, and the curved members have the effect of relieving the stiff appearance produced by the ordinary timbers in a

roof. *Plates* IV. and V. show the application of such a truss to (Example 39) a span of 50 feet. It is obvious that such a truss would not be used where walls are relatively low, for in such a case the building would appear "top heavy," nor with a slope of roof less than 45°. Such a truss, in fact, is intended for use in a building of lofty height, and generally of dignified appearance. In the case we are considering the walls are 30′ in height, the trusses are 10′ apart, and the slope is 45°. It will be seen that the absence of a tie rod joining the two lower joints causes a tendency to spread laterally, which is generally counteracted by buttresses placed outside the walls. These buttresses, again, are susceptible of architectural treatment, and form, as is well known, a feature of Gothic architecture. Where such a thrust is furnished by the buttresses, and the load is equally distributed all over the truss, all the members are not necessary, and some of them, such as IG, GW, MN, and WN (*Fig.* 1, *Plate* V.) are without any stress, but if the load is not equally distributed all over (as when wind is blowing on the roof from one side or the other, a matter which will be presently investigated), most of these members become subject to a certain amount of stress. The effect of the curved members BW, WG, WN, WU is the same as though they were straight, as indicated in chain-dotted lines. The curve in these timbers is given for architectural effect, and is not intended to indicate any advantage from a statical point of view. If the abutments or walls are capable of furnishing all the horizontal thrusts required, it will be seen that we have really a king-post truss, 2, 3, 4, supported on two inclined frame struts, shown by the dotted lines 1, 2 and 4, 6. We have now, therefore, to find what is the amount of horizontal thrust furnished by the abutments at 7 and 6, and we can do so in either of the following ways :—

(1). We see that if the whole load transmitted from the purlins to the truss on *either* side of the roof is equal to W, the truss 2, 3, 4 (*Fig.* 1, *Plate* V.) will have to bear W, and $\frac{1}{2}$W will be supported at each of the joints 2 and 4. Then $\frac{1}{2}$W rests upon 1—2 and upon 4—5, hence at the joint 2, $\frac{1}{2}W + \frac{1}{4}W = \frac{3}{4}W$ has to be supported on the inclined strut 7, 2. Draw *ab* vertically $= \frac{3}{4}$W (*Fig.* 4, *Plate* V.); draw *ag* parallel to the dotted line 7—2. From *b* draw *bg* horizontally; *bg* then represents the horizontal thrust at 7 or at 6. In the case in point, allowing a weight of 50 lbs. per square foot, *bg* equals 7,000 lbs.

(2). The second method is to multiply $\frac{3}{4}$W by the perpendicular

istance of 2 from AB, and divide by the height of the Point 2 above —6. This is the same as taking moments around Point 7.

We are now in a position to draw the stress diagram for the hole truss. For the joint at 7 we have three forces, viz.: the clined strut 7—2, the curved rib BW, and the vertical member or art of the wall AB. By this time we know the magnitude and rection of inclined thrust WA (=*ag* in *Fig.* 4), and we know the rections of the other two. Therefore we can draw the stress agram *wa*, *ab*, *bw* (*Fig.* 2, *Plate* V.). Then passing to the joint 1 we have *ba*, *ac*, *cd*, *db*, and so on in regular sequence, obtaining timately the stress diagram shown in *Fig.* 2, *Plate* V. An vestigation of this figure shows that most of the members are in mpression, the exceptions being the beams *bd*, *su*, and *lk*. The nounts of compression or tension in any members are comparatively nall.

If, however, we were to omit the curved ribs WB and WU, the fect would be that there would be no horizontal thrust brought pon the walls, but a large increase of stress upon the other members the truss. In such a case *lk* would have the greatest tension, and VG and WN, which previously were without stress, would now ave very considerable tension. The stress diagram is shown on ig. 3, *Plate* V. This shows the necessity of securing the joint at ne junction of WB and WG.

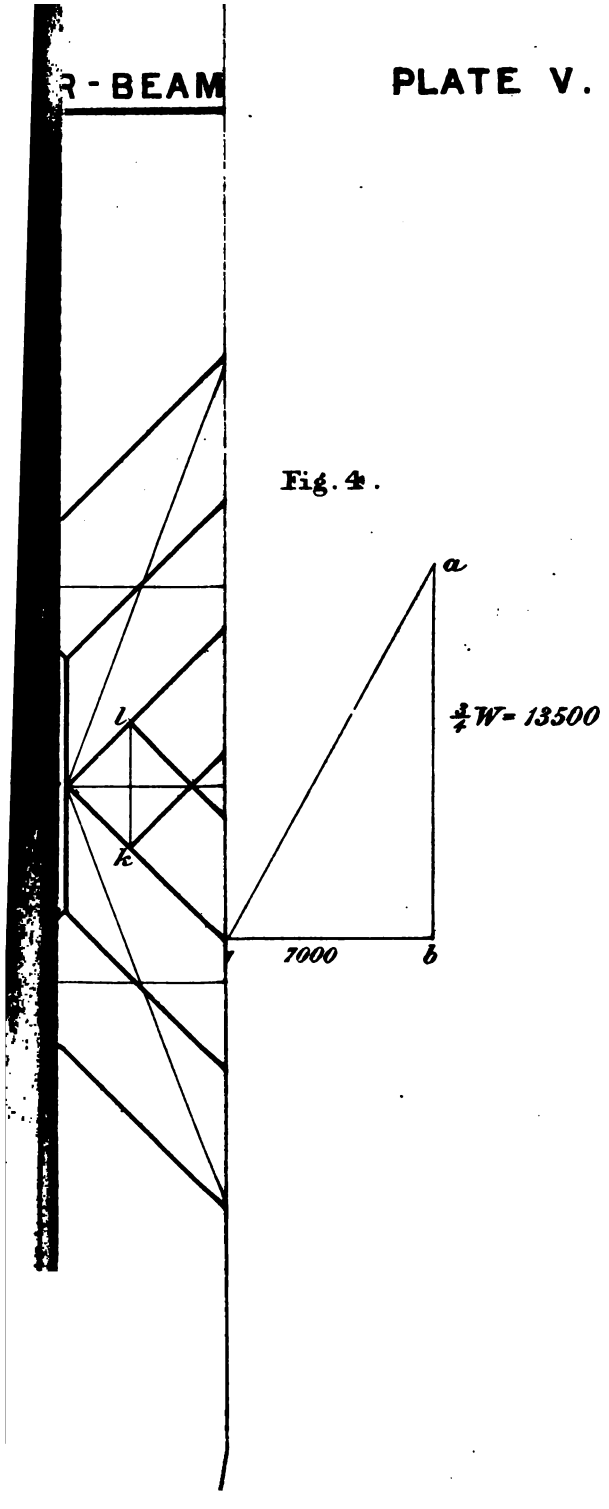
We have now to consider the effect of wind pressure upon such a tructure. Assuming that the building has been constructed on some ofty eminence where the friction of the ground has no retarding effect pon the wind pressure, which may be taken at 50 lbs. per square oot on a thin vertical plate, we see that the amount which will be ormal on such a roof will be very small indeed, for it is evident hat all that we need take account of is the positive front pressure, ince there is no possibility of a negative back pressure acting on he other side, *i.e.*, the interior of the building. The amount, herefore, of horizontal pressure on a vertical plane that we should ave to consider would be about 30 lbs. per square foot. From *Fig.* 11, p. 236, we see that the percentage of normal pressure on a 45° lope would be about 25 per cent, hence only about 8 lbs. per square oot would be the amount required. The stress diagram for wind ressure is shown in *Fig.* 2, *Plate* VI. In it we see that the bars *wg*, *d* and *lk* are in tension, the others either in compression or have no tress acting upon them at all.

The same effect, though to a greater extent, would be produced if

R-BEAM

PLATE V.

Fig. 4.



resses on the various members differs in no way from that which is been previously considered.

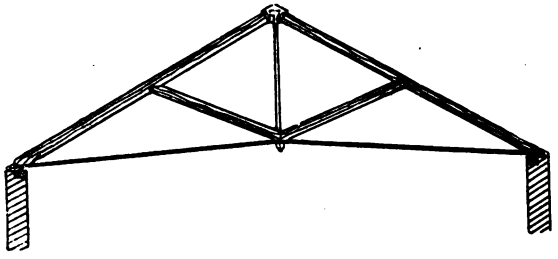


Fig. 247.

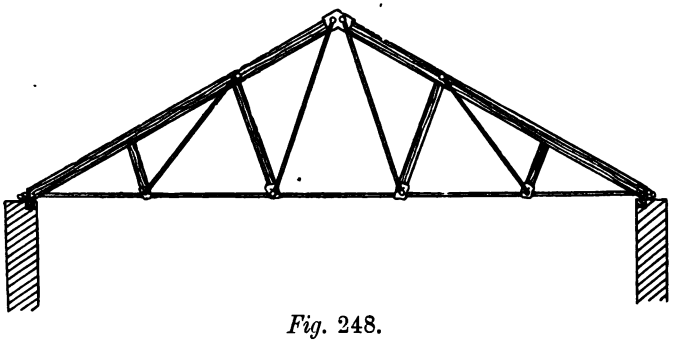


Fig. 248.

Iron and Steel Roofs.

Trusses formed of iron or steel can often be obtained from manufacturers ready made, by specifying the span, slope, distance apart of trusses and nature of roof covering. It is, however, most important that the principles on which these trusses are designed should be thoroughly understood, because, as will be seen, slight differences in design lead often to very great economy. It is necessary, therefore, to devote attention to this subject.

As iron or steel expands or contracts considerably under various changes of temperature, and as the span of iron or steel roofs is sometimes considerable, it is necessary to make some arrangements for allowing this expansion to take place at one end of the truss. Neglect of this precaution has sometimes led to very serious difficulty. The internal stresses which have been set up by the expansion of the metal and by the rigid attachment of the ends have been such

that the stresses in various members have been reversed. Tie bars have buckled under compressive force, and joints have shown a tendency to fail. In very large spans, such as those for railway stations, it may be necessary to place friction rollers under one end of the trusses, but in most ordinary cases, say up to 50 or 60 feet, it is sufficient if the free end of the truss terminates in a smooth horizontal plate resting upon another horizontal plate on the top of a wall, that end of the truss being free to slide, the other end being fixed.

In all roofs the temporary load may be brought upon one side only. Formerly, it was customary to consider this temporary load as produced by wind pressure only, but we have seen that in roofs up to 30° slope the wind pressure may be entirely neglected, and that for slopes up to 45° or 50° the vertical component of the wind pressure is comparatively small. On the other hand, it is quite possible that the whole of one side of the roof may be covered with workmen, either repairing or renewing the roof covering,* or removing accumulated snow. The effect of such partial loading will be to produce stresses upon the members of the roof which are not the same as would be produced by uniform loading all over. This subject has not been alluded to in connection with the ordinary forms of wooden roofs because the timber members which will be necessary for a uniform load all over would be equally capable of bearing an unsymmetrical temporary load + uniform permanent load. In the case of iron or steel roofs the matter is somewhat different, because these members must be designed for either tension or compression, and if it was found that any one member has to bear sometimes one force of stress, sometimes the other, the most suitable form of cross section must be adopted, so that either tension or compression may be equally well borne.

In the case, therefore, of iron roofs, we must consider the roof under at least four different aspects :—(1). Roof covered with a uniform load, permanent and temporary. (2). A roof covered with a permanent load all over and a temporary load on one side, both ends being fixed. (3). A roof loaded, as before in (2), on that side which

* The writer had on one occasion to renew the covering on one side of a large troop stable, where the slates had been laid with insufficient lap to prevent leakage when exposed to certain winds. The weather was bad, and it was a matter of urgent necessity that the horses should be kept out of the stable as short a time as possible. Every available workman was, therefore, put on the roof, which was thus partially loaded with a very considerable weight on one side.

free to move. (4). Roof loaded, as before in (2), but load on the posite side from that which is free to move. Stress diagrams must made out for each of these four cases, the results examined and ssibly tabulated, and the maximum stresses, whether in tension or mpression, may be adopted as the active loads which each member ust be designed to resist.

As regards the design of members, it is customary in the case of nsion bars to use either round or flat bar-iron. There is no object using any other form. If rivetted connections are employed, flat rs are commonly made use of, but if the connections are pin and ık, or gib and cotter, round bars are generally adopted. Compres-on members, however, may be of various forms. It has been pre-ously pointed out that both with tension and compression members is highly desirable that the member should be loaded concentri-lly with the axis of the member. Thus, a single angle iron is an nsuitable form for a compression member, and even a T-iron is not ite perfect. T-irons, however, are frequently used for principal fters, because the table of the tee gives a convenient place for the ring of purlins, and angle irons are frequently used as purlins, pecially with tiles or corrugated iron. Double bars of flat iron, nnected at intervals, as in Example 29, form a very convenient d economical strut, but are limited in their application. Channel ons, arranged in pairs, are sometimes used, but more frequently in ridges than in roofs.

In order to find the reactions produced by an unsymmetrical load, e have to consider the three cases of (1) both ends fixed; (2), loaded fixed end; (3), loaded at free end. In the first case, the whole rce coming one side of the roof may be considered to be uniformly istributed over that side and acting normally. The reactions at oth walls will be in direction parallel to this force, and inversely roportional to the distances AC, CB, C being the point where the ne of the resultant force intersects the horizontal span AB (*Fig.* 249). n the second case the reaction at the free end will be vertical, and us the reaction at the fixed end will be found by producing the ne P until it cuts the vertical AD at the point D. DB will then be e direction of the other reaction (*Fig.* 250). In the third case it ill be similarly obtained (*Fig.* 251). The stress diagrams for the ur cases are illustrated in *Figs.* 2, 4, 5 and 6, *Plate* VII.

In considering the various forms of iron roofs, we have to bear in ind two facts: first, that the length of the struts should be made as hort as possible; secondly, that the load which is brought upon the

strut should be as direct as possible. Hence if we load a strut, indeed any member, with a combined direct and transverse load, may obtain a stress intensity at one side or the other which v necessitate a larger section than is usually obtainable in the mark

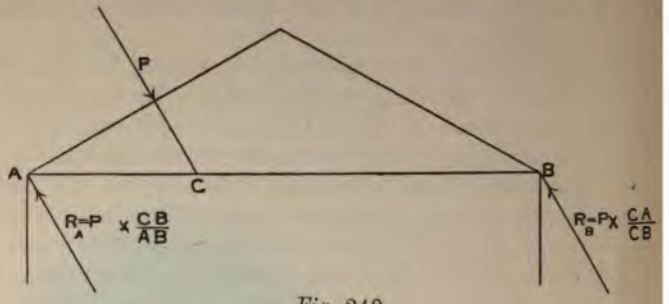


Fig. 249.

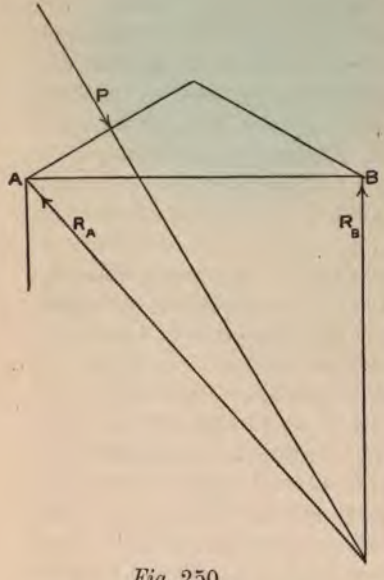


Fig. 250.

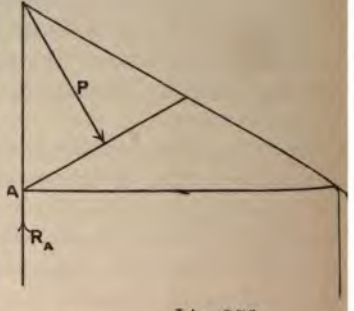


Fig. 251.

We have also to bear in mind the fact that unnecessary multipli tion of tension members, which may be theoretically advisable not practically economical, because there are limits to the mar sizes of bars which can be used.

Summarizing these rules, an iron truss should be designe

), with short struts ; (2), with the members under direct, and not mbined, stress ; (3), with all members, whether in tension or com- ression, under such stresses as may be easily met by market sections metal. To these we may add (4), the struts should be arranged as to have their axes in prolongation of the stresses brought on em. This is, of course, a matter which has already been con- dered in connection with the resistance of columns, but may here gain be alluded to. As an example, we may take the roof shown *Plate* VII.

EXAMPLE 40.—*Design a roof for a span of* 28′ *in the clear, of the rm shown in* Plate V., *distance apart of trusses* 8′, *slope* 30°, *covering ntless slates on* $\frac{3}{4}$″ *boarding, resting on purlins at* 2′ *internals. Material ild steel* (*Siemens-Martin*).

This form of roof is very common. It has the disadvantage that e principal rafters are under combined stress, but otherwise it ful- s most of the conditions for a good form of truss.

As the truss rests on walls, it will be necessary to add to the span, given in the clear, a certain amount on each side to allow for span tween centre to centre of bearings. It will be sufficient if we add on each side.* Hence span for calculation = 28 + 2 = 30′.

The weights coming on the roof are as follows :—

Roof covering 10·5 lbs. Purlins about 2·5 lbs. Increment for ight of the truss itself (approximately, taken from Hurst's ndbook) about 2 lbs. Total, 15 lbs. per square foot permanent d. Temporary load, workmen + snow = 40 lbs. per square foot rizontal = about 35 lbs. per square foot as measured on the slope. The total weight per square foot, therefore, may be taken at lbs.

The length of one principal rafter is nearly 17′; hence W = total ight on one side = 17 × 8 × 50 = 6800 lbs. Of this $\frac{1}{2}$W is borne the centre = 3400, $\frac{1}{2}$W at the apex = 3400, and $\frac{1}{4}$W = 1700 at the ves, while the reactions at each support = $\frac{3}{4}$W = 5100 lbs.

With these data we draw the stress diagram shown in *Fig.* 2, *ate* VII. The stresses for the permanent load alone can be deduced om this diagram by either taking another scale of forces, as given *Plate* VII., or by direct proportion of the weights 50 : 15. (On a ide rule these can, of course, be read off at once).

We now have to consider what would be the effect of the temporary

* This would allow a bearing of 2′ on each side. With a roof of this span is would be an excessive allowance, but it would be on the safe side.

load being brought on one side only. As a matter of fact, we shall find that the stresses produced by the permanent load + the temporary load all over are greater than any of the combinations of unsymmetrical loads + permanent load, but, as an example, it is as well to illustrate the method of investigation. Taking first the ends as fixed, we have to consider what is the total amount of oblique load, and how much of that oblique load is borne at each support. As regards the amount, we have seen that the total vertical load (temporary) from workmen and snow = 40 lbs., but if this be resolved normal to the roof, it will be reduced. If we take 30 lbs. as the normal pressure per square foot, it will probably be sufficient for the purpose, at least, of demonstrating the point at issue. The total force, therefore, on one side will be $17 \times 8 \times 30 = 4080$ lbs., of which $\frac{1}{2}$ acts in the centre, and $\frac{1}{4}$ at each end of the principal. The reactions in the first instance are found graphically by producing the central force till it cuts the line BL at X (*Fig.* 3, *Plate* VII.). Draw BY in any direction equal on any scale to 4080. Join YL, and from X draw XZ parallel to YL, then BZ = reaction at L and YZ = reaction at B. Having these reactions, the stress diagram shown at *Fig.* 5 can be drawn.

If we consider the oblique load to be acting on the fixed end, the other end being free to move, we see that the reaction at L will be vertical, the whole of the horizontal component of the oblique pressure being taken up by the reaction at the fixed end. Hence the resultant normal pressure and the two reactions will when produced meet in a point, as shown by the chain-dotted lines in *Fig.* 3, *Plate* VII., and a triangle with sides parallel to these directions will represent the various amounts, thus producing equilibrium. Draw *hb* parallel to the normal oblique force, and = 4,080 lbs. on any scale, then *ba* and *ah* (*Fig.* 4, *Plate* VII.) will represent the reactions at the points of support, if drawn parallel to directions of those reactions. With this polygon of loads the stress diagram can be easily drawn, and is shown in *Fig.* 4.

By a similar procedure the stress diagram for the loads when coming on the side of the roof where the free end is can be drawn and is shown in *Fig.* 6, *Plate* VII.

We have now to see whether any combination of the stresses on any of the diagrams of oblique load + the stresses on the diagram for the permanent load, are greater than the stresses for the permanent and temporary load all over, as in *Fig.* 2.

For convenience of reference, the loads are written on the bars;

ıse in *Fig.* 2 being both for temporary + permanent loads, and for manent loads only, according to the scales as shown below.

By examination of these we see that the maximum load in every ıe is when the roof is loaded *all over* with permanent and temporary d, and we also see that the effect of oblique loading is in no case ;h that it changes the nature of the load from tension to com-ession or otherwise.

We may therefore safely calculate the members upon the maxima esses shown in the diagram (*Fig.* 2, *Plate* VII.).

Tension Bars.—The stresses are as follows:—

$$AD = 16{,}000 \text{ lbs.} = 7{\cdot}14 \text{ tons.}$$
$$FG = 8{,}000 \text{ lbs.} = 3{\cdot}57 \text{ tons.}$$
$$AG = 8{,}400 \text{ lbs.} = 3{\cdot}75 \text{ tons.}$$

llowing 8 tons per square inch safe tension for Siemens-Martin ɜel, we have for AD, $\frac{\pi d^2}{4} \times 8 = 7{\cdot}14$, whence $d = 1{\cdot}05$, say $1''$. milarly FG $= \frac{3}{4}''$ and AG $= \frac{1}{16}\frac{3}{}''$.

Struts.—The compression on DF = 3,000 lbs. and the length of the rut between bearings = 28″. If we use two bars connected by stance pieces, which is a very good form of strut, we see that for actical fastening we would require a width of about 2″*. Assume at each bar is $2'' \times \frac{1}{4}''$. Then the ratio of length to least dimension the strut can be made as 28″:2″, by arranging the distance pieces the centre so that the least dimension there in any direction shall) 2″. From Table XXVI., page 175, we see that the value of the nstant n for this form of strut = 3, hence $n \times \frac{l}{h} = 3 \times 28 \div 2 = 42$. or mild steel, ends rounded, this value corresponds in the table) a value of 4·76 tons per square inch. As the cross section is $\times 2 \times \frac{1}{4}'' = 1''$, it will stand 4·76 tons, and it is actually required to ɜar 1·34 tons. Hence we see that the dimensions required for ractical fastenings are very much greater than those required for ieoretical strength. The distance pieces may be at 9″ intervals, e., there may be two of them in the length of the strut. The ıethod of working out this detail has already been fully gone into in ıxample 29, Chapter X.

Principal Rafters.—As the principals are loaded throughout their

* Using a pin and link connection (see Example 29, Chapter X.). A rivetted oint would not be used in a roof of this span.

length, they are under combined compressive and tra
If we consider the rafter as continuous from ridge to
that the point of maximum stress will be at the junctio
Here the effect of the bending will be to bring the u
the T-iron into tension and the lower fibres into comp
the rafter is already under compression from its positio
Hence the point of maximum compressive stress will b

Fig. 252.

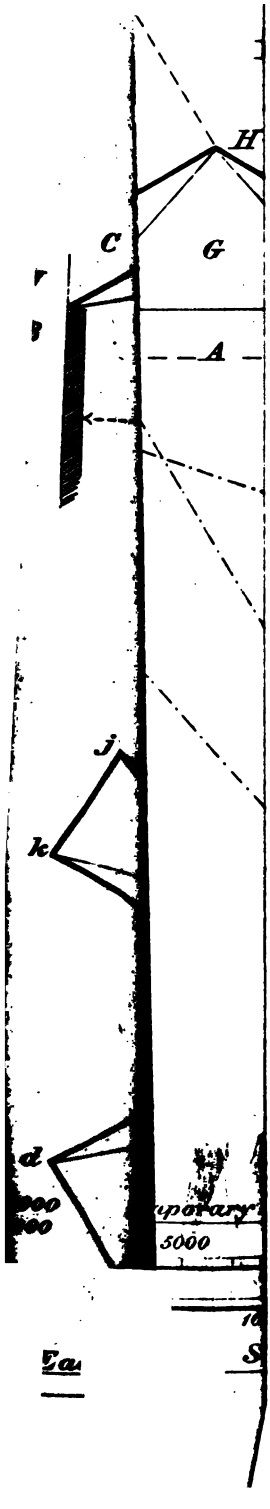
fibres of the T-iron. To find t
stress, we may proceed by trial
we may take a market section, an
the maxima combined stresses br
are greater or less than the mate
stand. Take a section $3'' \times 4'' \times \frac{1}{2}$
Fig. 252. From Table XVIII., pa
that the value of $I = 5{\cdot}04$, and
$M_{ff} = Wl \div 8$, where W = weight
rafter = 3400 lbs., and l = half the length of the rafte
Hence $Wl \div 8 = 3400 \times 100 \div 8 \times 2240 = 18{\cdot}8$, say 19 in
value of E in tons is 13,000. P = the thrust on the
the stress diagram = 17600 lbs. = 7·9 tons. Hence th
combined stress—

$$r = \frac{M_{ff} \times y}{I - \frac{Pl^2}{10E}} = \frac{19 \times 2{\cdot}67}{5{\cdot}04 - \frac{7{\cdot}9 \times 10000}{130000}} = 11{\cdot}3 \text{ tons per sq}$$

The ultimate strength of mild steel in compres
32 tons, the elastic limit being about 18. Hence we
section will give a factor of safety of nearly 3. This
theoretically considered advisable, although it is wl
allowed in practice. Besides, in the foregoing investig
taken a load all over the roof, which would very se
be called into play, and have, therefore, so far *begu*
certain margin of safety. Hence we may fairly consid
as perfectly safe.

The sizes given in Hurst's pocketbook for an *iron* ro
and span and load, *but with trusses at* 6 *feet apart*,
$FG = \frac{11}{16}''$, $AG = \frac{3}{4}''$, and principal rafter $= 3\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{1}{2}$
observed that all these are less than those just found.

It is unnecessary to point out that in this truss th
both the principals and struts is not employed to the l



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Method of Sections.

The stresses obtained by the stress diagram as exemplified in the regoing investigation are, of course, subject to the disadvantage at any error of drawing frequently is carried throughout the whole the calculations, and vitiates the whole of the results. It is viously important to have some method of independent check upon e correctness of the results. This check is supplied by what is lled the *Method of Sections*, introduced first by Professor Rankine, d elaborated by subsequent writers.

This method is based upon the following principles:—For forces ing in a plane, there are three conditions of equilibrium. If e forces are resolved in two directions at right angles to each ıer, then

(1). The algebraic sum of all the vertical components must = 0.
(2). „ „ „ horizontal „ „ = 0.
(3). „ „ „ forces about any axis perpen ular to the plane of the forces must = 0.

Of these principles, the last only need be used in the method in nt. Let us conceive the structure cut by a line, which need not essarily be straight, but should cut, if possible, only three bars, l conceive that forces are applied to the cut parts of those bars the direction of their axes, so as to keep the whole in equilibrium, en it is evident that those applied forces are = the stresses in the rs. To determine the stress in any one of these bars, it is only cessary to take moments of all the forces acting on the structure, *und the point of intersection of the other two bars.*

Let us apply this principle to the case of the roof in the preceding ample. Draw a line through e bars FE, FG, GA (*Fig.* 253), d conceive that there are rces applied to the ends of e cut bars to keep the frame the left of the line in uilibrium. To find the ress on GA, we may take oments round the point of tersection of FE and FG, , the apex of the truss. he forces acting on the vered part of the truss to the left of the dividing line are, besides the

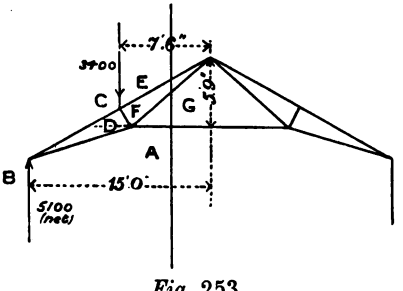


Fig. 253.

three forces EF, FG, and GA, the reaction at AB, and the weight at CE. Taking moments round the apex, which is the point of intersection of the bars EF, FG, we see that there are only three forces to be taken account of, as the other two pass through the apex, viz, the known forces AB = 5,100 and CE = 3,400, and the unknown force GA. Measuring the leverages of these three forces and noting which moments tend to turn the frame in a positive, and which in a negative, direction, we see that—

$$AB \times 15 + CE \times 7{\cdot}5 + GA \times 5{\cdot}9 = 0,$$

i.e.,
$$GA = 5100 \times 15 - 3400 \times 7{\cdot}5 \div 5{\cdot}9 = 8600.$$

From the stress diagram GA = 8400, which is near enough for all practical purposes.

Similarly, the stress in FG might be found by producing GA and EF to meet in a point X, and taking moments round X. In this case the moments of the external forces AB and CE would both be positive, and that of the unknown force FG negative.

This method of sections is not only a most valuable check on the accuracy of stress diagrams, but it affords sometimes a means of ascertaining the stress in a bar which is not easily determinable by the stress diagram method. For instance, in the truss shown in *Fig.* 254 (which, it will be observed, is a further development of the form investigated in the previous example, only, by giving additional points of supports in the principals, it tends to obviate the

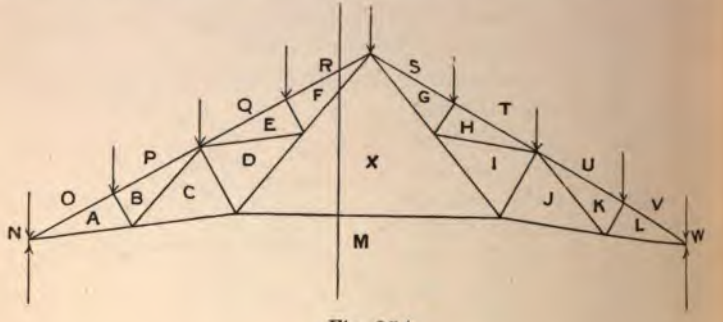


Fig. 254.

disadvantage of combined stress) the stresses in the central joint, where five bars meet, are indeterminate, because only two out of the five stresses are known. It is true these stresses may be found by symmetry, and by a little trial and error could be easily

fitted in. The method of sections, however, gives us an exact solution of the problem. For example, if we draw a dividing line to the left of the apex, as in the preceding example, cutting the bars RF, FX, XM, we can calculate the stress in XM in exactly the same manner as in the previous case. Having XM, we can find CD, and the whole of the stress diagram easily follows (*Fig.* 255).

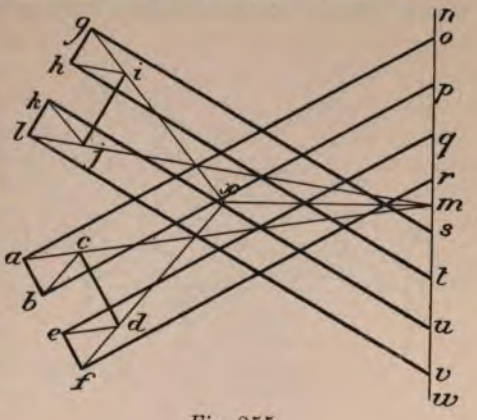


Fig. 255.

*Use of Stress Diagram in Modifying Designs of Roofs.**

The stress diagrams have hitherto been made use of for ascertaining the stresses in a roof of given form which has been designed. There is, however, a further and most important use of these diagrams which has not received much attention, and that is, since they indicate graphically the stresses produced in a design of a given form, they also indicate *how economy of design may be effected by modification of that form.* Alterations of a very slight nature in the inclinations of the bars, which may not affect the usefulness of the roof, may very largely affect the stresses coming on those bars, and thereby tend to economy of construction.

Take as an example of this principle the roof shown in *Fig.* 256. In this case the tie bar is inclined slightly upwards. The stress diagram is shown in *Fig.* 257. Now take the same roof, and make the tie bar horizontal, and draw the stress diagram to the same scale (*Fig.* 258).† It will be seen that the stresses in the most important bars are reduced. Go one step further, and draw the tie bar inclined downwards, and the other bars in such a way as to bring the minima stresses, and we get a roof (*Fig.* 259) evolved from the stress diagram shown in *Fig.* 260 which is theoretically economical.

The obvious objection to this is (1) that the downward

* This important practical use of stress diagrams has been pointed out to the writer by 2nd Lieut. R. L. Graham Fraser, R.E.

† Only as much of the diagram is drawn as is necessary to elucidate the principle under discussion.

inclination of the tie bars interferes with the head room of the interior, and (2) that it tends to increase slightly the length of the

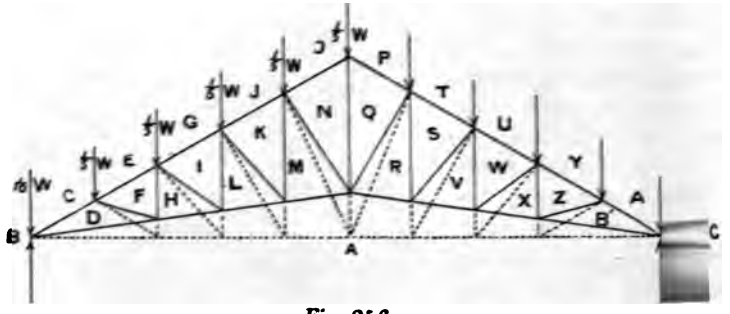


Fig. 256.

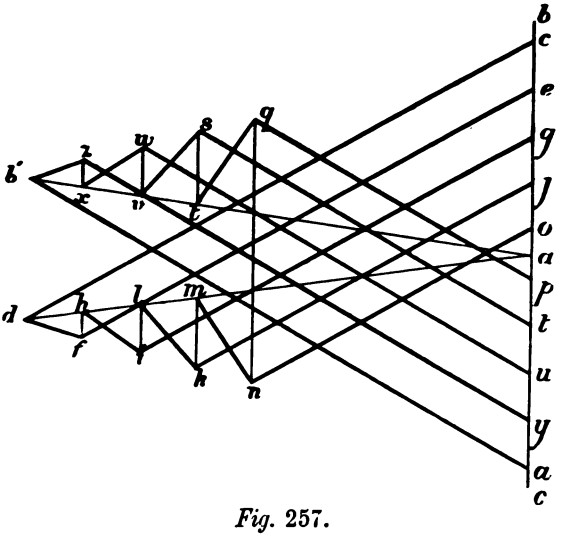


Fig. 257.

struts. As regards (1), there are many roofs where the head room is not a matter of urgent importance, and as regards (2), the size of the struts has frequently, as in the example worked out above, to be based on the practical width required for fastening, and is already larger than theory demands. Hence in most cases—even where there was no economy gained by reduction of stress—the extra size required would only be represented by the actual length increased, and not by any increase of cross section.

The smaller the angle between the principal rafters and the tie s at the points of ports, the greater the sses on all the members the roof (other things ng constant), hence the ı of the engineer in de- ning a roof should be so modify the inclinations he bars that the minima sses may be brought on

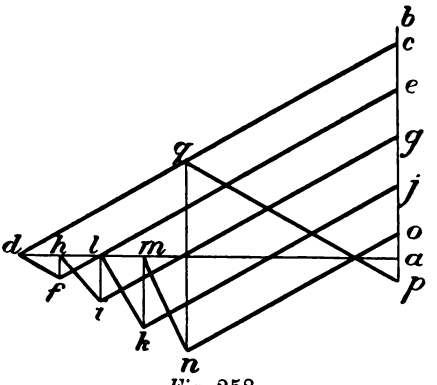


Fig. 258.

f *Fig.* 260 be compared h *Fig.* 257, the two ng on the same scale

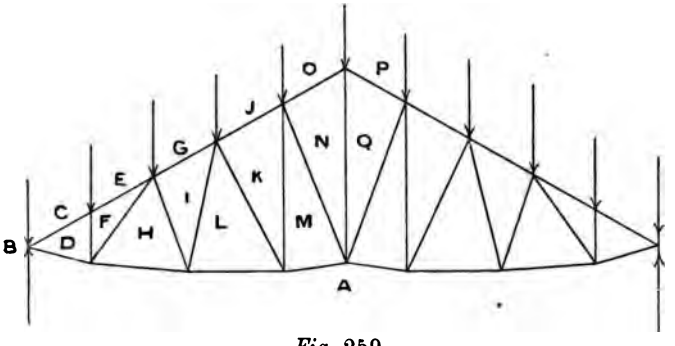


Fig. 259.

ıd for the same conditions of span, ad, etc., it will be seen (1) that the ress on the lower part of the prin- pal is reduced by about half; (2), at the stresses in all parts of the incipal in *Fig.* 260 are nearly con- ant, enabling a uniform section of aterial to be used without waste;), that the tension on some of the bars is reduced almost to *nil*, and others is about half what it is in *g.* 257; and (4) although the in- ned struts are of greater length,

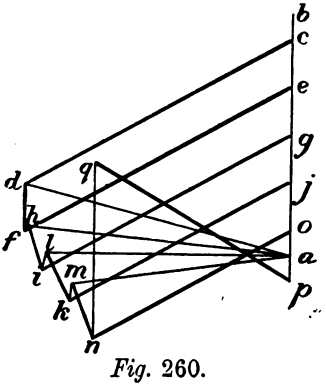


Fig. 260.

the stress coming on them is very largely reduced. If, for example we consider the span to be 48′, and the trusses at 8′ intervals, with a roof covering of countess slates, and material of the truss to be wrought iron (not steel), the scantling of the principals would be, in the case of *Fig.* 259, T-iron 3″ × 3″ × $\frac{3}{8}$″, whereas in the case of *Fig.* 256 it would need to be T-iron 4″ × 4″ × $\frac{1}{2}$″. The struts would all be of very light scantling in both cases, but the tie bars in *Fig.* 259 would be much lighter than in *Fig.* 256.

Application to Cantilevers.

Sometimes, in such cases as railway stations, roofs are on the cantilever principle. Here the stresses are very simply found by stress diagrams on the same principles as those already investigated. *Fig.* 261 shows such a roof, its stress diagram being shown on *Fig.* 262. It will be noticed that the stresses here are reversed from

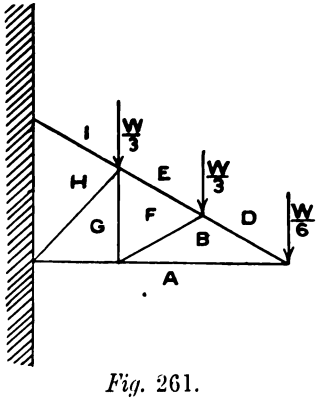


Fig. 261.

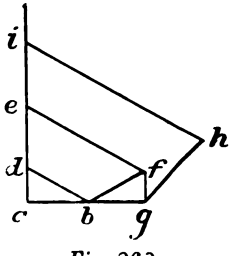


Fig. 262.

what they would be in the case of a roof supported at the eaves. The principal rafters are now in tension and the tie beam in compression.

CHAPTER XIII.

Foundations, Walls, Etc.

Primary Importance of Subject.—Nature of Soil.—Design of Foundations in Ordinary Soil.—Pile Foundations.—Stability of Walls.—Failure by Overturning, Crushing and Sliding.—Openings in Walls.—Stability and Design of Buttresses.—Stability of Ordinary Chimneys.—Tall Chimneys.—Empirical Rules for Retaining Walls and Arches.—Appendix.

The design of the foundations of any structure is a matter of the very greatest importance, but the consideration of the subject has been in this book deferred until we have thoroughly considered all matters connected with the transverse load of beams, compressive stress of struts, the lateral force coming on roofs, and other matters which bear upon the question. Without some knowledge of these it would be impossible to make a satisfactory examination of the external forces coming on foundations, and the internal resistance which they are capable of affording.

Nature of Soil.

The nature of the soil upon which a building is constructed is necessarily the first subject for consideration, and inasmuch as most ordinary soils are not incompressible, the usual object to be aimed at in designing foundations is to see that the settlement of the soil under the weight is uniform. Settlement must be expected, but the object of the design should be so to spread the weight over the area of the foundation that in no part shall there be a greater tendency to settle than in others. It is not intended here to enter into the various practical matters which must necessarily occupy an engineer's attention when he is actually constructing the foundations of a building, such as the digging of trial pits, obtaining samples of the soil, etc., but it may be pointed out that where there is any doubt about the supporting power of the soil, this may be easily ascertained by the application of a test load. For instance, a baulk of timber 12″ square may be placed vertically in a

bed of a depth corresponding to the proposed foundations. Pegs may be driven into the surrounding ground in lines radiating from the baulk, with their tops carefully levelled. The timber may then be loaded with weights until settlement is observed in the surrounding ground, as indicated by difference in the levels of the pegs. The weights will then indicate the load per square foot which the soil is capable of supporting. The safe load in tons per square foot for various classes of soil is given in the following table:—

TABLE XXX.

Approximate Safe Load in Tons per Square Foot which Soils of Various Kinds can Bear.

Description of Earth.	Approximate Safe Maximum Load in Tons per Square Foot.
Bog, morass, quicksand, peat moss, marsh land, silt	0 to 0·20
Slake and mud, hard peat turf	0 to 0·25
Soft, wet, pasty or muddy clay and marsh clay	0·25 to 0·33
Alluvial deposits of moderate depths in river beds, etc. ...	0·20 to 0·35
NOTE.—When the river bed is rocky and the deposit firm they may safely support 0·75 ton, but not more.	
Diluvial clay beds of rivers	0·35 to 1·00
Alluvial earth, loams and loamy soils (clay and 40 to 70% of sand) and clay loams (clay and about 30% of sand)	0·75 to 1·50
Damp clay	1·50 to 2·00
Loose sand in shifting river bed, the safe load increasing with depth	2·50 to 3·00
Upheaved and intermixed beds of different sound clays ...	3·00
Silted sand of uniform and firm character in a river bed secure from scour, and at depths below 25'.	3·50 to 4·00
Solid clay mixed with very fine sand	4·00
NOTE.—Equal drainage and condition is especially necessary in the case of clays, as moisture may reduce them from their greatest to their least bearing capacity. When found equally and thoroughly mixed with sand and gravel, their supporting power is usually increased.	
Sound yellow clay containing only the normal quantity of water	4·00 to 6·00
Solid blue clay, marl and indurated marl, and firm boulder gravel and sand	5·00 to 8·00
Soft chalk, impure and argillaceous	1·00 to 1·50
Hard white chalk	2·50 to 4·00
Ordinary superficial sand beds	2·50 to 4·00
Firm sand in estuaries, bays, etc.	4·50 to 5·00
NOTE.—The Dutch engineers consider the safe load upon clean firm sand as 5½ tons per square foot.	

TABLE XXX.—(*Continued*).

Approximate Safe Load in Tons per Square Foot which Soils of Various Kinds can Bear.

Description of Earth.	Approximate Safe Maximum Load in Tons per Square Foot.
Very firm, compact sand foundations at a considerable depth, not less than 20′, and compact sandy gravel	6·00 to 7·00
NOTE.—The sustaining power of sand increases as it approaches a homogeneous gravelly state.	
Firm shale, protected from the weather, and clean gravel ...	6·00 to 8·00
Compact gravel	7·00 to 9·00
NOTE.—The relative bearing powers of gravel may be thus described:—1. Compact gravel. 2. Clean gravel. 3. Sandy gravel. 4. Clayey or loamy gravel. Sound, clean, homogeneous Thames gravel has been weighted with 14 tons per square foot at a depth of only 3′ to 5′ below the surface, and presented no indication of failure. This gravel was similar to that of a clean pebbly beach.	

This table is taken from Mr. Newman's *Earth Slips and Subsidences.* Generally speaking, one may allow a safe load per square foot of 5 tons in rocky soil, 4 tons in thick clay, when dry, 1 ton in soft wet clay, 8 tons in gravel and coarse sand, 4 tons in sand when compact. Prof. Rankine has given the following formula:—

$$\text{Load per unit of area} = wh\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)^2,$$

where w equals weight of a unit volume (a cubic foot) of the soil, h is the depth in feet below surface, and α is the angle of the repose of the soil. This formula takes no account of cohesion. It is roughly approximate for most soils, and has been found to give results which are below those of experiment.

It is important that foundations should be laid at such depths as to be unaffected by frost. This is especially the case with clay. In Great Britain clay foundations should, therefore, be taken to a depth of about 4′; other soils may be taken to a depth of about 3′. When clay occurs in layers which are not horizontal, or where there are alternate layers of clay and rock, great care is necessary to prevent slipping. The sliding of one layer of rock upon

the clay may be guarded against by cutting a trench across the slope of the base and filling in with concrete.

The bearing power of clayey soils, however, can be much improved by drainage. Indeed, in all cases surface or subsoil water should be carefully drained away from any proposed foundations.

With regard to sand also, the question of water is of importance. Coarse gravel and sand forms one of the best foundations, whereas fine sand, saturated with water, is most treacherous. Sand, however, when prevented from escaping laterally, is one of the best foundations it is possible to have. The great suspension bridge of Brooklyn has its piers founded upon a layer of sand 2′ thick resting upon rock. Many other large structures are founded upon this material under proper precautions. When exposed to the scour of a river, foundations should be taken down to such a depth as to be beyond the scouring action. The bed of a river may be of excellent gravel and sand, yet it would never be safe to trust the foundations of a bridge to such a soil, although it would be quite safe to build a structure upon it apart from the scouring action of floods. The foundations for the large river bridges on the Sind-Peshin Railway were carried sometimes through 20 or 30 feet of the hardest and most closely-compact gravel and sand to solid rock below. These foundations have stood the test of repeated floods, but there is no doubt that, had they not been taken down to rock, the stability of the bridges would have been seriously imperilled.

In foundations springs are often met with, and give much trouble, not only because they wash out the mortar from the masonry or concrete, but because the force of the water acts in a contrary direction to the weight of the structure, and endangers its stability. This was very notably the case in the construction of the Barrage of the Nile. It was, however, successfully overcome by driving a pipe down at the site of each spring, conducting the water thence to the surface and filling round the edge of the pipe with concrete. Finally, the pipe was plugged up.

The soil, however, may be improved by increasing the depth. Ordinary soils will bear a greater weight at greater depths, because of the original weight that they have borne. This is especially the case with clay. Sometimes, also, the soil may be improved by ramming or by driving short piles in to it. These piles need not be more than about 6′ long and 6″ in diameter. They may be driven fairly close together, and the effect will be to compress the soil between them. This method of procedure, however, is not very applicable to

s other than clay. Sometimes it has been found very satisfactory deposit in the foundation trenches a layer of gravel.

Where there is a soft layer underlying a hard one, it is desirable leave about 8′ thickness of sound earth between, and where there e soft portions intervening between sound parts in a foundation ench, the soft parts should be arched over so as to relieve them of ll weight.

To ascertain the load supported by the foundation, we have to onsider the weight of the building itself, the movable or temporary oads coming upon it, such as the weight of the floors and snow on he roof, etc., and, lastly, the effect of unsymmetrical loading, such as would be produced by the force of the wind. It is usually sufficient to consider such of these weights as act upon *one foot in the length* of the wall of the building, and not necessarily to consider the structure as a whole. In so doing, of course, we neglect the assistance afforded by the cohesion of the parts of the wall on either side of the strip in question, but the neglect of this cohesion is on the safe side. In the case of tall chimneys or other towers, it is necessary, however, to consider the structure as a whole.

The manner in which the weight of the roof and floors is transferred to the walls must obviously be taken into consideration. The longitudinal walls of a building designed to hold heavy goods bear, as a rule, the whole weight of the girders which support the floors, and also carry the greater part of the roof. The foundations, therefore, of such longitudinal walls must be considered as bearing much greater weight than any of the cross walls in the building.

A layer of concrete is now the usual *bed of foundations* in ordinary soils. The breadth is usually about 1′ greater than the thickness of the brickwork above it, that is, an offset of 6″ is left at either side, so that after the concrete bed has been laid, the exact position of the brick walls may be accurately marked by leaving a good margin on either side. The London County Council give 4″ as the projection of the concrete beyond the footings. They also specify that the concrete shall be at least 9″ thick. Such regulations, however, are open to the obvious objection that no reference is made to the superincumbent weight, which should determine both the breadth and thickness of the concrete.

We have then to consider what should be the *breadth* and *thickness* of the concrete bed.

As regards the *breadth* there are two considerations, one practical, he other theoretical. The first is that a sufficient breadth must be

allowed to give a fair base for the superstructure, leaving a margin on either side for the accurate laying out of the work. The second and theoretical requirement is that the breadth must be sufficient to bring the weight per square foot, coming on the soil, within the safe limits of that which the soil can sustain.

For small buildings the first of these considerations governs the width of the foundations. The walls of dwelling houses are fixed not so much by considerations of stability as of comfort, and the minimum thickness in this country is, generally, 14″. An offset of even 4″ to the concrete bed beyond the walls will generally give more width than is necessary to satisfy the theoretical requirements of the weight on the soil being within safe limits, even with two-storied houses. But where the weight borne by the walls is considerable, it may be necessary to determine the requisite breadth by equating the weight per lineal foot of the wall and its maximum load with the proposed breadth of foundation, multiplied by the safe load of the soil per unit of area. The slope or angle at which the weight on a concrete bed can be distributed downwards may be safely taken at 60°, according to some of the best authorities. Most cases of concrete foundations, worked out on the rules given below, would result in a steeper slope. This is on the safe side.

To ascertain the *thickness* of the concrete is, however, not so easy, because there are certain unknown quantities in every case, such as the nature of the materials, the bonding, the quality of the concrete, the height of the footings. The aim of the engineer must be to extend the base of the structure in such a way as to distribute the pressure uniformly over the foundation area. We have already pointed out that it is necessary that the settlement must be uniform. **Hence the axis of the load, that is, the vertical line through the centre of gravity of the weight, should pass through the centre of the area of the foundation.** This, no doubt is self-evident, but the subject is so frequently neglected that it is necessary here to draw special attention to it. Unquestionably the most frequent cause of failure in foundations is due to the unequal loading which is brought to bear on the foundation area. All foundations should be so constructed as to press the ground *slightly concave upwards rather than convex upwards*, and endeavours should be made to bring the concrete bed into the condition of a beam of the inverted cantilever form (*Fig.* 263), where the projections on either side are cantilevers uniformly loaded. The calculation, therefore, of the thickness will depend upon the resistance to tension of the concrete

he lower layer. The ordinary formula for a uniformly-loaded ·ilever of rectangular section is

$$\frac{Wl}{2} = \frac{rbd^2}{6}\text{; or } W = \tfrac{1}{3}\frac{rbd^2}{l},$$

vhich r = the Modulus of Rupture of the concrete, the value of ch may be taken from Table XIII., Chapter III.

XAMPLE 41.—*Design the foundation for a three-storied warehouse, as on on* Fig. 264, *with brick walls built on loamy soil, bearing* 1·5 *tons square foot, and let it be assumed that the weight per foot run on the vlations is altogether* $6\frac{1}{2}$ *tons; the thickness of the wall of the lower ey is* 27 *inches.*

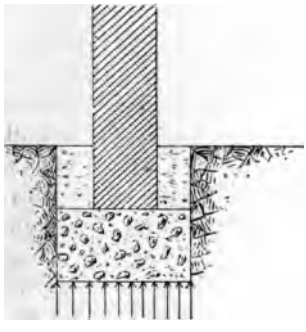


Fig. 263.

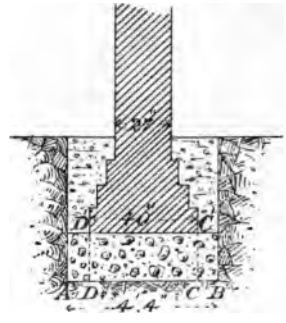


Fig. 264.

We have first to find the width of the foundations. Evidently is will be

$$\frac{6\frac{1}{2}}{1\frac{1}{2}} = 4'\ 4'' = 52 \text{ inches.}$$

ıe footings can be designed as in the figure, leaving 6″ clear on her side. Now if the brickwork is thoroughly well bonded d constructed, the projections AD and BC will be inverted ntilevers, bearing a uniform load upwards from the reaction of e earth equal to $1\frac{1}{2}$ tons per square foot, and tending to break the ncrete at DD′, CC′, then

$$l = \tfrac{1}{3}\frac{rbd^2}{l}\text{, where } W = \frac{6'' \times 1\frac{1}{2} \text{ tons}}{1' \text{ (length of the assumed piece of wall)}}$$
$$= \cdot 75 \text{ tons.}$$

$h = 12$ inches; $l = 6$ inches; r for concrete of 1 cement, 1 sand and 8 shingle = 133 (from Table XIII., Chapter III.).

Giving a factor of safety of 4, the concrete should be 18 inches thick. In this example we have taken a very poor class of concrete, but a low factor of safety. It would no doubt have been better to have taken a good concrete, say 1 cement, 1 sand and 5 shingle. The factor of safety adopted must depend upon the workmanship.

As concrete is cheaper than brickwork in many localities, it is often customary to dispense with the footings altogether and take the concrete from the foundation bed to the ground level. This is almost invariably done in small buildings, and, of course, the thickness of the concrete is far greater than that which is demanded by requirements of theory.

Where there are two walls of different weights meeting one another it is very necessary that the foundations should not be rigidly connected together. The effect of such construction is shown in *Fig.* 265, and the cracks which very frequently appear in buildings are doubtless caused by the shearing stress at the point of junction. Such cracks are generally seen, as shown in *Fig.* 266, where the resultant of the horizontal and vertical shearing stress has to be met

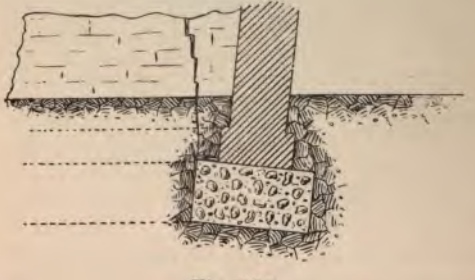


Fig. 265.

Fig. 266.

by the tensile strength of the material of which the wall is built. For instance, where the spire of a church is connected with the main body of the building, the *foundations* should not only be separate, but the *walls* should not be bonded into one another. This is also the case in dwelling-houses, where the pressure upon the walls is not

relatively so great as it would be in the case of a church; but it must be admitted that it is extremely difficult to get this carried out in practice. In America, where the art of designing foundations has been very much developed (on account of the erection of lofty buildings in Chicago, New York, and other cities), each pier, wall, etc., has its own foundation, and the interior walls are united to the exterior walls by anchors, which slide in slots. This is unquestionably a sound principle.

Inverted arches, as shown in *Fig.* 267, are frequently built to distribute the pressure coming upon the piers over a greater area.

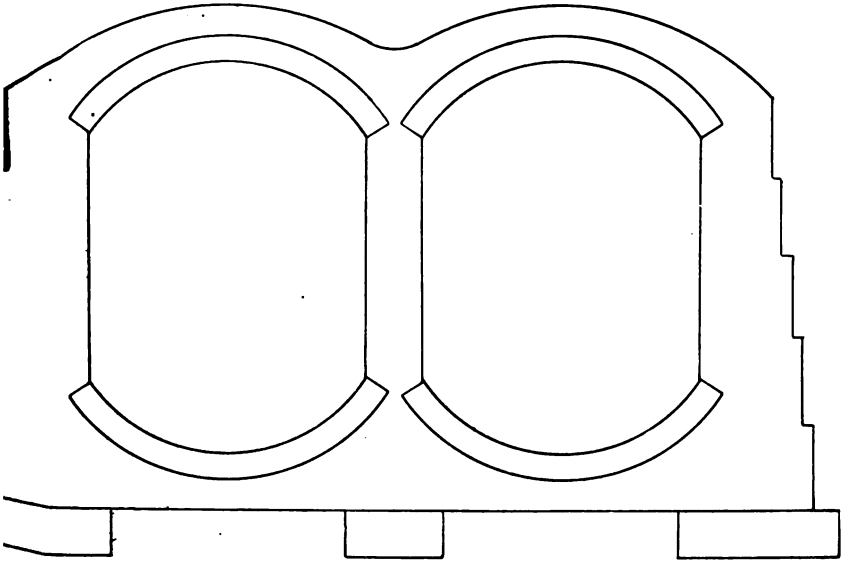


Fig. 267.

This method of construction is often used in connection with bridges where the bearing power of the soil is uncertain, but it is not a very satisfactory method, because the arch form induces lateral thrust, and it is difficult to make the areas of the different parts of the foundations proportional to the loads to be supported. Where the bridge spans a stream liable to floods, the effect of the inverted form is to cause a scour in the centre, and not to distribute the volume of water over the whole of the breadth of the span. The effect of this scour in the centre is sometimes serious, because

it tends to wear away the material of which the invert is built. This form of foundation is not, therefore, one to be usually recommended.

At the reconstruction of Aldershot, sheets of corrugated iron and of old felt from the dismantled huts of the old camp were often laid under the concrete in the foundations. This spread the weight over the soil, and protected the lower layers of the concrete from being damaged by contact with the ground.

Foundations on Piles.

There are two classes of piles whose bearing power has to be considered:—(1). Those whose lower end rests upon hard soil and whose bearing power is, therefore, the strength of the pile considered as a column. The soft earth round the pile will prevent lateral deflection to a great extent, hence the strength of the pile will be approximately the strength of the material. Professor Rankine gives 1,000 lbs. per square inch of section for such piles as the safe load which can be depended on. This is much the same as would be allowed for a short column of ordinary fir with a factor of safety of about 6. (2). The second case is that of piles driven through soft earth, and depending for their bearing power on the friction of the earth on the sides. Professor Rankine gives 200 lbs. per square inch in this case, but this takes no account of the length in the soil, and it is manifest that the side friction depends on the square surface, *i.e.*, length × perimeter, of the pile in the ground, and on the nature of the ground and the soil. Mr. Trautwine gives the following rules:—"The ultimate friction of piles, even with the bark on, and driven about 3 feet apart from centre to centre, probably never exceeds about 1 ton per square foot, even when well driven into dense moist sand or loamy gravel, nor more than 0·5 to 2·75 of a ton in common soils or clays, or 0·1 to 0·2 of a ton in silt or wet river mud, depending on the depth and density. The friction of cast-iron cylinders seems to be about 0·3 that of piles."

The ordinary terms for the specification of piles are somewhat as follows:—"Each pile should be driven until it sinks only $\frac{1}{4}$" after 10 blows of 15 cwt. falling 15 feet," or "$1\frac{1}{4}$" after 5 blows of 15 cwt. falling 30 feet." Specifications of this sort are open to much objection because in them no regard is paid to the depth already in the soil, or to the rapidity of the blows. It may be that a pile driven through soft mud has penetrated a hard substance below. The effect then of succeeding blows will be simply to injure the fibres of the

imber at the upper end, and perhaps to produce splits throughout ts length and seriously impair its strength.

On the whole, it is considered that, although there are many ormulæ which deal with the subject of the bearing power of piles, ret the conditions are so difficult to express with any mathematical recision that it is better in any case of doubt to determine the earing power of the pile by actual experiment. The question, hich will frequently arise, of when to stop driving a pile is a diffi- ult one to answer, because—

(1). It depends to a large extent upon the strata through which he pile has to be driven. "A long pile driven through gritty material nto softer underlying strata will have sufficient frictional resistance bear with perfect safety the required load, this resistance being duced to a minimum during the process of driving, but developed its full extent after an interval of rest; and in the reverse case, at of a firm soil underlying a soft stratum, the ability of a pile to ar a load would be small notwithstanding the greater resistance to rther penetration" (*Patton on Foundations*).

(2). The pile must be driven for a sufficient depth, under all rcumstances of any given case, to give steadiness or stability to the ructure. In the case, for instance, of the foundations of a river idge, the scouring action of the current must be taken into account.

(3). The same material offers so much difference of resistance to les driven under similar conditions that it is impossible to lay wn any general definite rules which would necessarily apply in y given case.

(4). The effect of hammering the end of the pile, and causing hat is called a broom, results in such loss of energy that no rule at does not take into consideration this varying factor is of very uch practical use.

We are thus driven to the necessity of experiments in each case. is ascertained from the experiments of others that piles from 30 40 feet in length, in even the softest soils, will bear from 10 to 12½ ns per pile. With a round pile 1 foot in diameter, this is the me as 200 to 250 lbs. to the square inch. In sand and gravel, les will carry to the full extent of the crushing strength of the nber, provided the depth in the ground is at least about 10 feet. is depth is that necessary to be beyond the reach of vibrations used by moving loads.

To sum up the foregoing, it may be said that experiment is of e greatest practical value, and that the following facts have been

established:—**(1). That when piles are driven in soft soil, the penetration of from 30 to 40 feet will be ordinarily sufficient, apart altogether from considerations of weight of hammer, height of fall, and penetration at the last blow. (2). In clayey sand or gravel, the depth required is only that necessary to get beyond the reach of scour or of vibrations caused by moving loads. (3). Continuous hammering of piles does frequently more harm than good.**

One of the most recent formulæ for pile-driving published in this country is that of Mr. Haswell,* founded on many experiments. The safe weight, according to this formula, borne by a pile driven till the final blow, produces a depression of $\frac{1}{2}''$.

$$l = \frac{4w8\sqrt{H}}{C},$$

where l = safe load in pounds,
w = weight of ram,
H = height in fall.

C is a constant between values of 3 and 6, according to the nature and condition of the soil, the character of the piles, and the excellence of their driving.

Stability of Walls.

In considering the stability of walls, we have to consider the various ways whereby a block of masonry, such as a retaining wall, a chimney, a dam, etc., may fail.

There are three ways whereby failure may occur:—(1). The action of any lateral force combined with the weight of the structure itself may tend to overturn the block on the side furthest from the application of the force. This is called *failure by overturning.* (2). The action of external forces combined with the weight of the block itself may be such that, while it does not overturn the block, it may yet cause so much force to be concentrated on the edge that the material there may not be able to withstand the pressure produced. This is called *failure by crushing.* (3). The third method of failure is that shown in *Fig.* 268,

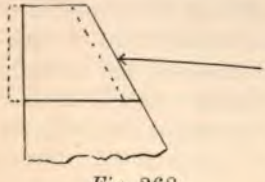


Fig. 268.

* *Minutes of Proceedings of Institution of C.E.*, March, 1894.

where the block tends to slide along some plane. This is called failure by sliding, and is analogous to failure by shearing in a beam.

We shall now consider these three forms of failure.

(1). As regards failure by overturning, we can see at once, and without much reasoning on the subject, that if the resultant of the weight and the external forces falls outside the section, either the block would overturn as shown in *Fig.* 269, or else some tensile force must be supplied in order to anchor it in position. This tensile force is in many cases supplied by the mortar in which the structure is bedded, and, as will be seen from examples, in comparatively unimportant structures it is quite safe to rely upon the tensile strength of the mortar thus brought into action. It must be remembered, however, that the strength of a chain is that of its weakest link, and that the weakest part of any masonry structure is the *adhesion* of the mortar, a subject which has already been alluded to in Chapter III. In ordinary brickwork built in cement and sand, not more than 30 lbs. to the square inch can safely be relied on, but the amount will vary with the materials used (see Table XI., Chapter III.). In large and important structures, where any danger to life may result from failure, it is undoubtedly risky to trust in any way whatever to the tensile or adhesive strength of the cement and materials, and hence **in all such structures it should be an axiom, never to be departed from, that the structure should be so designed that in no part of it is any tension brought into play under any circumstances whatever.** If when we have a structure composed of blocks of various kinds, and wish to ascertain whether it is safe against overturning, we may draw a vertical section of the proposed wall and lay off to scale the forces acting in various parts (*Fig.* 270). If the resultants of these forces cut the bed joints as shown at A, B, C, we are at all events sure that the structure is safe against overturning. The line which joins the points A, B, C, etc., is called the *Line of Resistance* of the section.

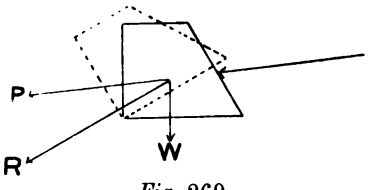


Fig. 269.

(2). *Failure by Crushing.*

A structure might be safe against overturning, but it is evident that we have practically to consider whether the resultant pressures

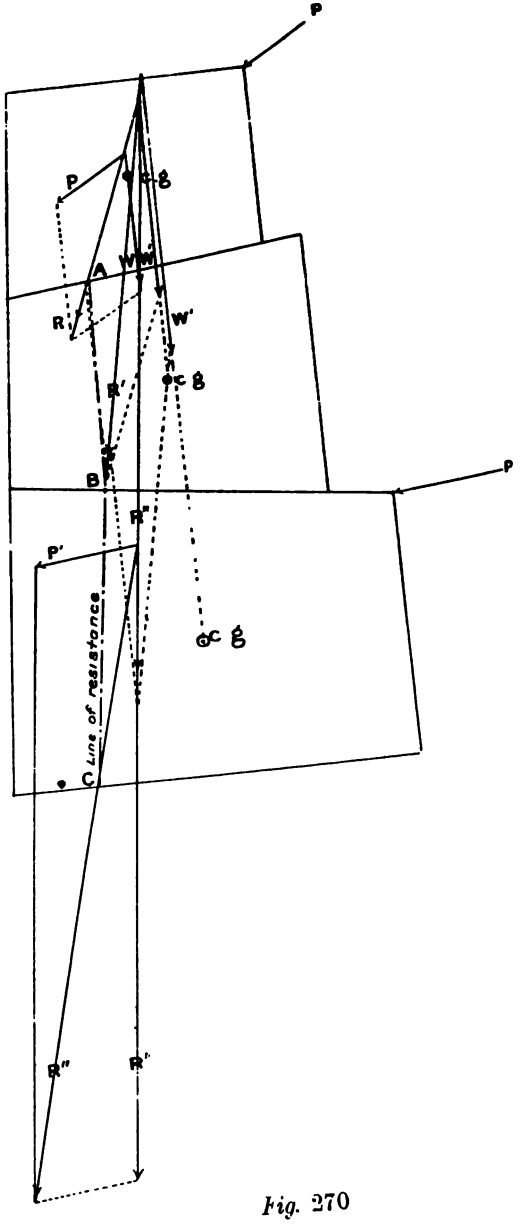


Fig. 270

at all points are brought so far within the limits of the wall that there shall be no tendency for the material to crush or even crack. It will perhaps assist us in the understanding of the matter if we consider the two blocks separated by a layer of elastic material, such as indiarubber. *Fig.* 271 represents the general effect when there is but a slight external force. The resultant pressure of that external force and weight is but little removed from the centre, and the indiarubber on the right side is only slightly more compressed than it is at the left side. In the lower part of the figure is shown a graphic representation of the forces, where the line *ab* represents the maximum pressure at any part of the base, and *cd* the minimum. Taking *Fig.* 272 to represent the pressure diagram derived as above, we have the following formula* :—

Let t = thickness of the joint AB.

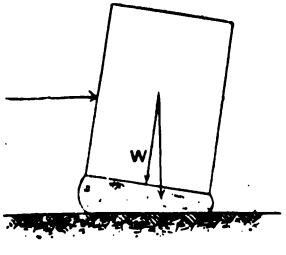
„ d = minimum distance of centre of pressure P from outer edge of joint.

„ N = resultant normal pressure = area A*ab*B.

„ $n = \frac{N}{t}$ = mean intensity of pressure if spread uniformly over t.

„ Y = maximum compression per square unit represented by B*b*.

„ y = minimum compression per square unit represented by A*a*.



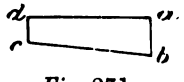


Fig. 271.

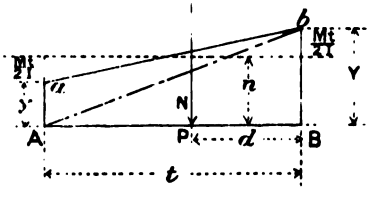


Fig. 272.

* From General Wray's *Application of Theory to the Practice of Construction.*

Then, taking moments about B of the whole figure AabB, and its component triangles AbB and aAb—

$$Nd = ntd = \frac{tY}{2} \times \frac{t}{3} + \frac{ty}{2} \times \frac{2t}{3},$$

whence $$y = \frac{3nd}{t} - \frac{Y}{2},$$

but $$n = \tfrac{1}{2}(Y + y),$$

hence $$y = 2n - Y.$$

Hence $$Y = 4n - \frac{6nd}{t} = \frac{2N}{t}\left(2 - \frac{3d}{t}\right) \ldots\ldots\ldots\ldots\ldots (\alpha),$$

$$y = \frac{6nd}{t} - 2n = \frac{2N}{t}\left(\frac{3d}{t} - 1\right) \ldots\ldots\ldots\ldots\ldots (\beta).$$

If I be the Moment of Inertia of the cross section in question (generally a piece of solid wall one unit in length, and therefore the section is a solid rectangle) then the maximum intensities of pressure at the inner and outer edges due to the horizontal component of the external thrust may be expressed in general terms as follows:—

Since $$M_f = \frac{rI}{y_o} \text{ and } y_o = \frac{t}{2},$$

hence $$r = \frac{Mt}{2I} \text{ and } Y = \frac{N}{A} + \frac{Mt}{2I} \ldots\ldots\ldots\ldots\ldots (\gamma),$$

$$y = \frac{N}{A} - \frac{Mt}{2I} \ldots\ldots\ldots\ldots\ldots (\delta),$$

N being the vertical pressure, A the area of the cross section, and M the moment of the horizontal component of the external force.

To apply the above formulæ, we have to find by graphic representation the position of the centre of pressure P in any given case, and the value N of the normal resultant. Substituting the values thus found we have a value of Y which may be compared with the safe resistance of the material.

So far we have considered the position of the centre of pressure as but slightly removed from the centre of the section. But if the lateral pressure is still so much further increased that the centre of pressure is now at $\frac{1}{3}t$ from the edge, we see that in Formula (α) $Y = 2\frac{N}{t}$ = twice the mean intensity of pressure, the state of affairs

ıg as shown in *Fig.* 273. At the edge nearest to that from which lateral pressure comes there is neither tension nor compression ; *Fig.* 274, where the assumed rubber joint is now relieved of all ssure at one side). If the lateral pressure is still further increased, sion will be brought into action, or if the point is incapable ɔearing any tension, part of it will be entirely relieved of pressure.

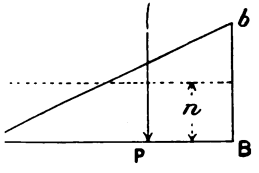


Fig. 273.

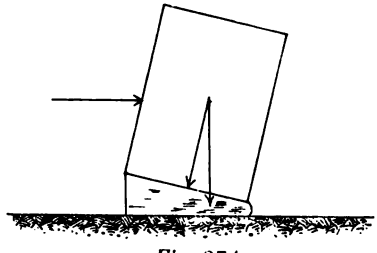


Fig. 274.

In all important structures this is a state of affairs which is nerally not admissible. Masonry can be little trusted to resist nsion, both because the cementing mortar has not a large tenacity, ıd because the adhesion of the mortar is still less. This is ɹrticularly the case when one has to deal with the pressure of ıter. Where such exists, the water will penetrate through the ıallest crack, and as the pressure of water acts in all directions ith an intensity which depends only upon the depth, it will be en that an upward pressure would be brought to bear upon the terior of the crack, and, therefore, upon the structure, of an ıount varying with the depth below the surface. The extent of e crack and that upward pressure will tend to bring an extra ushing stress upon the down-stream side of the wall. Hence the ndency will be to increase the tension at the up-stream side, to crease the crack that has already been caused, and still further to danger the stability of the structure. This subject will be further nsidered when treating of the design of dams in Part II.*

Broadly speaking, therefore, **the safe limit of stress in a ɔlid masonry structure, except of those of a comparatively ıimportant character, is reached when the resultant 'essure falls at the limit of the centre third of any joint,**

* It has been clearly pointed out by Professor Unwin in an article on the ɔuzey Dam.

or, what comes to the same thing, when it deviates from the centre a distance equal to one-sixth the thickness.

Some consideration must be taken, however, of the material of which the wall is composed. It has been pointed out* that "the axiom that the line of resultant pressure should always fall within the centre third is practical, but it diverts attention from the essential condition of constructive stability, that is, that the crushing stress on the outer face should not exceed the limit of safety." It is possible to conceive of a wall of considerable height where it would not be safe to allow of the resultant pressure to come near the limit fixed by the centre third near the base, if it were built of soft material.

In arches, the line of resultant pressure frequently comes outside the centre third, but the pressures which come upon an arch ring are not, as a rule, so great as would endanger the failure of the structure from crushing, provided the arch is built with due regard to the proper arrangement of the load.

Where the resultant pressure comes nearer to the edge than the centre third, it is possible to ascertain the maximum pressure by similar reasoning to that which we have just considered. For if d be the distance of the centre of pressure from the edge, it is evident that the pressure will be distributed over a distance $= 3d$, and if P be the resultant pressure, $P \div 3d$ will be the mean pressure, and $2P \div 3d$ will be the maximum pressure (*Fig.* 275). In this investigation the tenacity of the mortar is not considered; it would, no doubt, reduce the maximum pressure, but the amount of that reduction would vary with so many circumstances that it is not possible, practically, to ascertain it with any accuracy, and the neglect of such consideration is on the safe side.

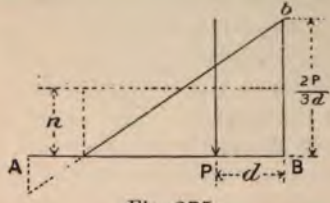


Fig. 275.

Position of Limiting Pressure in Chimneys, etc.—Hitherto in this investigation we have considered the effect of crushing force on a rectangular section, but it is evident that in the case of chimneys the cross section under consideration may be a hollow rectangle or a ring. In such circumstances the position of limiting pressure will not be expressed by the centre third of the

* By Major-General Sir John Ardagh, K.C.I.E., C.B., R.E.

base, but will vary with the value of I. In a solid circle the least value of d from the edge will be $\frac{3}{8}t$; in a thin hollow square* or rectangle it will be approximately $\frac{1}{6}t$; in a circular ring* it will be approximately $\frac{1}{4}t$ (*Fig.* 276). The simplest method of applying these principles in any particular case is to draw, on a sufficiently large scale, a section of the proposed structure, and to represent on that section the external forces acting upon it (wind, water pressure, thrust of arches, etc.), and then to test it for stability, seeing whether the resultant pressures fall within the safe limits at each joint of the structure. If it is found that the structure is not sufficiently stable, the dimensions or form may be altered accordingly, or *vice versa*.

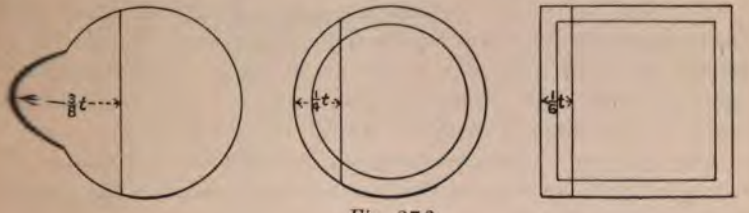


Fig. 276.

(3). *Failure by Sliding.*

We have seen that there may be a tendency on the part of two blocks resting on one another to slide along some joint under the action of external forces. In *Fig.* 277 the external force P may be resolved into two—one vertical and one horizontal, or one normal to the joint and one parallel to it. The vertical or normal force tends to press the blocks together; the parallel or horizontal force tends to make them slide along the joint, and this tendency to slide is resisted by the friction of the blocks and by the shearing strength of the mortar, the latter, however, being usually neglected.

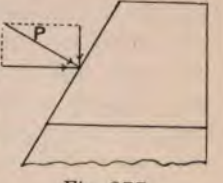


Fig. 277.

The law of friction of solid bodies, as determined by experiment, is, that the force required to cause motion in one body resting on another is a constant fraction, f, of the normal force pressing the two bodies together, the value of f depending on the nature of the surface in

* Where the thickness is small. The more accurate values are shown in the Appendix to this chapter.

contact, and not on the area. This constant or *coefficient of friction* is found by slowly tilting up the surface of one body until that resting on it begins to slide. Thus in *Fig.* 278, when the surface AA is gradually inclined, the body resting on it will not at first begin to slide because of the friction, but after it has passed through a certain angle, called the "angle of repose," it will begin to slide. The weight of the body may be resolved into two components, one acting normally to the plane AA, and the other acting along the plane AA. When motion begins, the component along the plane will just be equal to the friction, *i.e.*, $F = N \tan \alpha = W \sin \alpha$. This reasoning applies to every joint throughout the structure. Hence the third condition of stability is that, taking all the forces acting above any joint, the sum of their components parallel to the joint must not be greater than the sum of the components normal to the joint multiplied by the coefficient of friction of the material.

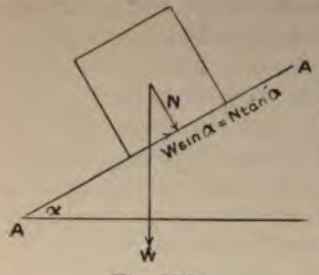


Fig. 278.

In other words, the angle which the resultant of all the forces acting above any two joints makes with the normal to that joint must not be greater than the angle of repose of the material.

It would appear desirable to allow a margin of safety. Therefore, to consider a structure safe, the direction of the resultant forces on any two joints should make with the normal to that joint a somewhat lesser angle than the angle proposed. It is proposed to make this angle not greater than $\frac{4}{5}$ *the angle of repose.* There is seldom any difficulty in doing this, because in all cases where sliding is likely to occur the joints of the masonry can be tilted so as to reduce the angle. This is the reason why the layers of a masonry in a retaining wall are built at right angles to the battered face of the wall, as in *Fig.* 279, and the joints in an arch abutment are built near the springing of the arch, tilted at an angle to the face of the abutment.

Openings in Walls.

Before going on to consider examples of design in the case of enclosure walls, buttresses, and chimneys, it may be as well here to say a little about openings in walls. It is sometimes necessary to break an opening in a wall, and the question arises how wide that opening may be without running the risk of the wall above coming

down. It will be seen that under these circumstances reliance has to be placed on the tranverse strength of the masonry above the opening. That masonry has to be considered as a beam fixed at both ends and loaded uniformly. The formula for such a case is

$$\frac{Wl}{12} = \frac{rbd^2}{6},$$

where W is the total weight distributed all along the span, l is the required length of the opening, r is the intensity of resistance to cross breaking at the extreme edge, b and d representing the height and thickness of the wall that is left.

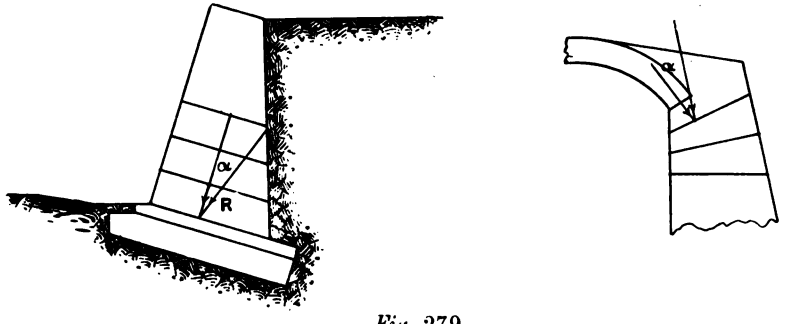


Fig. 279.

Take, for instance, the following:—

EXAMPLE 42.—*It is proposed to cut a* 5′ *opening through a brick wall* 2′ *thick, which rises* 2′ *above the opening* (Fig. 280).

The weight of the brickwork being taken at 112 lbs. per cubic foot, W $= 112 \times 5 \times 2 = 1120$ lbs.; $l = 60''$, $Wl \times 12 = 5{,}600$ inch-lbs. b in this case $= 24''$, and $d^2 = 24'' \times 24'' = 576$. Hence $r = \frac{5600}{2304} = 2{\cdot}43$ lbs. per square inch. It would be a very poor mortar that could not stand that amount of tensile stress, and hence it will be quite safe to make such an opening.

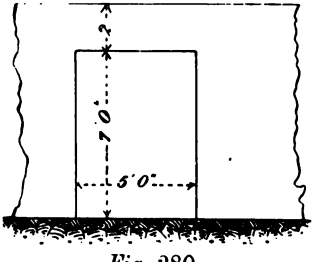


Fig. 280.

It will be seen that the higher the amount of masonry above the opening, the less tendency will there be for failure. This principle is of importance when one has to place a girder in a wall to support

the weight of the wall already in position with a view to the removal of the portion below the girder. It also shows the advantage of using a quick-setting mortar when it is desired that masonry, built for the first time above girders, shall become self-supporting at an early period.

Stability against Overturning.

EXAMPLE 43.—As an example of the stability in an ordinary case, we may take an enclosure wall 16′ high, 18″ wide, the brickwork weighing 112 lbs. per square foot, and built in a position where wind pressure is great. From what has been said in a previous chapter on wind pressure, it will be seen that the ground friction has a very considerable retarding effect upon the action of the wind, and we might, therefore, expect that there would be a greater force exerted by the wind on the upper layers of the wall than on the lower. Assuming that the centre of wind pressure is at about 10′ from the ground, and that its amount is equal to about 30 lbs. per square foot on the wall, we have a total wind pressure per foot run of the wall of $1 \times 30 \times 16 = 480$ lbs., acting with a leverage of 10′ (*Fig.* 281). Hence we may consider that, as far as the wind pressure is concerned, the wall in the position of a cantilever with a load acting at 10′ from the point of fixing, and an amount equal to 480 lbs. The bending moment equals $480 \times 120 = 57{,}600$ inch-lbs., and the Moment of Resistance equals $\frac{rbd^2}{6}$, where r represents the tenacity of the mortar per square inch, $b = 12''$, *i.e.*, the length of the portion of the wall under consideration, and $d = 18''$, *i.e.*, the thickness of the wall.

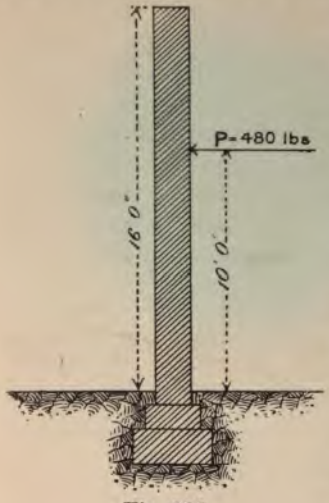


Fig. 281.

Hence $$\frac{rbd^2}{6} = \tfrac{1}{6}r \times 12 \times 324 = 648r.$$

The effect of the wind would be to produce a pressure at the

eeward edge and a tension at the windward edge of an intensity r ound by the equation

$$r = \frac{57600}{648} = 89 \text{ lbs. per square inch.}$$

There is, however, a pressure brought upon the base of the wall from he weight of the brickwork $= 16 \times 1{\cdot}5 \times 112 \div 18 \times 12 = 12{\cdot}4$ lbs. This must be added to the pressure at the one edge and deducted rom the tension at the other.

Hence the pressure at the leeward edge will be $89 + 12{\cdot}4 = 101{\cdot}4$ lbs., nd the tension at the windward edge will be $89 - 12{\cdot}4 = 76{\cdot}6$ lbs. ɔer square inch.

It will require a good cement to stand such a tension; ordinary ime mortar would be too weak, and hence it would be necessary ither to build this wall in good cement mortar or to apply ɔuttresses at intervals. The method of investigating the stability of ɔuttressed walls will be presently investigated. It may here be ɔointed out that considerable economy may be effected in nclosure walls by alternate panels and buttresses, as in *Fig.* 282, he material taken from the panels being used in the buttresses.

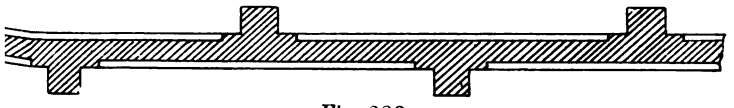


Fig. 282.

In the case just considered no account is taken of any support that ıay be derived from each end of the wall. If the wall be of con-derable length, it is clear that very little support may be expected om this source, but if the length be less than 10 times the eight, or if there be cross walls reducing the unsup-orted length, the tendency to overturn, and the corres-onding stress produced, will be much less.

*Stability and Design of Buttresses.**

Let us first consider the buttress as rectangular. Let ABXY ig. 283) be the section of the wall with a horizontal force T acting the top, and a rectangular buttress of known width cd (see Plan, ig. 284), but unknown depth $cb = x$, and let the weight of the wall

* From Tarn's *Mechanics of Architecture.*

and vertical component (if any) of inclined thrust = P. If w = weight of wall per unit cube, = thickness, l = length between any two adjacent buttresses, and h = height of wall, then P = $wlht$, and weight of buttress = $whx \times cd = Q = whxt$ if $cd = t$. Taking moments round the outer point of the centre third of ZX, the base of wall + buttress, we have (*Figs.* 283, 284)

$$T \times h = P\left\{\frac{t}{2} + x - \tfrac{1}{3}(t+x)\right\} + Q\left\{\frac{x}{2} - \tfrac{1}{3}(t+x)\right\}$$

and if $cd = t$—

$$= wlht\left\{\frac{t}{6} + \tfrac{2}{3}x\right\} + whtx\left\{\frac{x}{6} - \tfrac{1}{3}t\right\} \quad \ldots(\alpha).$$

This gives a quadratic equation in terms of x, which is thus easily found.

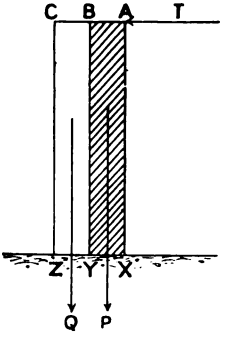


Fig. 283.

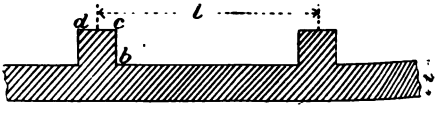


Fig. 284.

If we consider the buttress to be triangular, as in *Fig.* 285, the weight, L, is half what it formerly was if cb is the same as before, but, owing to the position of the centre of gravity, it acts now at a distance from the edge = $\frac{2}{3}x$, whereas formerly that distance was = $\frac{1}{2}x$.

Taking moments as before round the outer point of the centre third of the whole base of wall plus buttress—

$$T \times h = P\left\{\frac{t}{2} + x - \tfrac{1}{3}(t+x)\right\} + Q\left\{\tfrac{2}{3}x - \tfrac{1}{3}(t+x)\right\}$$

$$= wlht\left(\frac{t}{6} + \tfrac{2}{3}x\right) + whtx\left(\tfrac{1}{3}x - \tfrac{1}{3}t\right) \ldots(\beta).$$

This will give a value of x slightly less than that obtained from (a), when the buttress is rectangular. The economy of design is therefore manifest.

Although the relative values of x may be compared algebraically, it will be perhaps sufficient and more generally evident if the principle is applied to an actual example.

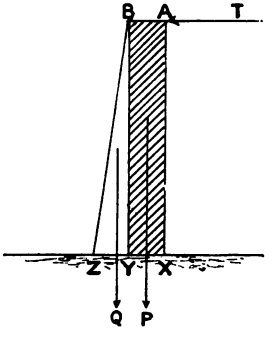


Fig. 285.

EXAMPLE 44.—*In the wall previously considered in Example* 43 *let there be buttresses as shown in* Fig. 286, 12′ *high, rectangular in plan and at* 10′ *intervals. Let the width of the buttress be* 18″, *and the weight per foot cube be the same as the rest of the walls, viz.,* 112 *lbs.*

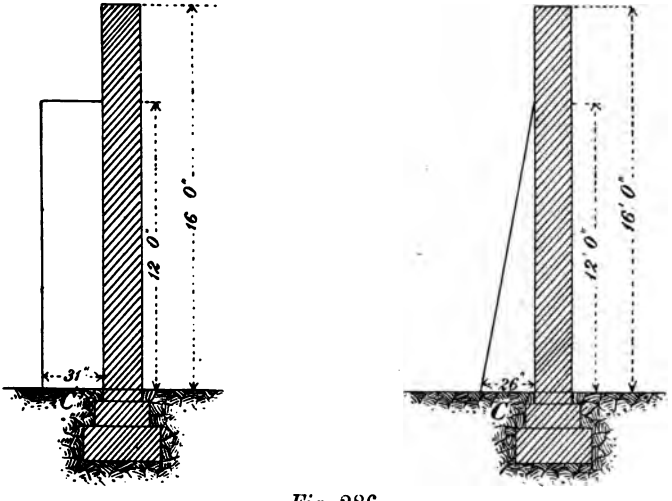


Fig. 286.

Then taking moments round C, the outer point of the centre third of the thickness of wall and buttress, we have the overturning moment of the wind pressure the same as in the previous example, except that we now consider 10′ in length of the wall, *i.e.*—

$h \times T = 16 \times 10 \times 30$ lbs. per square foot $\times$ 120″

$= 57600$ inch-lbs. $= 257$ inch-tons.

The weight of 10′ run of the wall

$$= wlht = 112 \times 10 \times 16 \times 1{\cdot}5 = 26880 \text{ lbs.} = \text{about } 12 \text{ tons.}$$

The weight of the buttress

$$whtx = 1{\cdot}5 \times 12 \times x \times 112 = 2016x \text{ lbs.} = {\cdot}9x \text{ tons.}$$

Hence from (α), since $t = 18''$—

$$257 = 12\left\{(3'' + \tfrac{2}{3}x) + {\cdot}9x\left(\frac{x}{6} - 6\right)\right\}$$

$$= 36 + 8x - 5{\cdot}4x + {\cdot}15x^2.$$

Hence $$x^2 + 17{\cdot}3x + 75 = 1474 + 75,$$

and $$x = 30{\cdot}7'', \text{ say } 31 \text{ inches.}$$

Let the buttress be triangular and equation (β) becomes

$$257 = 12\left\{(3 + \tfrac{2}{3}x) + {\cdot}45x\left(\frac{x}{3} - 6\right)\right\}$$

$$= 36 + 4{\cdot}3x + {\cdot}25x^2.$$

$$x^2 + 28x = 1472,$$

$$x = 26 \text{ inches.}$$

Hence the buttress in the latter case may be at least 5″ shorter than the former case, and contain less than half the brickwork.

This practical point is still further true when the weight per foot cube of the material is relatively great. This may be shown by the following:—

EXAMPLE 45.—*The hammer-beam roof in Example* 39 *is supported on walls* 2′ 6″ *thick, and* 30′ *high to the wall plate. The curved rib springs from a corbel* 18′ *from the ground. The material of the wall and buttress is granular limestone weighing* 156 *lbs., or* ·07 *ton per foot cube* (Fig. 271).

The horizontal thrust at the corbel is 7,000 lbs., and the wind pressure horizontal component at the wall plate 1,000 lbs (see Chapter XII., pages 270—272). This will give the maxima overturning stresses, and these will very rarely occur, as they are the stresses due to the maximum wind pressure, and to a load of workmen distributed all over the roof, a combination of circumstances which would seldom, if ever, occur.

1. Let the buttress be 2·5 long and the full height of the wall, *i.e.*, 30′, and be built of the same stone as the remainder of the wall.

The overturning moment is

$$\tfrac{1000}{2240} \times 360'' + \tfrac{7000}{2240} \times 216 = 836 \text{ inch-tons.}$$

The weight of the wall is $10 \times 30 \times 2{\cdot}5 \times {\cdot}07 = 52{\cdot}5$ tons. To this must be added the vertical components of the reactions, *i.e.*, 1,000 lbs. at the top of the wall and 13,500 lbs. at 12′ from the summit = 14,500 lbs., or 6·5 tons. Hence total pressure = 59·1, say 60 tons.

Weight of buttress $= 2{\cdot}5 \times 30 \times x \times {\cdot}07 = 5{\cdot}25x$ tons. Taking moments round the outer point of the centre-third, we have from equation (α)—

$$836 \text{ inch-tons} = 60\,(5 + \tfrac{2}{3}x) + 5{\cdot}25x\left(\frac{x}{6} - 10\right),$$

whence $$x = 32{\cdot}7, \text{ say } 33''.$$

2. If the buttress be triangular with other dimensions as before, we have the same overturning moment, and weight of wall, as before, but the weight of the buttress now is $2{\cdot}625x$. We have, therefore, substituting values from equation (β)—

$$836 = 60\,(5 + \tfrac{2}{3}x) + 2{\cdot}625x\,(\tfrac{1}{3}x - 10),$$

whence $$x = 18{\cdot}2 \text{ inches,}$$

which is not much more than half the former amount (see *Fig.* 287).

Practically it would be easier to build the buttress with offsets and sloping copings, as shown in *Fig.* 288, approximately to the line of the calculated triangle.

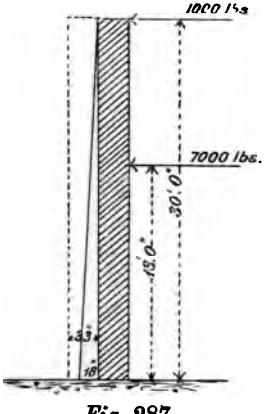


Fig. 287.

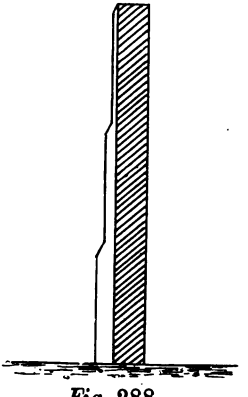


Fig. 288.

Graphically, the design thus made may be checked as follows:—Take a section through the wall and buttress, as in *Fig.* 289, and let the lines AB, CD represent in magnitude and direction the inclined forces coming on the wall.

First investigate the stability at the joint CE. The forces acting on that part of the wall are (1) the inclined thrust AB, and (2) the weight of the upper part of the wall (21 tons) acting at its centre of gravity. Completing the parallelogram of forces, we find that these two have a resultant, represented in magnitude and direction by FH, and with its point of application at a point K on the joint EC, which lies within the centre third of that joint. Passing next to the base XY, we see that the inclined force CD (6·8 tons), and the resultant of the forces above the joint CE, *i.e.*, F'H' or LO, have a resultant F'M, which, compounded with the dead weight of the lower part of the wall and buttress, 31·5 tons = LN, cuts the base at Z, which is within the centre third.

Stability of Chimneys.

As a chimney stack, either in an ordinary house, or when built separately in connection with a furnace, is of all structures peculiarly exposed to wind pressure, it is well to investigate its stability.

Here the full negative as well as positive pressure of the wind is developed. It may amount to as much as 50 lbs. per square foot of surface, though undoubtedly such pressure would be most exceptional. It is true that careless construction of the joints in brick chimneys frequently leaves projections in which the lifting force of the wind has fuller scope for action; and the common practice of corbelling near the tops of chimney stacks for the sake of ornament still further endangers the stability. If, therefore, we consider the pressure as amounting to 50 lbs. per square foot, it will no doubt be excessive, but, on the other hand, there are usually indeterminate factors which tend to instability, and which require a certain margin of safety. Hence 50 lbs. may be taken.

The overturning force of the wind has to be met by the resistance afforded by the weight of the whole structure, assisted by the adhesion of mortar. Taking the general equations

$$Y = \frac{N}{A} + \frac{Mt}{2I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\gamma),$$

and

$$y = \frac{N}{A} - \frac{Mt}{2I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\epsilon),$$

we can easily find the maximum and minimum pressures at any point.

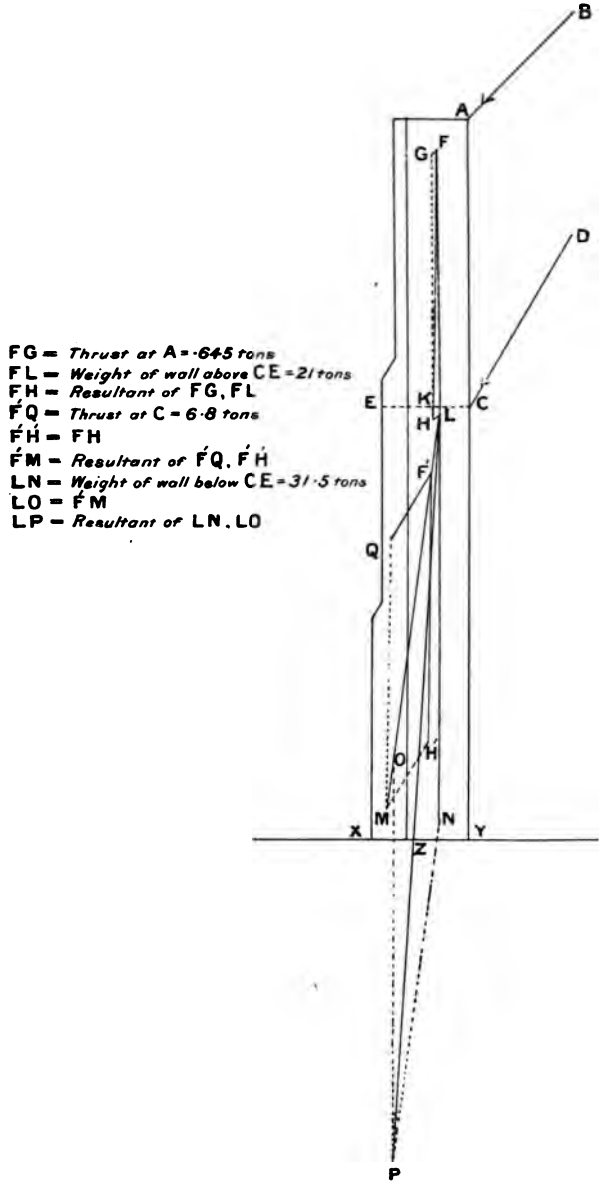

FG = Thrust at A = ·645 tons
FL = Weight of wall above CE = 21 tons
FH = Resultant of FG, FL
F'Q = Thrust at C = 6·8 tons
F'H' = FH
F'M = Resultant of F'Q, F'H'
LN = Weight of wall below CE = 31·5 tons
LO = F'M
LP = Resultant of LN, LO
B
A
F
G
D
E
K
H
L
C
F'
Q
O
H'
M
N
X
Y
Z
P

Fig. 289

Example 46.—*A chimney shaft, shown in* Figs. 290 *and* 291*, 8′ in height, 27″ broad and 4′ 2″ long, is subject to a wind pressure of* 50 *lbs. per square foot. There are two flues in it, each* 9″ × 14″. *Find the maximum and minimum pressures at each edge, the brickwork weighing* 112 *lbs. per foot cube.*

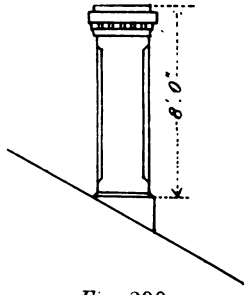


Fig. 290.

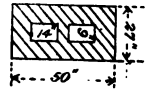


Fig. 291.

The area of the brickwork in cross section is

$$27 \times 50 - 2 \times 9 \times 14 = 1098 \text{ square inches} = A,$$

and the weight of the brickwork at the lowest section is

$$\tfrac{1098}{144} \times 8 \times 112 \text{ lbs.} = 6854 \text{ lbs.} = N.$$

Moment of wind pressure

$$= (8 \times 4{\cdot}1 \times 50) \times 48'' \text{ (half the height)} = 78600 \text{ inch lbs.} = M.$$

The thickness $t = 27''$ and $I = \frac{1}{12}bt^3 - 2\,(\frac{1}{12}b't'^3)$, where $b = 50$ and $t = 27$, $b' = 14$, $t' = 9''$.

$\therefore$ $I = 80389$ and $2I = 160778$,

$$Y = \frac{N}{A} + \frac{Mt}{2I} = \tfrac{6854}{1098} + \frac{78600 \times 27}{160778}$$

$$= 6{\cdot}26 + 13{\cdot}2 = 19{\cdot}4 \text{ lbs. per square inch,}$$

$$y = 6{\cdot}26 - 13{\cdot}2 = 7 \text{ lbs. per square inch nearly.}$$

It would be very poor mortar that could not stand so small a tensile stress, and poor brickwork that could not stand this amount of adhesion, but if it were considered desirable to eliminate even this small amount of stress, it would be possible to do so by increasing the width of the chimney stack, or under certain circumstances introducing an iron stay.

* It may be noted that in practice the flues are not usually as shown in *Fig.* 291, but with the 14″ parallel to the shorter side of the shaft.

The calculations for such an iron stay need not here be considered. ; is evident that it would either have to bear tension or compres- ion, according to the way the wind was blowing, and it would, herefore, be usually calculated as a long strut. To find the pressure t might be called upon to bear, we would first need to find what vind pressure per square foot the chimney, as designed, could bear, when the tension on the windward edge = 0. The difference between he wind pressure per square foot and the maximum of 50 lbs. dis- ributed over the whole area of the chimney would be the compres- ion coming on the bar. Thus in the case above let p be the wind ressure when $y = 0$.

Then $p \times 8' \times 4{\cdot}1' =$ the total wind pressure, acting with a leverage f 48″. Hence $M = 1584p$, the other values being as before, except hat $y = 0$. Hence in equation ($\dot{c}$)—

$$0 = 6{\cdot}26 - \frac{1584p \times 27}{160778},$$

hence $p = 23{\cdot}6$ lbs. per square foot.

The actual pressure expected being 50 lbs., the bar would have to e designed to bear a pressure of $50 - 23{\cdot}6 = 26{\cdot}4$ lbs. on every foot f exposed surface, *i.e.*, altogether $8 \times 4{\cdot}1 \times 26{\cdot}4 = 870$ lbs.

No calculations for sliding are necessary, for even with a chimney alf a brick thick, with the smallest size of flue (9″ square), for every ot of vertical surface exposed more than a cubic foot of brickwork, eighing 112 lbs., would have to be moved. The coefficient of iction for wet mortar is about ·5, hence the force resisting shearing ould be $112 \times {\cdot}5 = 56$ lbs. per cubic foot of brickwork moved. ence even the smallest practicable chimney would resist more ressure than could be brought against it by the wind, as far as liding is concerned.

Tall Chimneys.

It is sometimes necessary for military engineers to design tall himneys. In those that are circular in plan the wind pressure gainst the sides is about $\frac{1}{2}$ the value of that against a vertical plane f breadth = the diameter. The wind pressure may be considered s $\frac{1}{2}$ that acting equally all over a plane = the mean diameter, and aving a maximum intensity of 50 lbs. Neither of these assumptions re accurate, for we know that the ground friction will have a very reat retarding effect, and also the diameter near the top of the himney, where the wind pressure is greatest, will be less than near he base. The assumptions, however, are on the side of safety.

It is usually customary to design the chimney first, and then apply the rules for calculating the maximum and minimum pressures, finding the value of the maximum wind pressure that the chimney, as designed, is capable of sustaining, and comparing that with the maximum wind pressure which may be expected. The maximum intensity of pressure that may be brought to bear on the structure at any part is then investigated, and is compared with the resistance which the material can stand. Lastly, the pressure on the foundations should be investigated.

Where the chimney is octagonal, not circular, in section, the value of the Moment of Inertia of the cross section at the point where failure is most likely to occur, usually at the ground level, must be ascertained. This Moment of Inertia is easily found by taking I for the circumscribing rectangle of the octagon, and deducting the values of I for the triangles at the corners, as in Example 7, Chapter V. (*Fig.* 293).

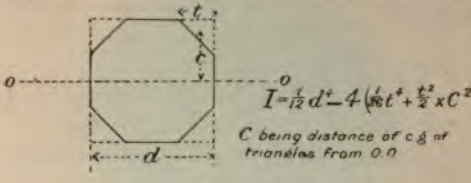


Fig. 292.

The steps to be taken in the investigation of the stability of a long chimney are as follows :—

(1). Ascertain the wind pressure that will reduce the compression to *nil* at the windward side. This should be calculated at the section of greatest stress. This will appear, as a rule, by inspection, but it is usually at the ground level.

(2). With this maximum wind pressure find the maximum pressure on the brickwork at the leeward side, and compare with the known strength of the material.

(3). Ascertain the centre of pressure of the combined weight of the chimney shaft and the wind force. The position of this centre of pressure in a shaft of circular section is found from the following formula, a proof of which is given in the Appendix to this Chapter :—

$$x = \frac{5D^2 + d^2}{8D},$$

where x = distance of the centre of pressure from the windward edge, D and d are the outer and inner diameters of the shaft. Similarly, in a square shaft where D and d are the outer and inner sides—

$$x = \frac{4D^2 + d^2}{6D}.$$

(4). The weight of the shaft acting at the centre of pressure, as found above, should be compounded with the weight of the foundations, and the centre of pressure on the foundations ascertained. From this the maximum weight on the foundations can be calculated, and compared with the bearing strength of the soil.

As an example, the following chimney, very similar to one erected at the Royal Arsenal,* Woolwich, and shown on *Plate* VIII., may be investigated :—

EXAMPLE 47.—*In the chimney shown in* Plate VIII., *find* (1) *the maximum wind force necessary to produce neither tension nor compression on the windward edge ;* (2), *the maximum pressure on the brickwork ; and* (3) *the maximum pressure on the foundations.*

As a preliminary, it is necessary to find out the weight of the shaft, the weight on the foundations when no wind is blowing, and the Moment of Inertia at the cross section where the stress due to wind pressure is greatest.

In this case it is evident that the section where the pressure is greatest is at the base of the shaft, *i.e.*, at the section B (*Fig.* 2), above the square footing above the ground line. For the purposes of calculation, however, we may consider the shaft continued uniformly to the ground, and investigate the pressure at the ground line.

The shaft is divided into four sections, each 25′ high. The volumes of these sections are 1,970, 1,380, 1,000, and 740 cubic feet respectively, or a total of 5,090 cubic feet of brickwork, which at 112 lbs. per cubic foot weighs 255 tons. The firebrick lining, which is only taken into account as adding to the weight on the foundations, weighs 10·6 tons, and the square base above the ground line weighs 12·3 tons. The footings weight 50 tons, and the concrete, at 130 lbs. per foot cube, weighs 94 tons. Above the concrete there is earth filling, weighing 20 tons, and below the concrete there are 20 granite blocks, which, at 160 lbs. per foot cube, weigh 13·4 tons. Thus the total weight on the foundation area

$$= 255 + 10{\cdot}6 + 12{\cdot}3 + 50 + 94 + 20 + 13{\cdot}4 = 455{\cdot}3 \text{ tons}$$

on an area of 400 square feet, or 1·14 tons per square foot uniformly distributed all over. When the wind is blowing at its maximum the

* The height of the actual chimney was 101′. This has been taken at 100′ for simplicity.

maximum pressure will be greater than this, a subject now investigated.

The value of the Moment of Inertia at the base of the = 0·7854 ($R^4 - R'^4$), where R and R′ are the radii of the shaft, a flue, = 6·8′ and 4·6′ respectively. Hence I = 1,390 (expressed in of feet).

(1). To find the wind pressure that will reduce the compressi the windward edge to *nil*. The wind pressure on a cylinder has been previously pointed out, one-half that on its diar plane. In the case before us, if we take the breadth of the diar plane as the *mean* diameter of the chimney, we shall be on th side, for the wind pressure near the ground, where the diame larger, is much reduced by ground friction, as we have seen i cussing this subject in Chapter XI. The mean diameter c chimney is 10·75′, and the height 100′, hence the area o diametral plane will be 1,075 square feet. Then if x b pressure on such a plane, $2x$ will be the actual force of the win

The overturning moment at the base of the shaft will be

$$1075x \times 50^* \text{ foot-lbs.} = 24 \times x \text{ foot-tons.}$$

The pressure on the base of the shaft = 255 tons, as we hav above, and the area of the ring of brickwork = 0·78 ($13{\cdot}66^2 - 9{\cdot}1^2$ square feet. Substituting these values in the equation

$$y = \frac{N}{A} - \frac{M \times t}{2I},$$

we have

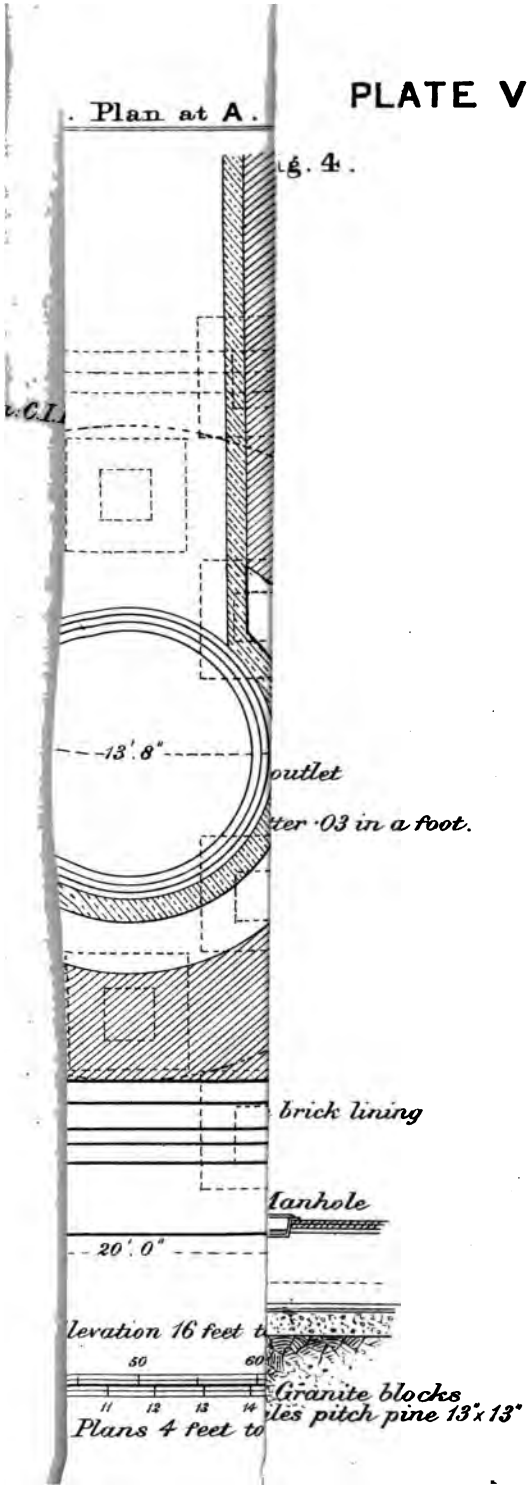
$$y = 0 = 255 \div 81 - 24x \times 13{\cdot}66 \div 2 \times 1390,$$

from which $x = 26{\cdot}7$ and the actual wind pressure = 53·4 lb square foot. So great a wind pressure would seldom, if ev experienced in any part of the British Isles, and certainly not i low-lying district of Woolwich, hence the chimney is per safe.†

* That is, taking the centre of pressure as half-way up. It would re somewhat lower if the wind pressure were equal all over. This will not case, so we are justified in taking it nearer the top.

† A chimney 162′ high was overturned by the tornado at St. Louis i 1896. Fracture took place where the thickness of the ring changed from 13″, 110′ below the top. The diameter at the base was 21′ 2″, at the top This was evidently built with an excessive economy, as 13″ is a very thickness to support a height of 110′.

PLATE VIII.


Plan at A.
g. 4.
C.I.
13'. 8"
outlet
ter ·03 in a foot.
brick lining
lanhole
20'. 0"
levation 16 feet t
50
60
Granite blocks
les pitch pine 13"x 13"
11
12
13
14
Plans 4 feet to

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ASTOR LENOX AND
TILDEN FOUNDATIONS.

(2). To find the maximum pressure on the brickwork at the ound line, we must substitute values in the formula

$$Y = \frac{N}{A} + \frac{M \times t}{2I}.$$

and A, I and t are the same as before, but

$$M = 24 \times x = 24 \times 26{\cdot}7 \text{ foot-tons.}$$

ence

$$= 255 \div 81 + 24 \times 26{\cdot}7 \times 13{\cdot}66 \div 2{,}780$$

$$= 6{\cdot}30 \text{ tons per square foot, or 98 lbs. per square inch.}$$

This gives a factor of safety of about 20 for stock bricks.

(3). To find the maximum pressure on the foundations, we must rst ascertain the position of the centre of pressure at the base of ie shaft. In the formula $x = 5D^2 + d^2 \div 8D$, the values of D and d re 13·66′ and 9·1′, hence $x = 10'$. Compounding the weight of the aft, including fire-brick lining, etc., first with the weight of the otings, and then with the weight of the concrete base, we find that ie position of the centre of pressure on the base is distant 8′ from e edge. Thus in the formula

$$Y = \frac{2N}{t}\left(2 - \frac{3d}{t}\right)$$

have N = total weight = 455 tons, t = width of concrete = 20′, d = 8′, d substituting values, Y = 36·4 tons on the end strip of the con- te 1′ wide and 20′ long, or 1·82 tons per square foot.

Most soils would be capable of bearing this safely.

In a square or octagonal chimney the procedure would be similar, t the value of I would vary with the form of the cross section, and e position of the centre of pressure would be at *about* $\frac{1}{4}t$ from the ward edge in both cases.

Empirical Rules for Retaining Walls and Arches.

The detailed investigation of the stability of retaining walls and arches is reserved for Part II. It is here proposed merely to give me empirical rules as established by good authorities, which would sufficient for design in ordinary cases.

Retaining Walls.

Sir B. Baker's rules are :—" Experience has shown that a wall, to stain earth having a level surface, whose thickness is $\frac{1}{4}$ height, and

has a batter of 1 to 2″ per foot on the face, possesses sufficient stability when the backing and foundation are both favourable. This allows a factor of safety of about 2, to cover contingencies." Sir B. Baker, as a result of his own experience, makes the thickness of retaining walls (at the base) equal to one-third of the height from the top of the footings.

General Fanshawe, R.E., some fifty years ago, gave the following rules for revetments :—Rectangular brick walls, retaining ordinary materials, should have the following thicknesses—32 per cent. vertical, 30 per cent. for 1″ to 2′, 28 per cent. for 1″ to 1′, 25 per cent. for 2″ to 1′.

The M.W.D. Handbook gives the following :—Front batter $\frac{1}{1}$, back batter $\frac{6}{1}$, top of wall not less than 2′ thick, and if surcharged, 3, thick. Many walls, however, on hill roads in India, have been built with a top width of 2′, both for surcharged and flat-topped walls, with a front batter of $\frac{4}{1}$, and with the back vertical. This is a very easily applied rule, *e.g.*, a wall 16 high has a base $2' + \frac{1}{4}^{6} = 6'$ wide at ground level.

The circumstances of the case must be taken into account. For instance, a wall on a salient should be given a greater thickness than one on a straight plan, and a wall to retain a railway, along which there may pass trains at a great speed, should also have a greater thickness than one in an ordinary road.

The constructive details of retaining walls are of much importance.

The foundations should be carefully constructed ; as a rule, a bed of concrete is given below the masonry. The drainage at the back of the wall should be carefully looked to. Water must not be allowed to lodge at the back, for if it does, it will not only add to the weight of the backing material, but will increase the lateral thrust at the back of the wall. If the soil is retentive of water, there should be a layer of stones or coarse gravel at the back of the wall, as far down as the lowest weep-hole, to act as a drain. With sandy soil, no backing is required, but, in all cases weep-holes should be given at the rate of one for every 3 or 4 square yards of surface. These weep-holes are generally formed of drain-pipes ; they should not be less than 7 square inches in sectional area, or they will soon choke. A drain should be constructed at the foot of the wall to carry off the water from the weep-holes.

Through bonding of the stones in the wall is important. The M.W.D. Handbook lays down that through bonds, from front to back, consisting of a single stone, or several stones put together,

;t be given at 5′ intervals along the face of the wall in every ;se. This is especially the case in walls built without mortar.)p iron is not good as a bonding material.

Giving a batter to the wall increases its stability, by throwing the ;re of gravity further back, but if carried too far the slope may ;uch that water trickling down the face enters the joints of the ;onry. The limit is generally 1 in 6, or 2″ to 1′.

A curved batter is sometimes given, but it involves cutting of the ;es, or extra thickness of the joints, neither of which are advisable inarily. On the other hand, it has increase of stability at the ts where pressure is greatest.

Counterforts are not recommended as adding to the stability of ;ining walls.

Arches.

The best rules, of an empirical nature, for the design of arches are far as is known by the writer) those of Mr. Trautwine. Those)lished in English textbooks concern only the depth of the arch ; at the crown, and the thickness of the abutment at the spring-, but do not take account of the thickness of the abutment itself, of the form of the arch between the springing and crown. Mr. ;utwine's rules, expressed in algebraic form, are as follows :—

If r = radius of the arch in feet (such a radius as will give a curve passing through the springings and crown of the intrados),

s = span of the arch in feet,

d = depth of the keystone in feet.

Then, for first-class cut-stone arches, whether circular or elliptic—

$$d = \frac{\sqrt{r + \frac{s}{2}}}{4} + \cdot 2 \qquad \text{(i.)}.$$

For second-class work this depth should be increased about $\frac{1}{8}$th; for brickwork or fair rubble about $\frac{1}{3}$rd, if the span exceeds 15 0′.

To find the radius in terms of the span and the rise. If a = rise ;et—

$$r = \frac{\left(\frac{s}{2}\right)^2 + a^2}{2a} \qquad \text{(ii.)}.$$

The above rules are based upon drawings and calculations made of es from 1 to 300 feet span, and of every rise.

It must be remembered, however, that it is better to make arch stones deep rather than shallow, as the structure is more stable and looks more durable. "A thin arch is as unsightly an object as too slender a column."

As regards the thickness of the abutments, Mr. Trautwine gives the following rules, which apply equally to the smallest culvert and the largest bridge, to any conditions of abutment, and of load:—"It gives a thickness of abutment which, without any backing of earth, is safe in itself, and in all cases, against the pressure when the bridge is unloaded." In the case of small arches, however, it depends on the resistance of the earth behind the abutments to an extent increasing as the span diminishes.

In practice, however, earth is always deposited behind the abutments as the work proceeds, so that this dependence on the earth pressure is quite sound.

If T = thickness of abutment in feet when the height does not exceed $1\frac{1}{2}$ times the base—

$$T = \frac{r}{5} + \frac{a}{10} + 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii.)}.$$

If of rough rubble, add 6 inches to secure full thickness in every part. In small bridges and large culverts of first-class railroads, subject to the jarring of heavy trains at high speeds, abutments from $\frac{1}{4}$ to $\frac{1}{2}$ thicker than the above have been given in many cases.

To ascertain the form of the abutment and arch ring, the following method is suggested:—

Let EO (*Fig.* 293) be the required span with given rise AI, and calculated thickness of arch ring at crown AT. Calculate the abutment thicknesses ON, EY by (iii.). From I, lay off IH = $\frac{1}{24}$ EO and join AH, then from N draw the indefinite line GNP parallel to AH. Make NG = $\frac{1}{2}$ IT, and from G draw a line touching the extrados of the arch at X (it is better, as will be shown in Part II. when treating of the theory of arches, to draw a curve passing through G and T. The greater the weights expected on the arch, the greater should be the curvature, because when loaded an upward thrust is induced at the haunch, and not at the springing). Then GX will be the top of the masonry filling above the arch, and this should be completed before striking the centering.

Next draw OP at $\frac{3}{2}$ from the vertical face OS. From the point P, where it cuts the line GNP, draw PS horizontal. If PS is on the ground line, nothing further is needed, but if the ground line is at

Q, make the base QU = SP + ¼ SQ, and draw the back UW parallel to GP. The line UW now becomes the back of the abutment. The additional thickness has reference rather to the pressure of the earth behind the abutment than to the thrust of the arch. In a very high abutment the inner line GP would give a thickness too slight to sustain this earth safely."

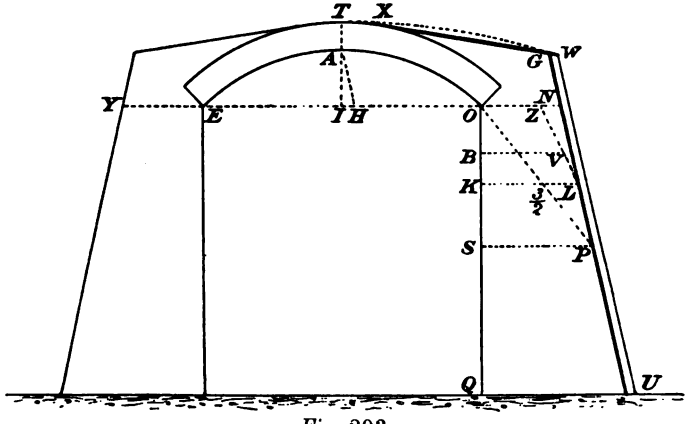


Fig. 293.

The width of base, further, should never be less than half the total height from ground to springing. If the above rules give a base less than half the height, the width must be increased accordingly, and the back, etc., drawn as above. When, on the contrary, the height of the abutment is less than the calculated thickness as found by (iii.), a slight reduction may be given as follows:—Make OK (*Fig.* 293) = ON and draw KL. Make OZ = ¾ of ON and draw LZ. Then for any height of abutment less than ON draw BV terminating in LZ. This BV will be sufficient base if the foundations are firm. The back of the abutment will be drawn upward from V, parallel to GP, and terminating at the same height as G or W.

A rough rule used in the English building trade for brick arches is:—Allow ½ a brick thickness in the ring for every 5′ of space, or fraction of 5′. Thus an arch 18′ span has 18 ÷ 5 = 3⅖, *i.e.*, 4 half-bricks = 18″ thick.

APPENDIX.

To find the centre of pressure in a cylindrical chimney of given outer and inner diameter, when the compression at the windward edge = O.*

This is equivalent to ascertaining the position of the centre of gravity of a wedge-shaped ring (*Fig.* 294), for the pressure is uniformly distributed by the mortar, from *nil* at the windward edge to a maximum at the leeward edge.

Now if the wedge were solid, the centre of gravity will be vertically above a point X, $\frac{2}{3}$ diameter from O. This may be proved mathematically by taking the sum of the areas of the vertical laminæ into which the wedge may be divided, and dividing by the volume.

But as the wedge is not solid we may consider the central portion only (*Fig.* 295). Assume this to be solid.

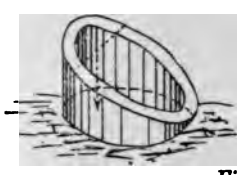
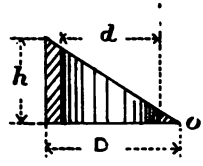


Fig. 294.



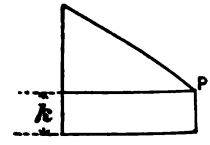


Fig. 295.

The c.g. of the upper portion would be $\frac{3}{8}d$ from P.

,, ,, bottom ,, ,, $\frac{1}{8}d$,,

Their weights are as

$$\frac{\pi d^3 m}{8} : \frac{\pi d^2 k}{4},$$

where m is the tangent of inclination

$$= \frac{h}{D}, \text{ and } k = \frac{h}{2}\,\frac{D-d}{D}.$$

The resultant c.g. acts at a distance from P

$$= \tfrac{5}{8}d\,\frac{\pi d^3 m}{8} + \tfrac{4}{8}d\,\frac{\pi d^2 k}{4} \div \frac{\pi d^2}{4}\left(m\,\frac{d}{2} + k\right) = \frac{d}{8}\,\frac{4D+d}{D},$$

i.e., distance of c.g. of central portion from P.

The distance from O

$$= \frac{D-d}{2} + \frac{d}{8}\;\frac{4D+d}{D} = \frac{4D^2+d^2}{8D} \quad \text{.....(i.).}$$

The volume of centre solid

$$= \frac{\pi d^3}{8}\,\frac{h}{D} + \frac{\pi d^2}{4} \cdot \frac{h}{2} \cdot \frac{D-d}{D} = \frac{\pi d^2 h}{8} \quad \text{.....(ii.).}$$

* Proof by Captain C. F. Close, R.E.

Now going back to the wedged-shaped ring, if X = distance of c.g. of ring from O—

$$\text{D} = \frac{\text{volume of centre part} \times \text{its distance} \div \text{X} \times \text{volume of ring}}{\text{volume of whole solid}}$$

$$= \frac{4\text{D}^2 + d^2}{8\text{D}} \times \frac{\pi d^2 h}{8} + \text{X}\left\{\frac{\pi \text{D}^2 h}{8} - \frac{\pi d^2 h}{8}\right\} \div \frac{\pi \text{D}^2 h}{8},$$

whence

$$\text{X} = \frac{5\text{D}^2 + d^2}{8d}.$$

Similarly, in a hollow square wedge, where the lengths of the sides are D and d—

$$\text{X} = \frac{4\text{D}^2 + d^2}{6\text{D}}.$$

CHAPTER XIV.

Design of Drains, Water and Gas Services.

Design of Service Pipes for Water Supply.—Design of Drains for Sewage and Rain-water.—Design of Gas Mains.—Illumination of Interiors.

As it is frequently necessary in buildings to arrange for water supply, sewerage, and gas supply, some information on the structural designs necessary for the efficient and economical working of these services is desirable.

It is proposed to limit the information in this chapter to the following heads:—I. Design of service pipes for water supply. II. Design of drains, either for storm water or sewage, of cylindrical pipes, either of stoneware or cast iron. III. Design of gas mains or service pipes. It is assumed that the reader is familiar with the laws of hydrostatics and of the physical properties of water and of gases.

The following table or data in connection with both water supply and drainage will be found useful.

Table XXXI.

Useful Data in connection with Water Supply.

Cubic feet per minute × 9000 = gallons per 24 hours.
,, ,, second × 540000 = gallons per 24 hours.
,, ,, ,, × 375 = gallons per minute.
1 cubic foot of water = 6·23 gallons = 62·35 lbs. = ·028 tons.
1 gallon of water = 10 lbs. = 16 cubic feet = 277·274 cubic inches.
Gallons of water × 224 = tons.
(Diameter)^2 ÷ 30 = gallons per foot run of full pipe.
(Diameter)^2 in inches = lbs. of water per yard run of full pipe.
Pressure in lbs. per square inch = ·434 × head in feet.
Head in feet = pressure in lbs. per square inch × 2·3.
Area of pipe = ·7854 × (diameter)^2.

To find volume of water any pipe or well will contain in 3 feet of length, square diameter in inches and divide by 10. Result will be in gallons.

I.—Design of Service Pipes for Water Supply.

The fundamental equations in all water supply problems are

$$Q = Av, \text{ (i.)}; \quad v = c\sqrt{rs}, \text{ (ii.)}; \text{ and } v = \sqrt{2gh}, \text{ (iii.)},$$

where Q = quantity in cubic feet per second (or any units).

A = area of cross section of liquid in square feet (or any units the same as Q).

v = velocity in feet per second (or any units the same as Q).

r = the hydraulic mean depth, or mean radius, *i.e.*, area of liquid ÷ wetted border or perimeter, in feet.

s = the sine of inclination, *i.e.*, height of fall ÷ length of channel, expressed in feet (or same units as the other expressions).

c = a coefficient varying with circumstances.

g = accelerating force of gravity.

If the value of v can be ascertained from either (ii.) or (iii.), it is evident that the quantity flowing in a stream of known cross section can at once be ascertained from (i.).

We shall first consider the value of v as represented by (iii.). This is the well-known expression for the velocity of a body falling freely, and it is equivalent to saying that the velocity with which a particle of water issues from an orifice in a vessel is the same as that which it would have acquired falling from a height equal to the depth of the water over the orifice. And as the pressure at a given depth is at once found (from Table XXXI.), we may find from Equation (iii.) the pressure in a pipe, of given area, necessary to produce a given discharge.

For example, let it be required to discharge 120 gallons per minute through an opening of 3″ diameter (the case of an ordinary fire hydrant). Find the necessary pressure in the pipe.

120 gallons per minute ÷ 375 = ·32 cubic feet per second. Area of 3″ pipe = ·7854 × 3^2 = 7·1 square inches = ·049 square feet. Hence

$$v = \frac{Q}{A} \text{ becomes } v = \frac{\cdot 32}{\cdot 049} = 6\cdot 5 \text{ feet per second,}$$

and as

$$h = \frac{v^2}{2g} = \frac{6\cdot 5^2}{64\cdot 4} = \frac{42\cdot 3}{64\cdot 4} = \cdot 66 \text{ (nearly) feet,}$$

the pressure in lbs. per square inch

= head in feet × ·434 (see Table XXXI.) = ·66 × ·434 = ·286 lbs. per square inch.

Now if the area of the pipe, instead of being 3", had been 1", the velocity of issue would then have been $\frac{\cdot 32}{\cdot 7854 \div 144} = 58{\cdot}5$ feet per second, and $h = \frac{v^2}{2g} = \frac{58{\cdot}5 \times 58{\cdot}5}{64} = 53{\cdot}5$ feet, and the pressure $53{\cdot}5 \times {\cdot}434 = 23$ lbs. per square inch.

In calculating the head of water (or pressure) required at the taps in a house or series of houses, it is advisable to allow an *initial* head from 5′ to 10′, or pressures of from 2·2 to 4·3 lbs. per square inch, according to circumstances. The greater the pressure the sooner will the baths, etc., be filled, but the greater the liability of the taps to leak.

Equation (ii.), *i.e.*, $v = c\sqrt{rs}$, is used for ascertaining the head required to overcome surface friction in the pipes. As r, or the *hydraulic mean depth*, is the area of cross section of the liquid ÷ wetted border, this, in pipes flowing full, will be the area of the interior of the pipe ÷ the interior circumference, *i.e.*, $\frac{\pi d^2}{4} \div \pi d = \frac{d}{4}$. The expression $s = \frac{h}{l}$, or head of water ÷ length of pipe. Equation (ii.) may, therefore, be written, in the case of pipes flowing full—

$$v = c\sqrt{\frac{d}{4} \times \frac{h}{l}}.$$

The value of c, from M. D'Arcy's experiments, for old pipes is

$$\sqrt{\frac{2g}{\cdot 010\,(1 + \frac{1}{12}d)}},$$

which we may call k.

Hence $$Q = Av = \frac{\pi d^2}{4} \times k\sqrt{\frac{d}{4} \times \frac{h}{l}},$$

or $$Q^2 = \frac{\pi^2 d^5 k^2}{64} \times \frac{h}{l}.$$

Call $$\frac{\pi^2 d^5 k^2}{64} = K.$$

Then $$Q^2 = \frac{Kh}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iv.)}.$$

Q is expressed in cubic feet per second in this equation, but in

actual practice is usually expressed in gallons per minute, so we may express this Formula (iv.) as follows :—

Let G = discharge in gallons per minute = 375Q. Then

$$\frac{G^2}{(375)^2} = Q^2 = \frac{Kh}{l},$$

or

$$G^2 = \frac{140625Kh}{l} = \frac{Mh}{l} \quad \ldots\ldots\ldots\

Example 48.—*A 200-gallon cistern in a house is situated at a vertical height of 35′ above the supply main, and the total length of the supply pipe is 120 feet. What would be a suitable size for the pipe if the pressure in the main is 100 lbs. to the square inch?*

In the case of cisterns, it is generally considered that the head should be sufficient to fill the whole in four hours. Hence

$$G = \frac{200}{4 \times 60} = \cdot 83 \text{ gallons a minute.}$$

A pressure of 100 lbs. per square inch = 100 × ·434 head = 43·4 feet head at the supply main. Of this 43·4 feet, 35′ will be required to raise the water to the vertical height of the cistern, leaving 8·4′ available to overcome friction, and to give initial velocity to the flow, as well as to overcome minor obstacles, such as bends, etc.

Assume a 1″ pipe. Then (from Table XXXII.)

$$M = 278 \cdot 72;$$

Equation (v.), $G^2 = \frac{Mh}{L}$, becomes

$$\cdot 83^2 = \frac{278 \times h}{120},$$

or

$$h = \frac{120 \times \cdot 695}{278} = \cdot 3 \text{ feet.}$$

We may use a smaller pipe, try a ¾″, M = 56·5. Then

$$h = \frac{120 \times \cdot 695}{565} = 1 \cdot 48'.$$

This would leave 8·4 − 1·48 = 6·92 feet for initial velocity of flow and minor losses, which will be just about sufficient. A ⅞″ pipe would perhaps be better than a ¾″, as giving a more ample margin.

It must always be remembered that in pipes running full, *pressure* and *head* are practically interchangeable terms, and that as hydrostatic pressure acts equally in all directions, the head at any point in a pipe is available for any branches taken from the pipe at that point. Thus in considering a scheme of water supply where there are branches at intervals, we may work along the main pipe from the outlets to the mains, or *vice versa*, calculating the head at each branch, and considering the quantities which may be required to pass through each portion as we go along. Thus in *Fig.* 296, supposing A, F, and D are points where water, in definite quantities, has to be supplied, through pipes of given size, we may,

in order to ascertain the head at the end of the system (I), either begin there or at the other end, *i.e.*, we may either find the head (h_1) required at I to overcome friction in IH while the whole quantity is passing, then the head (h_2) required at H to overcome friction in HE while the whole quantity, minus that required at D, is passing, and similarly h_3 at E and h_4 at C, finally summing up $h_1+h_2+h_3+h_4$ to give the total head at I, or we may start from A, first finding h_4 at C and working back to I.

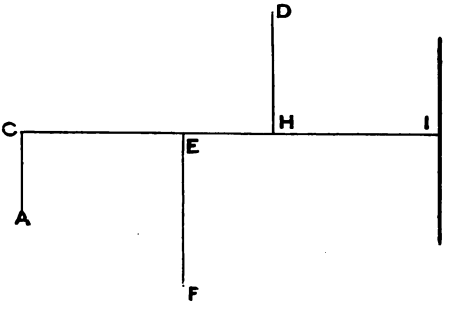


Fig. 296.

In this investigation we have ignored the head required to produce a given velocity v, and also that required to overcome resistance at the orifice of entry. In "long" pipes, *i.e.*, where the length of the pipe is more than 1,000 times the diameter, these resistances may be neglected. In "short" pipes, where the length is less than 1,000 diameters, these resistances must be taken into account. This subject is treated in a paper by Major Courtney, R.E., in the *R.E. Professional Papers* for 1896—"Notes on Indents for Pipes and other Stores for Waterworks."

Drainage.

This subject is divided into surface, or storm water, drainage, and sewerage. The principles of structural design are in both cases similar.

The cylindrical pipes down which the water or sewage flows may be full or partially full. The formula of D'Arcy, however, viz., $v=c\sqrt{rs}=\frac{\sqrt{2g}}{\zeta}\sqrt{rs}$, is equally applicable in either case.

As the value of $\zeta=\alpha\left(1+\frac{\beta}{r}\right)$, where α and β are constants varying with the nature of the surface of the channel, and r is the hydraulic mean depth, we see that no one value of the coefficient c will give satisfactory results. For various values of r, c has the following values:—

TABLE XXXIII.

*Values of Coefficients in D'Arcy's Formula for Channels of various classes, and for various Hydraulic Radii.**

Hydraulic Mean Depth.	Cement, Stoneware, or Planed Timber.	Ashlar or Brickwork.	Rubble Masonry.
·10	104·98	72·37	38·54
·12	109·61	76·98	41 76
·14	113·32	80·86	44·64
·16	116·36	84·20	47·23
·18	118·91	87·10	49·59
·20	121·07	89·65	51·76
·22	122·92	91·91	53·76
·24	124·54	93·93	55·62
·26	125·96	95·75	57·36
·28	127·22	97·39	58·98
·30	128·33	98·89	60·50
·35	130·65	102·10	63·94
·40	132·48	104·75	66·94
·45	133·96	106·92	69·59
·50	135·18	108·77	71·95
·55	136·20	110·36	74·07
·60	137·07	111·74	75·99
·65	137·82	112·95	77·74
·70	138·47	114·02	79·33
·75	139·04	114·97	80·80
.80	139·55	115·82	82·15
·85	140·00	116·59	83·40
·90	140·41	117·28	84·56
·95	140·77	117·92	85.65
1·0	141·11	118·50	86·66

* Calculated by Lieut.-Colonel E. C. S. Moore, R.E., from D'Arcy's formula and experiments.

l the H.M.D. in circular drains, the following table may be

TABLE XXXIV.

Hydraulic Mean Depth in Circular Pipes. D = diameter in feet.

ıvert	2 Area = D² ×	3 Wetted Periphery = D ×	H.M.D. =	
			4. D ×	5 Depth on Invert ×
	·7854	3·1416	·2500	·250
	·7707	2·6906	·2872	·302
	·7445	2·4981	·2980	·331
	·7155	2·3462	·3033	·359
	·6736	2·2143	·3042	·380
	·6318	2·0944	·3017	·402
	·5872	1·9824	·2963	·423
	·5404	1·8755	·2881	·443
	·4920	1·7721	·2776	·463
	·4426	1·6710	·2649	·482
	·3927	1·5708	·2500	·500
	·3428	1·4706	·2331	·518
	·2934	1·3695	·2142	·535
	·2450	1·2661	·1935	·553
	·1982	1·1592	·1710	·570
	·1535	1·0472	·1467	·586
	·1118	·9272	·1206	·603
	·07387	·7954	·09287	·619
	·04080	·6435	·06351	·635
	·01313	·4448	·03255	·651
	·00133	·2003	·00663	·663

he above table we can calculate the H.M.D. of a pipe of with any depth on invert. Take a 4″ pipe flowing 3″ deep.

Then 3″ = ·75 of diameter. Opposite ·75 we see in last column table the coefficient ·402. Hence

$$\text{H.M.D.} = \cdot 402 \times 3'' = 1 \cdot 2 \text{ inches} = \cdot 11 \text{ feet.}$$

From these data it is possible to calculate the velocity and discharge of sewers of ordinary size, when flowing at various depths and inclinations. For example, a 6″ pipe is flowing 3″ deep at a gradient of $\frac{1}{100}$. Find area of flow, H.M.D., velocity in feet per second, and discharge in gallons per minute. 3″ is ·5 diameter. Coefficient for ·5 is ·500. Hence

$$\text{H.M.D.} = \cdot 5 \times 3'' = 1 \cdot 5'' = \cdot 125 \text{ foot.}$$

Area of liquid

$$= (\tfrac{6}{12})^2 \times \cdot 3927 = \cdot 098 \text{ square feet} = 14 \cdot 13 \text{ square inches.}$$

Velocity in feet per second $= c\sqrt{rs}$, and $r = \cdot 125$, while $\cdot 125 = 110$ (from Table XXXIII.).

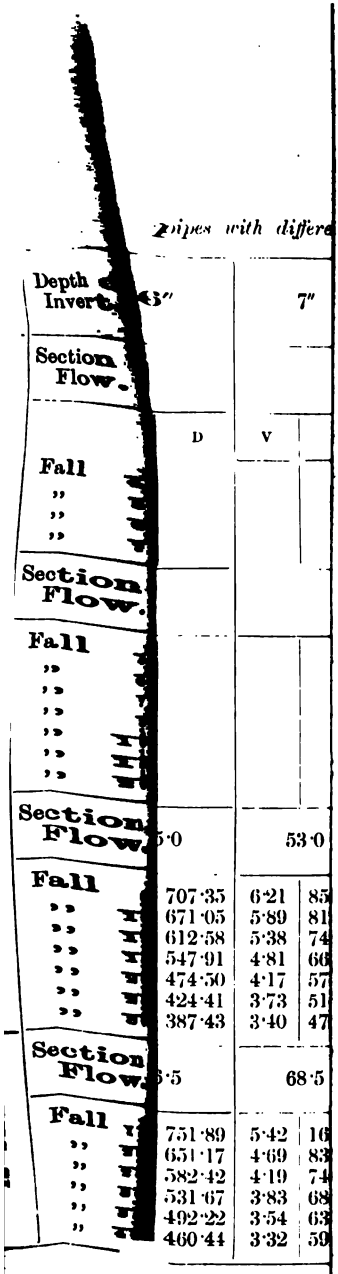
Hence

$$v = 110 \times \sqrt{\cdot 125 \times \tfrac{1}{100}} = 11 \times \cdot 35 = 3 \cdot 90 \text{ feet per second.}$$

Discharge in gallons per minute

$$= \text{A} \times v = 3 \cdot 9 \times \cdot 098 \times 375 = 143 \cdot 7.$$

Similarly, the various figures in Table XXXV. have been calculated.



pipes with differe

Depth Invert	3″		7″
Section Flow.			
	D	V	
Fall			
,,			
,,			
Section Flow.			
Fall			
,,			
,,			
,,			
,,			
,,			
Section Flow.	5·0	53·0	
Fall			
,,	707·35	6·21	85
,,	671·05	5·89	81
,,	612·58	5·38	74
,,	547·91	4·81	66
,,	474·50	4·17	57
,,	424·41	3·73	51
,,	387·43	3·40	47
Section Flow.	5·5	68·5	
Fall			
,,	751·89	5·42	16
,,	651·17	4·69	83
,,	582·42	4·19	74
,,	531·67	3·83	68
,,	492·22	3·54	63
,,	460·44	3·32	59

From inspection of Table XXXV. we see some important facts.

(1). That the point of maximum velocity in a sewer is not when it is flowing full, but when the depth on the invert is approximately $\frac{5}{6}$ of the diameter.

(2). That the maximum discharge is obtained, similarly, when the depth on the invert is about $\frac{11}{12}$ of the diameter.

(3). That where the fall and length are constant the discharge varies as $\sqrt{d^5}$. This is evident from the equation

$$Q = \frac{\pi d^2}{4} \times k \sqrt{\frac{d}{4}} \times \frac{h}{l},$$

where (if h and l are constants) Q varies as $\sqrt{d^5}$.

4. That in pipes of the same diameter the discharge varies directly as $\sqrt{\frac{h}{l}}$.

The minimum limit of velocity in a drain should be 3 feet a second. The maximum limit in a *stoneware* or brick drain should be 6 feet a second. Any drain that is laid at such a fall as to produce a greater velocity than 6′ a second should be of cast iron. If the velocity be below 3′ a second, the current will not remove solid matter, and the drain will gradually become choked, unless means of flushing are used.

In arranging any drainage scheme, the calculations for size of pipes should be based upon their maximum discharging capacity, and the pipes should be laid at such falls that a velocity of not less than 3′ a second is secured when the depth on invert = 0·25 diameter. Care should also be taken that the velocity of flow increases rather than diminishes, as the liquid flows down, for a check in velocity favours the deposition of solid matters which may be in suspension.

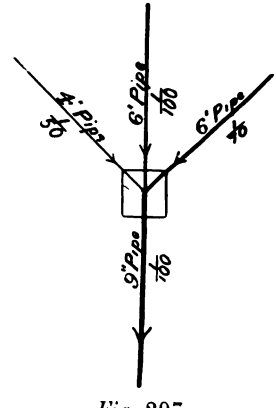


Fig. 297.

EXAMPLE 49.—*Three drains* 4″ *at* $\frac{1}{50}$, 6″ *at* $\frac{1}{100}$, *and* 6″ *at* $\frac{1}{70}$ *meet in a manhole. The slope of the proposed drain beyond the manhole is* $\frac{1}{100}$. *What should its size be?* (Fig. 297).

A 4″ pipe at $\frac{1}{50}$ discharges at its maximum 145·29 galls. per minute.
„ 6″ „ $\frac{1}{70}$ „ „ „ 370·64 „ „
„ 6″ „ $\frac{1}{100}$ „ „ „ 310·09 „ „

Total discharge ... 826·02

From Table XXXV. we see that a 9-inch pipe at $\frac{1}{100}$ discharges 910 gallons a minute. So a 9″ would do all right.

Suppose, however, that the fall had been $\frac{1}{50}$, then the discharge from a 9″ pipe could be found from that given for a slope of $\frac{1}{100}$ by considering the ratios of $\sqrt{\frac{h}{l}}$, *i.e.*, as $\sqrt{\frac{1}{50}} : \sqrt{\frac{1}{100}} :: x : 910$, *i.e.*, 1,300 gallons a minute. This is more than is required, and a smaller pipe would, therefore, be large enough. To find the size of such a pipe we take the ratios of $\sqrt{d^5}$, $1300 : 826 :: \sqrt{9^5} : x$, *i.e.*, as $243 : x$, hence $x = 154$, which gives a value to the diameter of about $7\frac{3}{4}$″. Stoneware pipes are not made of this size, but the velocity of flow would exceed 6′ a second, and hence a cast-iron pipe would be required, and, of course, cast-iron pipes can be made of any specified diameter.

There is a slight advantage always in point of greater velocity by using a small pipe for a given discharge. For instance, a 6″ pipe at $\frac{1}{100}$ has a maximum velocity of 4·54 feet per second when the depth on invert is 5″, and discharge 298 gallons per minute. When a 12″ pipe is laid at $\frac{1}{100}$ and is discharging 298 gallons per minute, the depth on invert is $3\frac{1}{4}$″ and velocity is 4·4 feet. Hence there is no gain, and, of course, greater expense, in putting large pipes in a system where smaller pipes can possibly be used. Another reason why a small pipe is to be preferred to a large pipe is that a given quantity of water occupies a greater space, *i.e.*, length of pipe, in the small pipe, and therefore tends to force in front of it any obstruction. For instance, 10 gallons of water occupies 18·7 feet in length of a 4″ pipe, while it only occupies 3·7 feet in length of a 9″ pipe, hence the force of the water is much more likely to sweep away obstacles in the former case.

In estimating the quantity of sewage that may be expected down a drain, it is customary to consider that half of the total daily allowance of water (20 gallons per head) is discharged into the drains in 6 hours. This will give a definite quantity of gallons per minute, and the drains should be arranged with such falls and of such a size that, if possible, this amount may have a velocity of not less than 3′ a second when flowing $\frac{1}{4}$ full. In some cases it will not be possible

to arrange this without going to excessive depths, say more than 16′ to 20′, and under these circumstances means of artificial flushing (by automatic flushing tanks or otherwise) must be resorted to.

Where storm water has to be discharged into the same drains as the sewage, the quantity to be allowed for will depend not only on the maximum rainfall in a given time, but on the nature of the surface to be drained. In England, $\frac{1}{2}''$ per hour is usually considered the maximum rainfall (though this is no doubt often exceeded), and the following percentage of the total rainfall is allowed from surfaces of various kinds as follows :—Roofs, 100 per cent; flagged surfaces, 40 per cent; paved surfaces, 20 per cent; gravel with clay subsoil, 1 per cent; and grass plots, 2 per cent.

It is no doubt more sanitary to have the storm drains separate from sewers, but frequently the "combined system" is adopted. In such cases the maximum discharge of the drain is limited by the storm water, the minimum by the greatest daily flow of sewage. At suitable points, *e.g.*, where the drains go near water-courses, storm overflows should be provided, so as to minimize the risk of the drains being flooded in heavy rain, and thus enable the size to be kept down. It is needless to say that with the combined system the ventilation of the drains must be most carefully attended to.

The following table (prepared by H. Sowerby Wallis, F.M.S.), is useful for showing the yield of rain-water from roofs of various sizes with varying rain falls.

TABLE XXXVI.

Daily Yield of Water from Roofs of Various Sizes, with Varying Rainfall.

Mean Rainfall.	Loss from Evaporation.	Requisite capacity of Tank.	Mean daily yield of Water.	Ditto. Wettest Year.	Ditto. Driest Year.
Inches.	Per Cent.	Cube Feet.	Gallons.	Gallons.	Gallons.
Area of House 10′ × 20′ = 200 square feet.					
20	25	100	4·3	6·7	3·2
25	20	135	5·7	7·5	3·9
30	20	145	6·8	9 4	4·5
35	20	155	7·9	11·0	5·0
40	15	165	9·7	13·1	7·2
45	15	170	10 9	14·2	8·8
Area of House 15′ × 20′ = 300 square feet.					
20	25	150	6·4	9·9	4·8
25	20	200	8·6	11·3	5·6
30	20	225	10·3	14·2	6·9
35	20	235	12·0	16·7	7·7
40	15	245	14·5	19 6	10·8
45	15	250	16·4	21·4	13·1
Area of House 20′ × 25′ = 500 square feet.					
20	25	250	10·7	16·6	8·0
25	20	335	14·3	18·7	9·7
30	20	375	17·1	23·6	11·4
35	20	390	19 9	27·7	12·7
40	15	405	24·2	32·8	18·0
45	15	415	27·3	35·7	21·8
Area of House 20′ × 50′ = 1,000 square feet.					
20	20	500	22·8	30·1	15·5
25	15	665	30·3	40·0	20·6
30	15	740	36·4	50·3	24·4
35	15	785	42·4	59·0	27·0
40	15	815	45·6	62·0	34·1
45	15	835	48·5	63·6	39·1
Area of House 25′ × 80′ = 2000 square feet.					
20	20	1010	45·6	60·2	31·0
25	15	1330	60·6	80·0	41·2
30	15	1480	72·7	100·5	48·7
35	15	1570	84·8	118·0	54·0
40	15	1630	91·1	123·0	67·9
45	15	1700	96·9	126·6	78·2

Where tables given in engineering pocketbooks, for water supply and drainage, are based on the formula known as "Neville's," they are not to be depended on, because that formula ignores the variations in the coefficient c caused by variations in the hydraulic mean depth, and by the nature of the surface of the channel.

Gas Supply.

The rules for the sizes of gas pipes are based on much the same principles as those for water pipes, with some important differences which will be noticed in due course. The quantity required at various parts of a building is settled, either by such regulations as those of the Barrack Synopsis, which allows a certain amount of gas per cubic space of interior, or by consideration of the necessary illumination in a given case. In order to utilize gas to the best advantage, it should be delivered at certain pressures, which will vary with the class of burner and the quality of the gas. For flat-flame burners and London 16-candle gas, the best pressure is usually taken at $\frac{7}{10}$ of an inch, pressure being measured by the height of a column of water which the gas will support, measured in tenths of an inch.

Now at the gas works the pressure in the mains is from 2·5″ to 3″. The difference, therefore, between this and the pressures at the burners is absorbed by skin friction in the pipes. *Pressure*, therefore, in gas design is analagous to *head* in water-supply calculations.

It may here be remarked that gas fires, stoves, engines, etc., require higher pressure, as a rule, than gas burners for lighting. This would need to be taken into account when designing a building where such fittings are required.

The pressure of the gas, the length and the diameter of the pipe, are the varying factors on which the quantity delivered depends. The quantity varies (1) directly as the square root of the pressure; (2), inversely as the square root of the length; and (3) directly as the $2\frac{1}{2}$ power—some authorities say as $2\frac{1}{4}$ power—of the diameter.

The following formula is given in Hurst's Handbook:—

Let D = diameter of pipe in inches.
G = specific gravity of the gas.
L = length of pipe in yards.
P = initial pressure in inches.
Q = quantity of gas in cubic feet per hour.
M = a constant value from 1,000 to 1,350.

Then

$$Q = M\sqrt{\frac{D^5 \times P}{G \times L}}.$$

From this we see that if M, D, G and P are constants, Q varies as $\sqrt{\frac{l}{L}}$, and if M, D, G and L are constants, Q varies as $\sqrt{p}$. Thus we may by simple proportion utilize the tables in Hurst's Pocket-book to ascertain the loss of pressure by friction, of any given quantity passing through any given length. Thus from the tables, we find that a 3″ pipe 100 yards long will supply 2,323 feet of gas at 1″ pressure. Suppose we have a 3″ pipe 400 yards long required to supply 1,600 feet of gas, and we want to know what pressure would be required, we first find the *possible quantity* that could pass through 400 yards of 3″ pipe at 1″ pressure, for if Q′ be that quantity, it is evidently

$$= 2323 \times \sqrt{\tfrac{100}{400}} = 1{,}161 \text{ cubic feet.}$$

And if 1,161 cubic feet absorbs 1″ pressure, the pressure P absorbed by 1,600 feet

$$= 1'' \times \frac{1600^2}{1161^2} = 1{\cdot}9,$$

or, in one expression—

$$P = 1600^2 \times 1 \div 2323^2 \times \tfrac{100}{400}.$$

Every foot of rise or fall in a pipe increases or diminishes the initial pressure by ·01 of an inch.

Illumination.

In addition to ascertaining the size of the pipes, it is frequently necessary to apply the principles of design to the arrangement of the light sources so as to provide for effective illumination. This, of course, applies to all sources of artificial light, and not only to gas.

Light is measured in England by *candle power*, the standard being a candle, which will consume 120 grains of sperm per hour, and of which six go to a pound. The illumination depends simply upon the quantity of light falling upon a given surface, and has nothing to do with the nature of that surface, though the effect of that surface may be that some of the light will be absorbed, and some transmitted or reflected. It is analogous to rain falling on grass plots, roofs, etc., where in one case the ground absorbs much of the rainfall and in another case gives it off.

The unit of illumination which is now usually taken in England is the *candle foot, i.e.*, the illumination produced by one standard candle at a distance of one foot, on a plane at right angles to the horizontal rays of light emitted by the candle. The illumina-

tion varies inversely as the square of the distance, one candle at one foot producing as much illumination as 4 candles at 2 feet. One candle at 2 feet will, therefore, have $\frac{1}{2^2} = 0{\cdot}25$ candle-feet illumination, 16 candles at 8 feet will have $\frac{16}{8^2} = 0{\cdot}25$ candle feet, providing always the plane of illumination is at right angles to the direction of the rays. The total amount of illumination in candle feet is represented

by $$\frac{I}{D^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i.)},$$

where I represents the total candle power of the light source, and D the distance in feet.

When the surface is not at right angles to the direction of the rays, it is evident that the illumination will be reduced. In *Fig.* 298 let the parallel lines represent a series of light rays emanating from some source, and falling upon a unit surface AB at right angles to them. If AB be pivotted at A and moved to the position AB′,

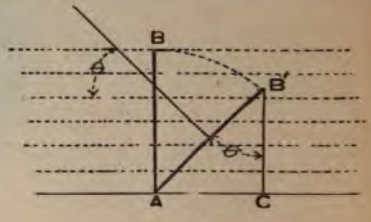


Fig. 298.

then the light falling on the surface will be B′C, and if θ be the angle between the direction of the light rays, and the normal to the plane $B'C = AB \cos \theta$. This may be expressed as follows:—**The intensity of illumination, which is received obliquely, is proportional to the cosine of the angle which the luminous rays make with the normal to the illuminated surface.**

Luminous surfaces *appear* to be of the same brightness at any point. This apparent intrinsic brightness is the same at all distances, thus a line of gas lamps lighting a street, when viewed from a direction nearly in the line of the length of the street, appear equally brilliant to the eye within a considerable limit of distance.

Let us further consider the effect of obliquity on a horizontal plane below a light source. Let I be the luminous intensity or candle power of the light source in a radial direction, let h be the height of the light source vertically above the plane, and x be the horizontal distance of any point C from the light source. Then the slant distance $= \sqrt{h^2 + x^2}$, and if a unit portion of the plane at C were

turned so as to be at right angles to the ray, the illumination there would be $\frac{I}{h^2+x^2}$ (from Equation (i.) above). The illumination e, therefore, on the horizontal plane (the angle of incidence of the light ray being α) is

$$e = \frac{I \cos \alpha}{h^2+x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii.)}$$

(*Fig.* 299).

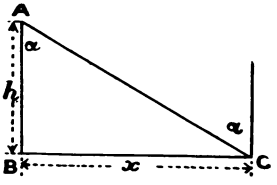


Fig. 299.

This formula may be transformed as follows:—

Since $h = \sqrt{h^2+x^2} \cos \alpha$,

if we multiply by $\frac{h}{h}$ or $\frac{h}{\sqrt{h^2+x^2}\cos\alpha}$,

$$e = \frac{Ih}{(h^2+x^2)^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii.)},$$

or since $\cos^2 \alpha = \frac{h^2}{h^2+x^2}$,

if we multiply by $\frac{\cos^2\alpha}{\cos^2\alpha}$ or $\frac{\cos^2\alpha\,(h^2+x^2)}{h^2}$,

$$e = \frac{I \cos^3 \alpha}{h^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv.)},$$

or since $\sin^2 \alpha = \frac{x^2}{h^2+x^2}$,

$$e = \frac{I \sin^2 \alpha \cos \alpha}{x^2} \quad \ldots\ldots\ldots\ldots\ldots \text{(v.)}.$$

From Formula (iv.) we see that the variation of lighting in a horizontal plane, where the rays are of equal intensity in all directions, is proportional to $\cos^3 \alpha$. If AB be the vertical height and AC the slant height, Formula (iv.) might be written

$$\frac{I \times AB}{AC^3}.$$

From (v.) we can ascertain the best height to place the light source so as to get a maximum intensity of illumination at a distance x, *provided the light rays are of equal intensity in all directions radiating from the source.* This is not the case in any gas burners. The angle works out to 54° ·44′, or $h = 0{\cdot}707x$.

Inasmuch, however, as there are very considerable variations in the radial intensity of gas lamps, it is necessary to ascertain, by means of a radial photometer, the value of that intensity at various angles, with various pressures of gas, with and without reflectors, globes, or any other means of improving the intensity in any direction. Radial photometry is as yet but little practised, yet there appears to be no reason why it should be more difficult to establish for any given lamp, pressure, etc., than the ordinary horizontal photometry.

Assuming that the values of the intensity at various angles have been ascertained, and are shown, as in *Fig.* 300, by the curve *mn*;

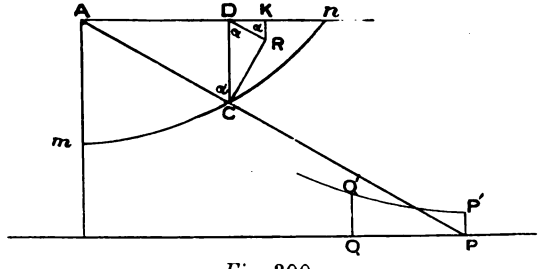


Fig. 300.

let the height above the proposed horizontal plane on which illumination is required be h. Any point P on that plane is lighted by a ray of intensity ascertained to be AC = I, and the value of the illumination from Formula (iv.) is $\frac{I \cos^3 \alpha}{h^2}$. Through C draw CD vertical to the horizontal line ADK, and CR at right angles to AC. From D draw DR parallel to AC, meeting CR at R, and from R draw RK parallel to CD. Then

$$KR = DR \cos \alpha = DC \cos^2 \alpha = AC \cos^3 \alpha.$$

Hence

$$c = \frac{KR}{h^2}.$$

By representing the unit of luminous intensity (candle power) by a line h^2, the illumination in candle feet is = KR. This may,

refore, be laid off at PQ, and if the same is done for other linations we have a curve QQ′, etc., giving a graphic representation the illumination on the given horizontal plane produced by the ht source at A. The ordinates PQ, P′Q′, etc., represent the ndle-feet illumination at the points P, P′, etc. If there are two more light sources the total illumination produced at any point ll be the sum of the ordinates.*

By this means it is possible to find the illumination produced in y place, either the interior of a building or an open space outside, r any arrangement of lamps of known radial photometric value. interiors this illumination will be largely augmented by reflection an extent which is not calculable. The influence of reflection ay, however, be taken into account by fixing a low standard of inimum illumination to be permitted.

It is thus possible to arrange the known light sources in the most onomical positions, both vertically and horizontally, to produce in y given space the best illuminating effect. In such buildings as urches, reading rooms, gymnasia, etc., this is very important. A ndard candle at 2 feet distance, or 0·25 candle-feet illumination, sufficiently good to enable a person of fair eyesight to look up the ins in a Bradshaw. Where there is prospect of much reflection light, *e.g.*, from whitewashed walls, etc., it would be quite justifiable fix upon an even smaller limit of minimum illumination, say 0·16 ndle-feet.

The following table, prepared by Prof. Vivian Lewes, gives the otometric value of various classes of gas burners at various angles ese values were taken by a Dibdin's radial photometer in a darkened om:—

* This graphic method of representing illumination is that proposed by Maréchal, of Paris, and published in *The Journal of Gas Lighting*, 16th bruary, 1897.

Table XXXVII.

Luminating Value of Various Burners at Angles below the Horizontal Terms of Candles per Cubic Foot of Gas Consumed. (16-*candle Gas*).

Angle.	Name of Burner.						
	London Argand.	Union Jet.		Batswing.		Regenerative.	Incandescent.
		No. 5.	No. 7.	No. 5.	No. 7.		
Horizon.	3·2	1·67	2·09	1·90	2·43	3·4	16·6
10	3·19	1·67	2·08	1·84	2·42	4·8	16·6
20	3·07	1·55	2·03	1·73	2·38	5·3	16·1
30	2·72	1·45	1·88	1·61	2·25	5·8	11·1
40	2·38	1·40	1·64	1·47	2·18	6·0	8·8
45	2·18	1·30	1·50	1·38	2·12	6·2	8·0
50	2·12	1·27	1·50	1·38	2·05	6·5	6·6
60	1·20	1·23	1·50	1·30	2·07	6·7	2·0
70	unreadable	1·23	1·37	1·23	2·00	7·7	unreadable
80	,,	0·76	1·25	0·76	1·75	8·0	,,
90	..	0·30	0·87	0·76	1·37	8·0	..

For exterior lighting, moonlight is a useful standard. It is between from $\frac{1}{60}$ to $\frac{1}{100}$ of a candle foot in Great Britain, when the moon is full and high in the heavens.

The illumination of a vertical plane, the height of which is small compared with the height of the light source, is $= \frac{I \sin a \cos^2 a}{x^2}$. It is, of course, *nil* below the light, and at 45° is equal to the illumination on a horizontal plane.

It is sometimes useful to know this, so as to arrange external lamps with a view to their giving sufficient light to enable lettering on a wall, etc., to be read.

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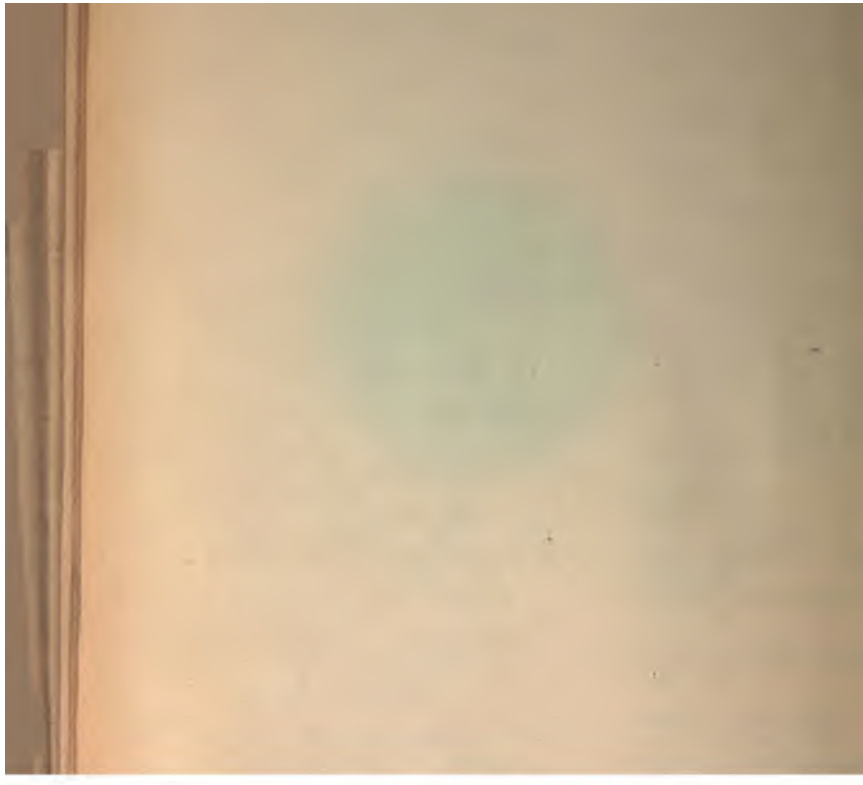
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